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SUBOPTIMAL PROCESSING IN SYNTHETIC APERTURE SONAR

TRAITEMENTS SOUS-OPTIMAUX EN SONAR A SYNTHESE D'OUVERTURE

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RESUME

L'exploitation d'une ouverture synthétique (plutôt que physique) pour l'imagerie acoustique des fonds marins peut présenter divers avantages, et notamment ceux d'une flexibilité accrue, d'un plus faible encombrement du système sonar et d'un meilleur rapport signal-bruit.

Encore faut-il que tout le système soit conçu de manière à exploiter ces avantages tout en minimisant l'impact des désavantages qui peuvent y être associés: images multiples ambiguës, faible vitesse de couverture spatiale (mapping rate) et sensibilité aux incertitudes de la trajectoire et de la propagation.

Dans ce contexte, on présente les résultats d'une recherche théorique effectuée récemment pour le compte du Centre de recherches pour la défense Atlantique (Dartmouth, N.-E., Canada) sur des méthodes sous-optimales de traitement visant à pallier aux problèmes énumérés ci-dessus. On examine en particulier les traitements d'enveloppe, en bande large, et le traitement par moyenne et l'on donne des expressions approchées de la performance. Celles-ci sont illustrées par l'exemple d'un sonar latéral à immersion profonde.

NOTATION

A	Rate of unambiguous area coverage (m^2/s)
B	Bandwidth (Hertz)
c	Speed of Sound in water (taken as 1500 m/s in example)
D	Horizontal antenna extension (m)
f	Frequency (Hz)
L_{eff}	Length of Synthetic Aperture (m)
pps	Pixels per second (s^{-1})
prf	Pulse repetition frequency (s^{-1})
Q	Quality factor
R	Range (m)
R_{max}	Maximum range (m)
R_{min}	Minimum range (m)
R_{ss}	Minimum range for beam sharpening by coherent averaging

SUMMARY

Using a synthetic (rather than physical) aperture for the sea floor imaging can present several advantages, in particular those of increased flexibility, reduced system dimensions and better signal-to-noise ratio.

The sonar system must however be designed in order to exploit these advantages, minimizing at the same time the associated problems: ambiguous multiples, small mapping rate and sensitivity to trajectory and propagation uncertainties.

A study was recently completed for the Defense Research Establishment, Atlantic (Dartmouth, N.S., Canada), on sub-optimal processing methods dealing with the above problems. Envelope, wide band and averaging processing methods are examined and approximate expressions for their performance are derived. The paper is illustrated by the example of a deep-tow side-scan sonar.

SNR	Signal-to-noise ratio (dB)
t	time (s)
V	Translational speed of sonar (m/s)
W	Swath width (m)
α	Constant (see eq. 2)
β	Constant (see eq. 6)
β_D	Azimuthal half-power beamwidth (rad)
β_{eff}	Half-power beamwidth of synthetic aperture (rad)
δ_a	Azimuth resolution (m)
δ_r	Range resolution (m)
γ_r	Spatial oversampling factor (see eq. 7)
λ	Wavelength (m)
v	Number of pings coherently averaged
ϕ	Phase (rad)



1.0 INTRODUCTION

Although synthetic aperture methods are widely recognized, used and reported in airplane and satellite borne radar, their application to sonar is not widespread. For example, the open literature we consulted revealed no example of full scale sea floor imaging applications. The existing synthetic sonar literature mostly describes model tank and theoretical work, and is much less abundant and advanced than the corresponding radar literature.

The reasons for this state of affairs include the following:

- (i) Coherence problems related to the difficulty of keeping a straight trajectory and/or appropriately monitoring the position of the sonar platform during the relatively long time necessary to form the synthetic aperture;
- (ii) Relatively small mapping rates, due in particular to the relatively slow speed of sound in water;
- (iii) Coherence problems related to the propagation of sound through an inhomogeneous medium.

After a brief review of the synthetic aperture sonar concepts, this paper introduces several methods, some of which are thought to be novel in synthetic aperture sonar, to alleviate the problems cited above. The methods presented have in common the fact that they are both suboptimal with respect to and more robust than the conventional synthetic aperture processing.

