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ANGULAR ACCURACY IN SEQUENTIAL DETECTION OF RADAR TARGETS

G.Corsini, E.Dalle Mese, G.Marchetti, L.Verrazzani

Istituto di Elettronica e Telecomunicazioni, Via Diotisalvi 2, Università degli Studi di Pisa, 56100 Pisa,Italy

RESUME

Les radars avec l'emploi des antennes "phased-array" devient de plus en plus intéressantes à cause des avantages qu'ils présentent en comparison des radar conventionnels. En particulier l'usage d'un array à balaiage électronique avec un essai séquentiel à rapport de probabilité (SPRT) pour la découverte des cibles, permit un appréciable épargne de puissance au de temps, comme on a démontré en litérature. Toutefois, malgré les nombreux travails sur les methodes séquentielles par la découverte des cibles avec les radars, il n'existe pas à présent une analyse détaillé des techniques pour l'évaluation angulaire.

Ce travail a pour sujet le problème de l'évaluation angulaire des cibles découverts au moyen des radars SPRT à balaiage électronique. Dans la prémière partie nous avons calculé les limites de Cramer-Rao pour toute évaluation angulaire non polarisée. Ensuite nous avon analysé l'évaluation à maximum vraisemblance de la position angulaire du cible dans le cas particulier, mais très intéressant, où le pas du balaiage est égale à la moitié de l'angle d'ouverture à -3 dB du diagramme de rayonnement. Nous avon complété notre étude avec une analyse numérique qui montre que la technique d'évaluation proposée est trés efficient.

SUMMARY

Phased-array radars have increasing interest for the advantages achievable with respect to conventional radars. In particular the use of electronically scanned array in connection with a sequential probability ratio test (SPRT) for target detection allows appreciable power (or time) savings, as it was pointed out in the technical literature. However, despite the numerous papers on sequential methods in radar detection, a detailed analysis of angular estimation is not available.

This paper deals with the problem of angular estimation of targets detected by means of the SPRT in phased-array radars. Firstly, we derive and discuss the Cramer-Rao bounds of any unbiased angular estimate. Secondly, we analyse the maximum likelihood estimate (MLE) of the target angular position in the particular but significant case where the scan step size is equal to half the -3 dB beamwidth. The analysis is completed by numerical computation which shows the effectiveness of the proposed estimation technique.

I. INTRODUCTION

Phased-array radars have increasing interest for the advantages achievable with respect to conventional radars. Basically, these advantages are due to the use of the electronically steered array as an adaptive sensor of the search volume. This aim can be achieved by adaptively sharing the total time and/or energy among the search directions. In addition we can have an adaptation of the radar functions during the time assigned for a specific purpose. Modern studies on multifunctions phased-array radars are devoted to fully exploit the capabilities offered by this type of antenna.

As an example of the first type of adaptation we mention the use of a sequential test for target detection. In a sequential procedure the test length is a random variable, whose value depends adaptively on the signal-to-noise ratio (SNR) of the received waveform. Among the sequential tests, the sequential probability ratio test (SPRT) is particularly attractive, owing to its optimality properties /1/. Since the pioneering work of Wald /1/, many papers has been written on the analysis of the SPRT applied in radar target detection; recently this test was used with success in an experimental multifunction phased-array radar project /2/.

A problem which arises in phased-array radars is to obtain an accurate estimate of the target angular position; in fact a rough estimate based on the mean value of the successive angular coordinates of the antenna boresight is not reliable because of the usually large scan step size. If a SPRT is used for target detection, it is natural to expect an improvement in the angular estimate by utilising properly the test length that gives information on the SNR. Notice that this approach is not possible in conventional radars where the detection test length is fixed. The problem of angular accuracy in conventional radars has been extensively treated in the technical literature, both for analog and digital signal processing. However, despite the great number of papers on sequential target detection, little effort was devoted to the angular estimation problem. The only relevant papers on this topic are by a team of russian researchers /3,4/, which present poor results not suitable for comparison and design purposes; however their work is an useful starting point for further developments.

