AN ESTIMATE ON UPPER BOUND ON SIGNAL-TO-QUANTIZING NOISE RATIO FOR SPEECH DPCM SYSTEM WITH NONADAPTIVE QUANTIZER

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RESUME

En fixant comme critère une approximative loi de densité de probabilité du signal à l'entrée du quantificateur gamma, on considère une supérieure limite du rapport signal à bruit de quantification dans le système du signal de la parole codé en MICD (Modulation par Impulsions Codées Différentielles) avec le quantificateur non adaptatif. Le gain 16,77 dB obtenue pour le codage avec l'entropie suppose ε-entropie pour les bits à la sortie du quantificateur.

Le titre:

ESTIMATION D'UNE SUPÉRIEURE LIMITE DU RAPPORT SIGNAL À BRUIT DE QUANTIFICATION DANS LE SYSTÈME DU SIGNAL DE LA PAROLE MICD AVEC LE QUANTIFICATEUR NON ADAPTATIF

SUMMARY

Subject to the constraint that the quantizer input is approximately gamma probability density, consideration is given to an upper bound on signal-to-quantizing noise ratio for speech DPCM system with nonadaptive quantizer. The 16,77 dB gain for entropy coding assumes ε-entropy of the bit stream of the quantizer output.

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I. INTRODUCTION

Differential pulse code modulation DPCM is an efficient way to encode highly correlated analog signals into binary form suitable for digital transmission, storage or input to a digital computer. Any signal whose power spectrum is not uniform possesses redundancy due to correlation in the signal. On the other hand, when the probability density of the signal to be transmitted is not uniform, redundancy is present, too. One of the ways to remove the correlation present in the signal is to use DPCM.

Much of the redundancy in a speech signal is eliminated when it is encoded into digital form by DPCM. Additional coding of the DPCM output using entropy coding techniques can result in a further increase in the signal-to-quantizing noise ratio without increasing the transmission rate.

To transmit continuous speech signals digitally it is necessary to approximate the signals by a finite number of discrete values, i.e. to quantize the signals. For a given source it is desirable to keep the transmission rate as low as possible within the permissible quantizing error. This lower bound is considered to be given by the ε-entropy.

Let in a digital communication system the information source generate the random signal ξ which is encoded with an accuracy ε, where $\varepsilon > 0$. The random signal η at the encoder's output and the generated signal ξ belong to the same space. Their associated probability belongs to the same class of distribution W_ε which is determined by ε. Then, ε-entropy $H_\varepsilon(\xi)$ is an infinum of the mutual information for two random signals, one of which is given, i.e.

$$H_\varepsilon(\xi) = \inf_{P_{\xi\eta} \in W_\varepsilon} I(\xi; \eta) \qquad (1)$$

Assume that a signal x arising from the source is expressed by sample values as well as that the mean-square error per signal sample should not exceed ε^2. When $x = (x_1, x_2, ..., x_M)^t$ is an M-dimensional random variable and has a sufficiently smooth density function $p(x)$, the ε-entropy of x is given by

$$H_\varepsilon(x) = M \log_2 \frac{1}{\varepsilon} + L\{p(x)\} - \frac{M}{2} \log_2 2\pi e \qquad (2)$$

where $L\{p(x)\}$ is the differential entropy of x [1].

The work reported here, contains theoretical results of an estimate on upper bound on signal-to-quantizing noise ratio for speech DPCM system with nonadaptive quantizer. At first, considering signal-to-quantizing noise ratio, two DPCM systems i.e. DPCM system without and DPCM system with entropy coding are compared. Further increase in the signal-quantizing noise ratio without increasing the transmission rate is possible if we assume ε-entropy of the bit stream of the quantizer output.

II. DPCM SYSTEM WITH ENTROPY CODING

DPCM system with nonadaptive quantizer and entropy coding will be considered. Adaptation of the quantizer can be accomplished by making an estimate of the standard deviation of the past sample values and dividing the signal by this standard deviation before it is quantized. The adaptive quantizers have considerably less idle channel noise than nonadaptive ones and are able to operate optimally over a wide range of input signal levels. Unfortunately, adaptation increases the entropy. On some of the encoders the adaptive feature is not used.

There is some evidence that suggests that for speech, the DPCM quantizer input is more peaked than the Laplacian function and may be more accurately represented by a gamma distribution [2]. Models obtained from histograms measured at the input to the quantizer of a digital communication systems using Nyquist samples of phonetically balanced phrases are the gamma probability density, too [3].

Consider nonadaptive DPCM system with entropy coding a block diagram of which is shown in Fig.1. The input signal S(t) whose mean-square value is σ^2 is sampled at twice its bandwidth W to produce a sequence of sample values $S_0, S_1, ... = \{S_i\}$. At the same time the predictor makes an estimate of each sample value on the basis of those that have preceeded it. These estimates are the sequ-

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ence $\hat{S}_0$, $\hat{S}_1$,... =$\{\hat{S}_i\}$. Each estimate is sub-tracted from the actual sample value, produ-cing a difference or error sequence e_0,e_1, ... ,$\{e_i\}$, where $e_i = S_i - \hat{S}_i$. The prediction process in DPCM causes the members of the sequence e_i to be independant. The quanti-zer represents each value of e_i by the near-est quantizing level. The DPCM quantizing levels are entropy coded before transmission. The quantizing levels are transmitted and used by the receiver to reconstruct the ori-ginal analog signal. The difference between each e_i and the quantizing level used to re-present it is called quantizing error q_i. The quantizing levels actually transmitted are the sequence $e_i + q_i$ and this is the sequence used by the decoder to reconstruct the analog signal. The quantizing error in the reconstructed signal is quantizing no-ise q (t). The decoder output is S (t) + + q (t).