This paper, which does not contain experimental results is nonetheless experimentally oriented in that it proposes practical processing schemes, that have not, to the author's knowledge, been tried yet in connection with synthetic aperture sonar.

It is largely based on work (Heering, 1982) which the author did for the Defense Research Establishment Atlantic.

2.0 BASIC PRINCIPLES OF SYNTHETIC APERTURE SONAR

2.1 FUNDAMENTAL RELATIONS

In the following, only the relations necessary for the understanding of the remainder of the paper, are presented. For more details, the reader is referred to Culsona (1970 & 1975), Tomiyasee (1978) and Lee (1979), in addition to the work cited above.

In a synthetic aperture sonar (strip mapping mode) the aperture is generated by the radiating element sequentially taking positions along a line at consecutive times. See Fig. 1 for the geometry. A synthetic aperture of length L_{eff} corresponds to a half-power beamwidth

$$\beta_{eff} = \lambda / (2L_{eff}) \quad (1)$$

where the factor 2 in the denominator results from the angle selectivity being provided in the transmission as well as in the reception process.

Further, we observe that L_{eff} corresponding to range R is determined by the width of the horizontal beam of the physical horizontal aperture D of that sonar at the range.

$$L_{eff} = \alpha R \lambda / D \quad (2)(*)$$

where $\alpha \leq 1$ accounts for the shading of the synthetic aperture.

Since

$$\delta_a = \beta_{eff} R \quad (3)$$

one has

$$\delta_a = D / (2\alpha) \quad (4)$$

(4) indicates an azimuth resolution that is independent of range or wavelength.

The range resolution of the sonar is, as usual

$$\delta_r = c / (2B). \quad (5)$$

2.2 SAMPLING CONSTRAINTS AND MAPPING RATE

The maximum pulse repetition frequency (prf) is limited by the range ambiguity constraint, i.e., the condition that echoes from the "previous" transmission not be received within the range gate of "this" transmission. (Fig. 1)

$$\text{prf} = \beta c / (2(R_{max} - R_{min})) \quad (6) \\ \approx \beta c / (2W)$$

assuming small grazing angles, $R_{max} - R_{min} \approx W$, the swath width of the sonar; $\beta \leq 1$ accounts for not all of $(\text{prf})^{-1}$ being utilized for reception. R_{max} and R_{min} are generally determined by the vertical beamwidth of the sonar and the geometry.

The minimum pulse repetition frequency corresponds to the minimum spatial sampling frequency along the sonar trajectory that will not produce aliasing. It can be shown (Cutrona 1975, Lee 1979) that

$$\text{prf} = 2\gamma / D \quad (7)$$

where $\gamma > 1$ is a spatial oversampling factor.

Equation (7) is similar to the maximum spacing condition between the elements of a conventional array to avoid grating lobes. It can be interpreted as requiring one "ping" at least every half length of the physical array, along its trajectory. Eq. 7 shows that the array length acts like a spatial low-pass filter on the acoustic field along the sonar trajectory. Finally, it is useful to observe that eq. 7 is based on narrow-band signal conditions.

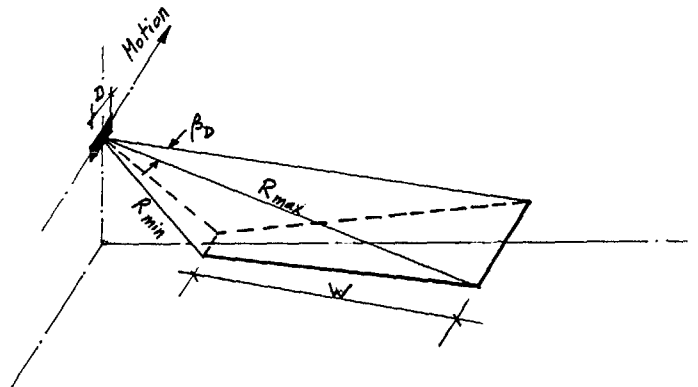


Figure 1: Basic Geometry

(*) This and the following expressions are small angle approximations

The unambiguous mapping rate A results from eq. (6) and (7)

$$A = VW \cong \frac{cD\beta}{4\gamma} \quad (8)$$

for the geometries mentioned in the comments to equation (6).