The aim of this paper is twofold. Firstly, we derive and discuss the Cramer-Rao bounds of any unbiased angular estimate in radars which use a SPRT for target detection. Secondly, we analyse the maximum likelihood estimate (MLE) of the target angular position in the particular but significant case where the scan step size is equal to half the -3dB beamwidth. The analysis is completed by numerical computation which shows the effectiveness of the proposed estimation technique. For clarity of presentation we grouped in section 2 the necessary mathematical preliminaries on SPRT and the definitions used in the sequel of the paper.

2. PRELIMINARIES

Target detection and simultaneous estimation of the azimuth θ_t may be accomplished with an accuracy better than the antenna mainbeam traverse aperture α if the search volume is scanned discretely with a step $\Delta\theta \leqslant \alpha/2$, so that a target has $M=\alpha/\Delta\theta \geqslant 2$ opportunities to be detected. An application of the lobe switching concept, developed in the earliest of the tracking radars, suggests to use a step $\Delta\theta=\alpha/2$ and to determine the ratio between the amplitudes of the signals received from two noncoincident antenna patterns so that all the parameters are removed, except angle of arrival. In particular if $\hat{\gamma}_1$ and $\hat{\gamma}_2$ are the MLE of the SNR's at the decision instants of the SPRT, relevant to two consecutive beam positions θ_1 and θ_2, the target bearing θ_t may be estimated by solving the equation:

$$(1)\quad \hat{\gamma}_1/\hat{\gamma}_2=g(\theta_1-\hat{\theta}_t)/g(\theta_1+\alpha/2-\hat{\theta}_t) \quad ,$$

where $g(\theta)$ is the normalized one-way traverse power radiation pattern. Thus the estimate $\hat{\theta}_t$ is determined through the knowledge of the power radiation pattern, that we assume to be expressible in factorized form $g(\theta)w(\phi)$: here θ and ϕ are the azimtuh and elevation angle. In Sect. 3 we will show that the solution of eq. (1) gives the MLE of θ_t.

In this paper we will consider two detection techniques: coherent and incoherent square-law detection. For many radar situations where coherence is an unlikely hypothesis the square-law detector is commonly used because it is optimum for all SNR's in the case of pulse-to-pulse Rayleigh fluctuating pulse trains; besides, it is also optimum for constant cross-section radar targets when the SNR is low.

In order to evaluate the accuracy of the estimate $\hat{\theta}_t$, we need some results from the analysis of the SPRT. The approximated analysis by Wald /1/ is not always applicable in radar detection problems, so we must resort to an exact analysis which we carried out in detail in report /5/. Here, after a brief summary of the SPRT structure, we recall only the significant formulas relevant to the above estimation problem.

The SPRT is defined by: observe an infinite sample sequence of independent and identically distributed (i.i.d.) random variables $\{v_i\}$, and at stage $k \geqslant 1$ decide between two simple hypotheses H_0 and H_1 according to the following rule:

- accept H_0 if $Z_k \leqslant B$,
- accept H_1 if $Z_k \geqslant A$,
- continue by observing v_{k+1} if $B<Z_k<A$,

where the stopping bounds are two real numbers $A>0$, $B<0$ and $Z_k=\sum_{i=1}^{k} \ln[p(v_i|H_1)/p(v|H_0)]$ is the logarithmic likelihood ratio at stage k. The total number of inputs $v_1 \ldots v_n$ observed before coming to a decision, i.e. the length N of the test, is a random variable.