Entropy coding is a source encoding technique. This is a variable-length proce-dure that assigns shorter code words to highly probable symbols and longer code words to less probable symbols. When the symbols to be transmitted are independant, it is possible to generate codes such that the average word length of these codes is approximately equal to the entropy of the symbols. To realise efficient coding when using entropy coding, it is necessary to keep the quantizing error fixed and obtain the quantization characteristics that will minimize the entropy. Entropy coding is ad-vantageous only when the difference between

for gamma distributed signals than for the Laplacian and Gausian distribution [4]. The minimum mean-square quantizing error is

$$\sigma_q^2 = \frac{2}{3N^2} \left[\int_0^v p^{4/3}(y) \, dy \right]^3 \qquad (3)$$

where $N=2^n$ is the total number of quantizing levels, while $p(y)$ is an even function repre-senting the probability density of the input to the quantizer [5]. The quantizer input for nonadaptive DPCM systems is approximate-ly gamma, i.e.

$$p(y) = \frac{k}{2\Gamma(m)} \left(k|y| \right)^{m-1} exp\left(-k|y|\right) \qquad (4)$$

where $\Gamma(.)$ denotes the gamma function. The parameter m determines the shape. The value m=0,5 produces the best fit to the data, while

$$k^2 = \frac{0,75}{\sigma_y^2} \qquad (5)$$

where σ_y^2 is the variance of the quantizer input signal [2,3].

The major assumption is that the limi-ting degradation factor is generally quanti-zing noise. The distortion criterion to be used will be the squared-error criterion. As v in (3) becomes large, we obtain the appro-ximation for the mean-square value of the quantizing noise
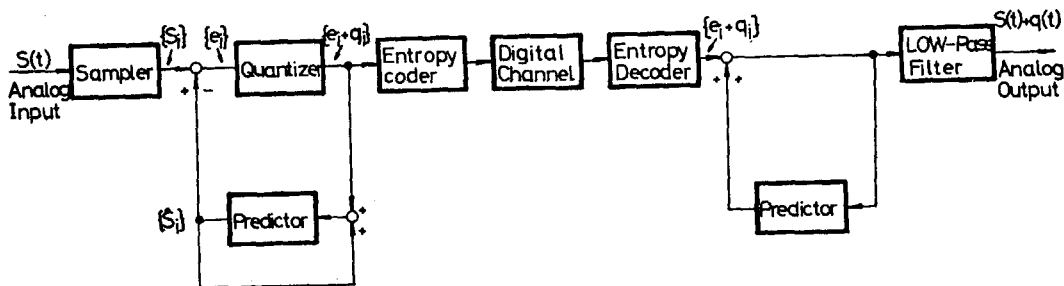
$$\sigma_q^2 = \frac{A}{N^2} \sigma_y^2 \qquad (6)$$

Fig.1. BLOCK DIAGRAM OF NONADAPTIVE DPCM SYSTEM WITH
ENTROPY CODING

the number of bits require to represent a sample value n and the average code length is considerable. This difference is more

where the constant A has the form

$$A = 4 \cdot 3^m \frac{\left[\Gamma\left(\frac{m+2}{3}\right) \right]^3}{\Gamma(m)}$$

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For m=0,5 we obtain A=5,67.

In the observed DPCM system, the quantizer is designed to minimize $\sigma_{\xi}^2/\sigma_{y}^2$ for a fixed number of quantizing levels N. For the DPCM system with entropy coding, the quantizer is designed to minimize $\sigma_{\xi}^2/\sigma_{y}^2$ when the entropy of the quantizer output is fixed. For both DPCM systems this results in minimizing $\sigma_{\xi}^2/\sigma_{y}^2$ and therefore maximizing the signal-to-noise ratios for a fixed bit rate in the channel.

III. SIGNAL-TO-QUANTIZING NOISE RATIO

Let us now calculate the bit rate C used to transmit the quantizing levels through binary channel for the DPCM system without entropy coding. It will be

$$C = 2 f_m \log_2 N \qquad (7)$$

where $2 f_m$ is the sampling rate. From equations (6) and (7), it will be

$$C = f_m \log_2 A \cdot \frac{\sigma_y^2}{\sigma_\xi^2} \qquad (8)$$

For the DPCM system with entropy coding, the bit rate in the channel is equal to the sampling rate multiplied by the entropy of the quantizer output H, i.e.

$$C = 2 f_m H \qquad (9)$$

When the number of quantizing levels gets large, the entropy of the quantizer output is

$$H \simeq H_i - \log_2 \Delta_q \qquad (10)$$

where H_i is the entropy of the quantizer input, while Δ_q is the step size of the uniform quantizer. The mean-square quantizing error is related to the step size by

$$\varepsilon_q^2 = \frac{1}{12} \Delta_q^2 \qquad (11)$$

On the other hand, the entropy of the quantizer input is

$$H_i = - \int_{-\infty}^{\infty} p(y) \log_2 p(y) \, dy$$

$$= \frac{1}{2} \log_2 \frac{4 e \sigma_y^2}{3 \lambda} \qquad (12)$$

where the Euler's constant $\lambda = 1,781$ while e=2,718.