Finally, the number of pixels per second (pps) generated by the synthetic aperture sonar is derived from (3, 4, 8)

$$\text{pps} = \alpha\beta B/\gamma \quad (9)$$

pps is, apart from weighting factors, an explicit function of B only. This is consistent with the fact, shown in Cutrona (1975), that, for equal azimuth resolution and equal trajectory parameters, the mapping rates of conventional and synthetic aperture side scans are equal.

It is shown in Cutrona (1975), Lee (1979) and Heering (1982) that the unambiguous mapping rate A can be increased by a factor n with respect to its value given by (8) with no adverse effect on δ_a by a variety of methods that are all essentially equivalent to operating n synthetic aperture sonars in parallel. This is physically understandable because of the equivalence of the output and input data rates indicated by (9).

2.3 SIGNAL PROCESSING

The synthetic aperture signal processing consists in "focussing" the synthetic aperture on each of the points to be imaged. It can be easily seen that, in the Fresnel zone of the synthetic array (where the conventional synthetic aperture sonar operates), the range y to a target point at $(0, R)$ is (see Fig. 2)

$$y \cong R + x^2/(2R) \quad (10)$$

valid for

$$-\beta_D R/2 < x < \beta_D R/2 \quad (11)$$

The synthetic aperture focussing on the target point $(0, R)$ consists in summing the returns obtained at all x satisfying eq. 11 aligned according to eq. 10. In the case when the "range walk" (see e.g. Tomiyasu, 1978) is not important, i.e. when

$$x^2/(2R) \ll c/b \quad (12)$$

the focussing can be carried out by adjusting the phase (assumed coherent at the transmission) of the received signals by

$$\Delta\phi(x) = 2\pi x^2/(\lambda R) \quad (13)$$

The quadratic dependence of $\Delta\phi$ on x indicates that the sequence of signals from the target point is linearly modulated in frequency. The focussing, which can be performed by optical or digital systems, is essentially equivalent to correlation processing of a linear FM.

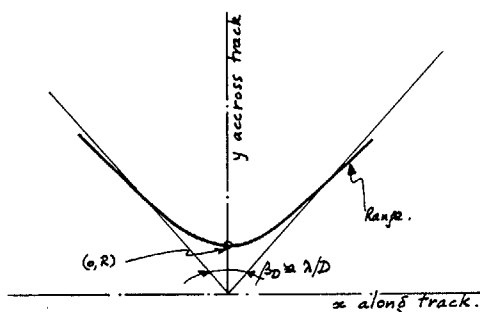


Figure 2:
Range History

2.4 OTHER CONSIDERATIONS

i) Focussing for non-ideal trajectory. The focussing operations indicated by eq. 10 and 13 can be satisfactorily carried out only if the actual time delay (or phase) history is known and compensated for with sufficient accuracy for every point of the map. The maximum residual phase error should not exceed 0.1 to 1 rad depending on the imaging quality required. (see e.g. Tomiyasu (1978), Damoulakis et al (1980)). It is shown in Tomiyasu (1978) that depending on the kind (R, x) and order of derivative (R, R, R) etc of the errors, various image defects result, that include image shifts, defocussing etc. These errors may be caused i.a. by

- Imperfect knowledge of (and compensation for) the actual (in general, non straight) track of the sonar platform. This is probably the most difficult problem in synthetic aperture sonar (see e.g. Kirk (1975) and Damoulakis et al (1980), in particular because of the relatively long (many seconds) aperture times necessitated by the conventional (optimal) processing. It is shown in Heering (1982 and 1983) that for realistic synthetic apertures, better track recovery is likely to be obtained with the help of state-of-the-art current meters than state-of-the-art accelerometers. In addition, autofocussing methods could probably be used too, since they are applied in synthetic aperture radar (Damonlakis et al, 1980).
- Medium inhomogeneities. These may destroy the phase coherence of the received echo and frustrate attempts at forming a synthetic aperture. A possible suboptimal solution is proposed in chapter 5. It should be noted that several workers (Christoff (1976), Williams et al (1976), Fitzgerald (1978), Spindel et al (1978), have recently shown that the medium may exhibit very stable propagation characteristics at frequencies ranging from 15 Hz to 100 kHz and ranges from a few tens of m to several hundreds of km. The problem may thus not be as severe as was once thought.

ii) Signal-to-noise ratio. Generally speaking, it can be said that the signal-to-noise (and to reverberation) ratio increases in proportion to the number of echoes integrated, except in incoherent synthetic aperture, where the increase is proportional to the square root of that number. Signal-to-noise ratio is not considered further in this paper.