In radar sequential detection the receiver must decide if the raw data v_k, normalized with respect to the standard deviation of the noise and received from a set range resolution cell, either contain random noise only (hypothesis H_0) or contain random noise plus signal samples $s_k(\Gamma)$ (hypothesis H_1). The vector Γ represents a set of unknown but nonrandom parameters specifying the signal. A "standard" value Γ_s must be assigned to Γ in order to determine the likelihood ratios; Γ_s is specified through the requirement that the standard signal is detected with prefixed probabilities of false alarm P_{FA}, and of detection P_D. If the hypothesis H_1 is composite, the test is defined in terms of the average likelihood ratio. The optimum character of SPRT derives from the following properties: the SPRT is closed (the test terminates with probability one) and requires on the average no more observations than any other test procedure having equal or smaller probabilities of error (false alarm probability and dismissal probability).

The distribution of N and the reliability characteristics of the sequential procedure can be determined through the probabilities that at the n-th stage the logarithmic likelihood ratio exceedes the decision thresholds A and B. N is a discrete random variable which can assume only positive integral values with probability mass function:

$$(2)\quad P(N=n|\Gamma)=P(Z_n>A|\Gamma)+P(Z_n<B|\Gamma)=\int_{-\infty}^{B} f_{Z_n}(z|\Gamma)dz+\int_{A}^{\infty} f_{Z_n}(z|\Gamma)dz \quad ,$$

where $f_{Z_n}(z|\Gamma)$ is expressed by the recursive convolution:

$$(3)\quad f_{Z_n}(z|\Gamma)=\int_{B}^{A} f_{Z_{n-1}}(\alpha|\Gamma)f(z-\alpha|\Gamma)d\alpha \qquad n>1$$

$$f_{Z_1}(z|\Gamma)=f(z|\Gamma)$$

and f(.) is the probability density function (p.d.f.) of the i.i.d. random variables $X_i=Z_i-Z_{i-1}$ denoting the likelihood ratio's increment. Eq. (3) signifies that the n-th stage is reached only if no decision is made at the (n-1)-th stage. Thus P_{FA} and P_D are given by:

$$(4)\quad P_{FA}=\sum_{n=1}^{\infty}\int_{-\infty}^{B} f_{Z_n}(z|\Gamma,H_0)dz \quad ,$$

$$(5)\quad P_D=\sum_{n=1}^{\infty}\int_{A}^{\infty} f_{Z_n}(z|\Gamma,H_1)dz \quad ,$$

the average number of observations required to terminate the test (average sample number) and its variance are:

$$(6)\quad \overline{N}=E\{N\}=\sum_{n=1}^{\infty} nP(N=n|\Gamma) \quad ,$$

$$(7)\quad \sigma_N^2=\sum_{n=1}^{\infty}(n-\overline{N})^2P(N=n|\Gamma) \quad .$$

3. MAXIMUM ANGULAR ACCURACY

In this Section we derive Cramer-Rao bounds on the error variance of any unbiased angular estimate. Our approach follows closely the one outlined in /3/. Let us define:

- $v_{ij} \triangleq$ i-th observation made when the antenna is pointing to the j-th direction;
- $M \triangleq$ number of adjacent directions where the target is present;
- $N_j \triangleq$ number of observations in the j-th direction, required to end the test;
- $\theta_j \triangleq$ antenna boresight angle corresponding to the j-th direction;
- $g_j \triangleq g(\theta_j-\theta_t)$= normalized traverse antenna one-way power pattern;
- $\gamma_j \triangleq \gamma_0 g_j$= SNR relevant to the j-th direction, where γ_0 is the SNR in the maximum gain direction;
- $\gamma_s \triangleq$ standard SNR for which the SPRT is designed.

Let us represent by $\Gamma=(\theta_t,\gamma_0)$ the vector of the unknown parameters to be estimated. We are concerned with a multiple parameter estimation problem, for which some general results are available.