Thus, the entropy of the quantizer output becomes

$$H = \frac{1}{2} \log_2 \frac{e}{9\lambda} \cdot \frac{\sigma_y^2}{\varepsilon_q^2} \qquad (13)$$

The corresponding bit rate is

$$C_1 = f_m \log_2 \frac{e}{9\lambda} \cdot \frac{\sigma_y^2}{\varepsilon_q^2} \qquad (14)$$

Since the variance σ_y^2 is the same for DPCM systems with and without entropy coding, from equations (8) and (14), we obtain

$$A \frac{\sigma_y^2}{\sigma_\xi^2} = \frac{e}{9\lambda} \cdot \frac{\sigma_y^2}{\varepsilon_q^2}$$

or

$$\varepsilon_q^2 \simeq 0,03 \sigma_\xi^2 \qquad (15)$$

The equation (15) gives the relationship between the quantizing noise powers, when the two observed systems operate at the same bit rate. As

$$\frac{\sigma_y^2}{\varepsilon_q^2} = \frac{9\lambda A}{e} \frac{\sigma_y^2}{\sigma_\xi^2}$$

when the bit rate is large, a quantizing system using entropy coding can achieve a signal-to-quantizing noise ratio of $10 \log \frac{9\lambda A}{e} =$ =15,24 dB greater than a system without entropy coding.

IV. UPPER BOUND ON SIGNAL-TO-QUANTIZING NOISE RATIO

Further increase in the signal-to-quantizing noise ratio without increasing the transmission rate is possible if we assume ε-entropy of the bit stream of the quantizer output. For a gamma model probability density, the ε-entropy of the quantizer output is, using equation (2)

$$H_{\varepsilon} = \frac{1}{2} \log_2 \frac{2 \, \sigma_y^2}{3 \lambda \pi \varepsilon_y^2} \qquad (16)$$

where ε_y^2 is the mean-square quantizing error. Using uniform quantization, gives $\varepsilon^2 \approx \varepsilon_y^2$. In this case, the bit rate in the cahnnel is equal to the sampling rate multiplied by the ε-entropy of the quantizer output, i.e.

$$C_2 = 2 f_m H_{\varepsilon} \qquad (17)$$

or

$$C_2 = f_m \log_2 \frac{2 \, \sigma_y^2}{3 \lambda \pi \varepsilon_y^2} \qquad (18)$$

Comparing the quantizing noise of the system in which we assume ε-entropy of the bit stream of the quantizer output and the system without entropy coding, we have

$$A \cdot \frac{\sigma_y^2}{\sigma_y^2} = \frac{2 \, \sigma_y^2}{3 \lambda \pi \varepsilon_y^2} \qquad (19)$$

Finally, it will be

$$\varepsilon_y^2 = \frac{2}{3 \lambda \pi A} \cdot \sigma_y^2 \qquad (20)$$

or

$$\varepsilon_y^2 \simeq 0,021 \, \sigma_y^2 \qquad (21)$$

The equation (21) gives the relationship between the quantizing noise powers when the two systems operate at the same bit rate. As

$$\frac{\sigma^2}{\varepsilon_y^2} = \frac{3 \lambda \pi A}{2} \cdot \frac{\sigma^2}{\sigma_y^2}$$

when the bit rate is large, a quantizing system using ε-entropy of the bit stream can give a signal-to-quantizing noise ratio of

$$10 \log_{10} \frac{3 \lambda \pi}{2} = 16,77 \text{ dB}$$

greater than a system without entropy coding. The 16,77 dB gain is an upper bound on signal-to-quantizing noise ratio for speech differential pulse code modulation system with nonadaptive quantizer.

V. CONCLUSION

Theoretical estimate on upper bound on signal-to-quantizing noise ratio for speech DPCM system with nonadaptive quantizer is presented subject to the following constraints: (1) the quantizer input is approximately the gamma probability density, (2) the limiting degradation factor is quantizing noise, (3) the distortion criterion used is the squared-error criterion and (4) the bit rate is large.

Three speech DPCM systems are compared concerning the signal-to-quantizing noise ratio:DPCM system with and without entropy coding as well as DPCM system with ε-entropy of the bit stream of the quantizer output. The speech DPCM system using entropy coding can achieve a signal-to-quantizing noise ratio of about 15,24 dB greater than a system without entropy coding. On the other hand, the speech DPCM system using ε-entropy of the bit stream of the quantizer output, gives a signal-to-quantizing noise ratio of about 16,77 dB greater than a system without entropy coding.

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