3.0 BEAM SHARPENING BY COHERENT AVERAGING

3.1 MOTIVATION

Incoherent averaging of consecutive returns is extensively used in radar and sonar in order to improve detection performance. Instead, we examine here the coherent averaging of ν consecutive returns in order to improve azimuth resolution. It is assumed that no phase corrections are applied to the returns and that the transmissions are phase-coherent.

The processing method proposed here is unfocussed in a manner similar to the "unfocussed synthetic antenna" proposed by Cutrona (1970) but the choice of the averaging interval leads here to a particularity simple realization.

3.2 PERFORMANCE EXPRESSIONS

If ν consecutive echoes are coherently averaged, this corresponds to

$$L_{eff} = \nu w / prf = \nu D / (2\gamma) \quad (14)$$

L_{eff} is now independent of the range: it is simply the distance covered by the platform during ν pulses. Therefore, by (1)

$$\beta_{eff} = \gamma \lambda / (\nu D) \quad (15)$$

We conclude from (15) that, neglecting γ , the sonar beamwidth $\beta_D = \lambda/D$ has been sharpened by a factor ν .

The minimum range R_{ss} at which this form of averaging can be used without focussing can be easily derived from (14) by imposing a $\lambda/4$ limit on the maximum difference between the shortest and longest round trip distances to the array. This results in

$$R_{ss} = \nu^2 D^2 / (4\gamma^2 \lambda) = L_{eff}^2 / \lambda \quad (16)$$

which is a simple far-field condition. Eq. 16 shows that whereas modest directivity gains only can be obtained at short range, larger gains can be obtained at larger range (where they are most needed).

3.3 CONSTRAINTS

The constraints expressed in eq. 6 and 7, and the resulting mapping rate (8) apply to the present case.

3.4 ONE IMPLEMENTATION

The azimuth processing can be carried out quite simply in the exponential averager schematically represented in Fig. 3. It has the advantage of requiring the storage of only one complex word per range resolution cell, and of having an acceptable sidelobe structure.

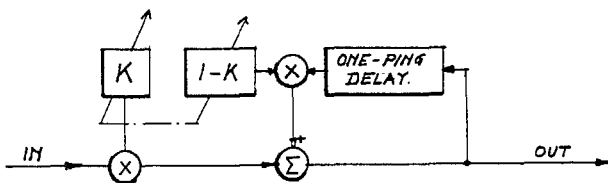


Figure 3: Coherent Averaging Implementation (one range resolution cell).

The "effective" number of pings integrated is easily verified to be

$$\nu_{eff} = 1 - 1/\ln(1-K) \quad (17)$$

This processor should be simple to implement as an add-on to existing side-scan sonars (see example in Chapter 6). For this to be possible however,

- (i) Ambiguity conditions (see section 3.3) must be satisfied.
- (ii) The sonar must be phase coherent.
- (iii) Platform and medium result must be sufficiently stable. This is likely to be realized for reasonable sharpening factors.

This is thus a suboptimal but robust synthetic aperture method, that could probably be applied to some side-scan sonars as an add-on. The sharpening factor ν_{eff} can be simply controlled (through K) as a function of the regularity of the platform trajectory, which, in many cases, could be ascertained from the existing platform sensors (e.g. compass).

4.0 BROADBAND SYNTHETIC APERTURE SONAR

4.1 MOTIVATION

In all synthetic aperture sonar and radar papers we have seen, a narrow-band implementation is postulated or implied. The motivations for examining broad-band applications are several:

- (i) Broad-band sources are available (e.g. boomers, parametric arrays) for underwater sound.
- (ii) If focussed synthetic aperture processing is feasible, then broadband imaging with constant azimuthal resolution can provide classification data not otherwise available.
- (iii) Wide band synthetic aperture processing can remove ambiguities associated with spatial undersampling. This can be used to increase the mapping rate given by (9), possibly in combination with other methods. This is in intuitive conformity with (10).