The computation of the Cramer-Rao bounds /6/ requires the knowledge of the logarithmic likelihood function which, in the hypotheses of i.i.d. variables, becomes:

$$(8)\quad \Lambda(\Gamma)=\sum_{j=1}^{M}\sum_{i=1}^{n_j} \ln p(v_{ij}|\Gamma) \, ,$$

where $p(v_{ij}|\Gamma)$ is the conditional p.d.f. of the observations. In order to proceed we need an expression for $p(v_{ij}|\Gamma)$ which depends on the detection model considered. According to the discussion in Sect. 2, the common models used are:

1. Coherent detection
2. Incoherent detection with nonfluctuating target
3. Incoherent detection with Rayleigh fluctuating target.

The probability density functions of v_{ij} are respectively /7/

$$(9)\quad 1.\ p(v_{ij}|\theta_t,\gamma_0)=(1/\sqrt{2\pi})\exp[-(v_{ij}-\gamma_j/\sqrt{\gamma_s})^2/2]$$

$$(10)\quad 2.\ p(v_{ij}|\theta_t,\gamma_0)=v_{ij}\exp[-(v_{ij}^2+\gamma_j^2/\gamma_s)/2]I_0(v_{ij}\gamma_j/\sqrt{\gamma_s})$$

$$(11)\quad 3.\ p(v_{ij}|\theta_t,\gamma_0)=[v_{ij}/(1+\gamma_j^2/2\overline{\gamma}_s)]\exp[-v_{ij}^2/2(1+\gamma_j^2/2\overline{\gamma}_s)].$$

Notice that in the above equations the dependence on θ_t is throught γ_j. In addition eq. (11) was obtained by averaging with the probability density function $p(\gamma_s)=(1/\overline{\gamma}_s)\exp(-\gamma_s/\overline{\gamma}_s)$ which corresponds to Swerling II fluctuating model /7/. Therefore, $\overline{\gamma}_s$ represents the standard average SNR. Finally it is worthy to recall that the observations v_{ij} in eq. (9) are the sampled output of a matched filter, while in eqs. (10) and (11) they are the sampled output of a matched filter followed by an envelope detector.

The derivation of the Cramer-Rao bounds in case 2 is a quite formidable task; however asymptotic results can be obtained from the solution of cases 1 and 3; in fact if the SNR is very small the probability density function (10) can be approximated by (11) and if the SNR is very large the probability density function (10) can be approximated by (9). For these reasons, let us consider cases 1 and 3 only. This is not a serious limitation from a practical point of view because, as it was pointed out in Sect. 3, the optimum receiver in case 3 is a good approximation to the optimum receiver in case 2.

For any unbiased estimate the minimum variance of the error is given by the Cramer-Rao inequality /6/. We have:

1. Coherent detection

$$(12)\quad \overline{\sigma}^2_{\hat{\theta}_t}(coh)=\gamma_s/[\gamma_0^2(\sum_{j=1}^{M} g_j'^2\,\overline{N}_j)(1-\rho_c^2)] \, .$$

with

$$(13)\quad \rho_c^2=(\sum_{j=1}^{M} g_j g_j'\overline{N}_j)^2/[(\sum_{j=1}^{M} g_j^2\overline{N}_j)(\sum_{j=1}^{M} g_j'^2\,\overline{N}_j)]$$

and $g_j' \triangleq (dg_j/d\theta_t)$;

2. Incoherent detection (Swerling II)

$$(14)\quad \sigma^2_{\hat{\theta}_t}(incoh)=\overline{\gamma}_s^2/\{\gamma_0^4(1-\rho_i^2)[\sum_{j=1}^{M} g_j^2 g_j'^2\overline{N}_j/(1+\gamma_j^2/2\overline{\gamma}_s)^2]\} \, ,$$

$$(15)\quad \rho_i^2=\frac{[\sum_{j=1}^{M} g_j^3 g_j'\overline{N}_j/(1+\gamma_j^2/2\overline{\gamma}_s^2)^2]^2}{[\sum_{j=1}^{M} g_j^4\overline{N}_j/(1+\gamma_j^2/2\overline{\gamma}_s)^2][\sum_{j=1}^{M} g_j^2 g_j'^2\overline{N}_j/(1+\gamma_j^2/2\overline{\gamma}_s)^2]} .$$

Eqs. (12-14) have been numerically solved with the following assumptions:

- the normalized traverse power pattern is gaussian: $g(\theta)=\exp[-2.77(\theta/\alpha)^2]$,
- the SNR in the maximum gain direction is the standard SNR ($\gamma_0=\gamma_s$),

- the scan step size equals $\alpha/2$ and $\alpha/3$ corresponding respectively to M=2 and M=3.