This last concept (iii) is not unknown in acoustics (see Kock (1980)) or optics (e.g. Farhart (1977)), but has, to the best of our knowledge, never been suggested to or applied to synthetic aperture formation.

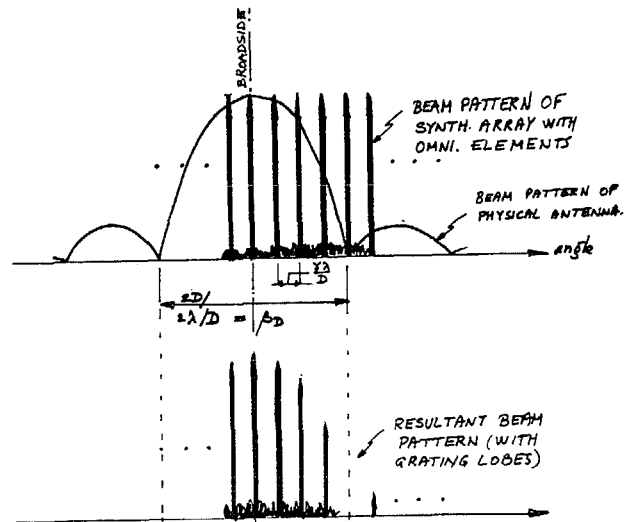


Figure 4: Array and Antenna Beam Patterns ($\gamma=0.25$)

4.2 RELATIONS

The ambiguities that occur in the conventional synthetic aperture processing when $\gamma < 1$ can be interpreted (Tomiyasu, 1978) as being caused by the grating lobes of the synthetic array (occurring at $\gamma\lambda/D$, $2\gamma\lambda/D$ etc from broadside) being inside the main lobe (of width $2\beta_D = 2\lambda/D$) of the horizontal beam pattern of the physical antenna. (see Fig. 4).

Since the angular position of the grating lobes is frequency-dependent, it is clear that wideband operation will result in smearing of the grating lobes. In particular, the situation where the 2nd grating lobe at the highest transmitted frequency is at or beyond the 1st grating lobe at the lowest transmitted frequency corresponds to lowest frequency $< \frac{1}{2}$ highest frequency, or

$$Q < 0.7 \quad (18)$$

where Q is the "Q" of the transmitted pulse. This situation is shown in Fig. 5, which illustrates the fact that no ambiguities can occur when the transmitted pulse has less than one period.

All the relations (e.g. 1-9) describing the operation of the synthetic aperture sonar remain valid with the only difference that γ is now allowed to take values < 1 when condition (18) is verified. The energy associated with the ambiguous images is, however, not cancelled, but spread out. The resulting increase in "reverberation" level may or may not be acceptable depending on the imaging application.

4.3 IMPLEMENTATION

Wide-band synthetic aperture is essentially equivalent to the presence of range walk in the received echo. The echo can either be divided into narrow-band components, processed in the conventional way and coherently recombined, or else the alignment can be made in the time domain. In the latter case, however, there will be a reduction of signal-to-noise ratio if no account is taken of the variation of L_{eff} as a function of frequency.

4.4 DISCUSSION

- (i) The implicit assumption was made, that the frequency response of the target "point" is constant over the band considered. This is, in general, untrue, but may be sufficiently verified in important practical cases.
- (ii) The practical limits of undersampling of the synthetic aperture have not been investigated.

Both questions should be further studied, and we think that the best approach would be simulation or experiment.

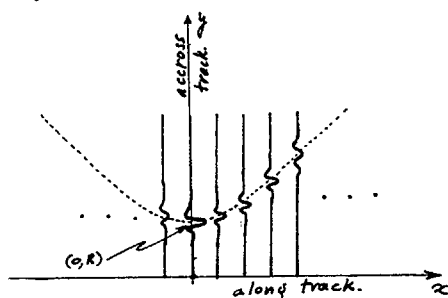


Figure 5: Typical Wide-Band Returns from Target at $(0, R)$.

5.0 INCOHERENT SYNTHETIC APERTURE

5.1 MOTIVATION

For application where the phase coherence of the received signal cannot be assured (because of incoherent transmission, propagation or poor track recovery) a form of synthetic aperture processing can still be used. It has to be based, however, on the delay information contained in the signal envelope. The azimuth resolution of this type of processing will, of course, be degraded with respect to conventional processing, as is to be expected of a method that uses only a part of the information available in the received signal.

5.2 PROCESSING AND AZIMUTH RESOLUTION

In this section, the principle of the signal processing and an approximate derivation of the obtainable resolution will be given for an isolated target. The reader will find more details in Heering (1982).

In the incoherent synthetic aperture, the azimuth resolution is obtained by processing the information contained in the time delay of the received signal, which varies as a function of the position of the sonar along its track; this is the range walk phenomenon discussed in Section 2.3. The situation is thus, in principle, similar to the wide-band case of chapter 4, with the important difference of the non-linearity introduced by the envelope extraction. This renders the accurate analysis of the performance of this processing scheme quite complicated, in particular in the presence of multiple targets.

The processing of the envelope-detected signal is carried out along the lines of section 2.3. We can then estimate $\delta_a/2$ as the along-track distance over which the response from an isolated target drops from its maximum to $1/2$ of its maximum value(*). When the transmitted signal is a short CW pulse, this results, under many practical conditions in

$$\delta_a \approx \sqrt{2 Q D} \quad (19)$$

This expression is similar to azimuth resolution expressions obtained in Section 2 and 4. In the particular case of $Q \approx 0.7$ (see eq. 18), eq. 19 reduces to

$$\delta_a = D \quad (20)$$

which should be compared to eq. 4.

5.3 REMARK

The expression (19) is only approximate. We would expect it to apply best in high SNR, low target density environment. The practical performance of the proposed method could probably best be established by simulation and real data analysis.

(* In Heering (1982), a slightly different definition is used for resolution, resulting in a different coefficient in (19).



6.0 EXAMPLE

We give as an example a conceptual synthetic aperture sonar system generally based on an existing deep-tow side-scan system, the Sea Marc I (made by I.S.E.). The parameters given are approximate.

• Parameters

$D = 1.5 \text{ m}$
 $\alpha = 0.5 ; \beta = \gamma = 1$
 $c = 1500 \text{ m/s}$
 $R = 1000 \text{ m}$ (range for which L_{eff} , δ_a etc. are calculated)
 $W = 1000 \text{ m}$
 $f_0 = 30000 \text{ Hz}$
 $Q = 5$
 azimuth resolution of $D @ 1000 \text{ m} \approx 33 \text{ m}$.

• Conventional S.A.S.

$\text{prf} \approx 0.75 \text{ Hz}$ by (6)
 $A \approx 560 \text{ m}^2/\text{s}$ by (8)
 $V \approx 0.56 \text{ m/s}$ by (7)

This value, a little over one knot, is close to the speed at which deep-tow systems are towed.

$L_{\text{eff}} = 16.7 \text{ m}$ by (2) : this value is not large because R is relatively small.

$\delta_a = 1.5 \text{ m}$ by (4)

$\delta_r = 0.25 \text{ m}$

$\text{pps} = 3000$

• Beam sharpening by coherent averaging:

$\beta_D = 1.91^\circ$

$L_{\text{eff}} = v \cdot 0.75 \text{ m}$ by (14)

$R_{\text{SS}} = v^2 \cdot 11.25 \text{ m}$; @ $R = 1000 \text{ m}$ $v_{\text{max}} = 9$ by (16)

$\beta_{\text{eff}} = 1.91^\circ / v$ by (15)

e.g for $v=4$, $L_{\text{eff}} = 3 \text{ m}$ should be easy to keep straight

within $\lambda/4 \approx 1.2 \text{ cm}$; $\beta_{\text{eff}} \approx 0.5^\circ$; $\delta_a \approx 8.3 \text{ m}$ by (3).

• Broadband synthetic aperture - is not possible because $Q=5$ (see eq. 18)

• Incoherent synthetic aperture

$\delta_a \approx 10.6 \text{ m}$ if the conditions of validity of (19) are met.

7.0 CONCLUSION

This paper has introduced several suboptimal but robust schemes for synthetic aperture processing. We think that they could well contribute to the practical applications of synthetic aperture sonar, by helping to attenuate some of its problems.

8.0 ACKNOWLEDGEMENTS

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