Some results obtained through the method developed in /5/ are reported in Figs. 1 and 2, relevant respectively to cases 1 and 3. From the above figures and a number of other numerical results, we can make the following remarks:

- the normalized minimum standard deviation of the angular estimate increases as the SNR increases in the incoherent case, in opposition to the behaviour related to the coherent case; this apparently surprising result can be explained as follows. As the SNR increases the average number of observations decreases so that, in the incoherent case, the estimate $\hat{\bar{\gamma}}_0$ becomes poorer, due to target fluctuations. Consequently the estimate $\hat{\theta}_t$ becomes poorer too;
- even if it is not apparent from Fig. 2, the normalized minimum standard deviation has an upper and lower bound; they are equal respectively to 0.255 and 0.06 for θ_t/α=0.25 and M=2;
- the accuracy in the incoherent case is better than that attainable in the coherent case for low SNR's; this occurs because the average number of observations is higher in the first case /5/.

4. MAXIMUM LIKELIHOOD ESTIMATE

In order to obtain an estimate of θ_t, suitable for practical implementation, let us consider the MLE, because of its optimality characteristics. A general solution of the problem, when M directions are considered, is very hard and does not lead to a simple solution /4/. Thus in this Section we will deal with the case M=2; this is not a severe limitation since high values of M are unpractical, both for the increase in the search volume scan time and for the difficulties to obtain very small scan steps. In addition the maximum accuracy for M=2 is satisfactory. Maximum likelihood estimate of the target bearing $\hat{\theta}_t$ and of the signal-to-noise ratio $\hat{\gamma}_0$ are obtained by solving the system:

$$\partial\Lambda/\partial\theta_t\big|_{\hat{\theta}_t}=0 \quad , \quad \partial\Lambda/\partial\gamma_0\big|_{\hat{\gamma}_0}=0 \quad ,$$

which can be written as:

$$(16)\quad \begin{aligned} &\sum_{j=1}^{2}\{\frac{\partial}{\partial\theta_t}\sum_{i=1}^{n_j}\ln p[v_{ij}|\gamma_j(\gamma_0,\theta_t)]\}=0 \,, \\ &\sum_{j=1}^{2}\{\frac{\partial}{\partial\gamma_0}\sum_{i=1}^{n_j}\ln p[v_{ij}|\gamma_j(\gamma_0,\theta_t)]\}=0 \,. \end{aligned}$$

It is not difficult to show that system (16) is equivalent to the following system:

$$(17)\quad \begin{aligned} &\frac{\partial}{\partial\gamma_1}\sum_{i=1}^{n_1}\ln p[v_{i1}|\gamma_1(\gamma_0,\theta_t)]=0 \,, \\ &\frac{\partial}{\partial\gamma_2}\sum_{i=1}^{n_2}\ln p[v_{i2}|\gamma_2(\gamma_0,\theta_t)]=0 \,. \end{aligned}$$

In order to solve system (17) it is necessary to specify the p.d.f. p[.] . In the case of coherent detection with a straightforward computation we obtain:

$$(18)\quad \sum_{i=1}^{n_1}v_{i1}=n_1\sqrt{\gamma_0}g(\theta_1-\theta_t) \;,\; \sum_{i=1}^{n_2}v_{i2}=n_2\sqrt{\gamma_0}g(\theta_2-\theta_t) \;.$$

Thus the solution $\hat{\theta}_t$ of system (18) must satisfy the equation:

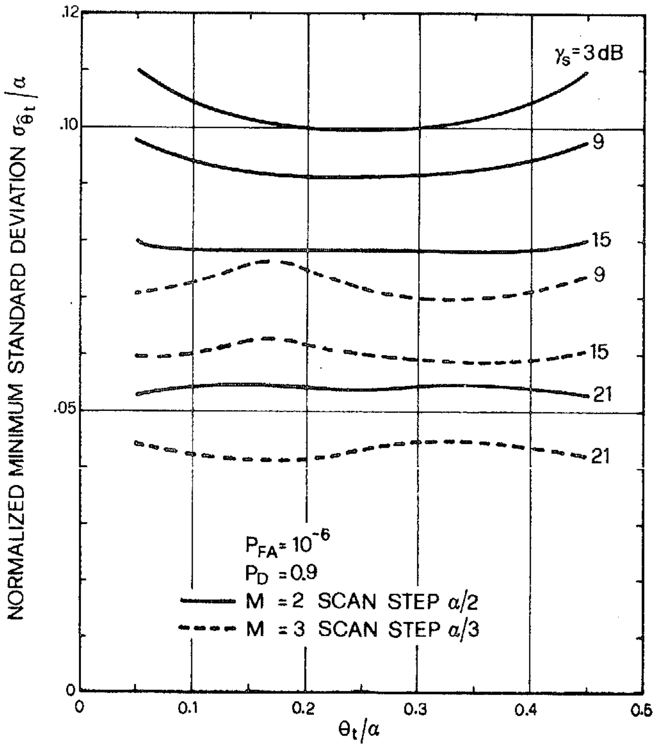

Fig. 1: Maximum angular accuracy (coherent detection)

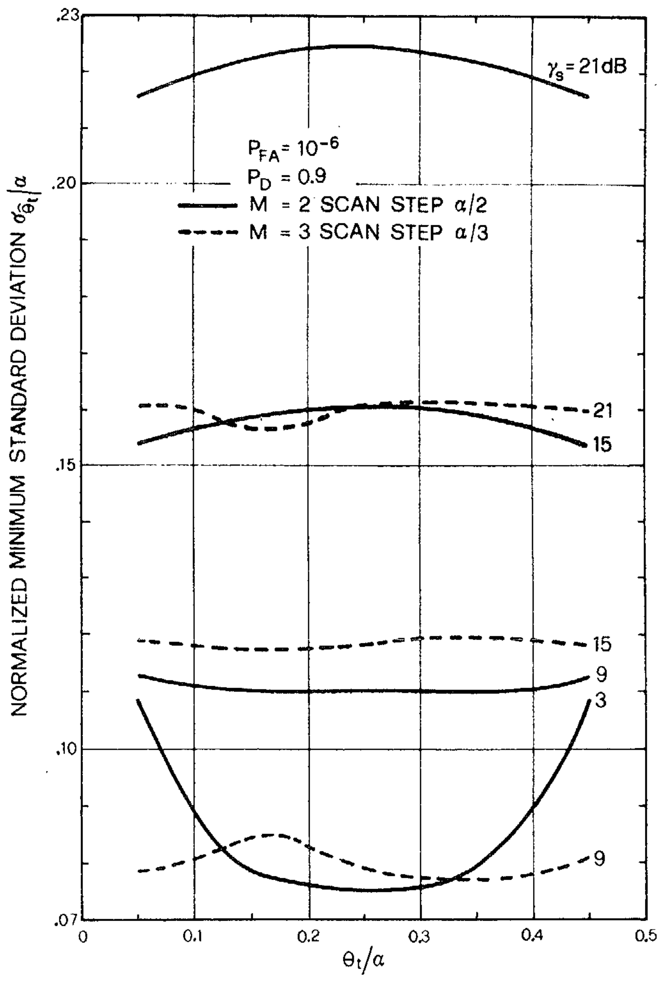

Fig. 2: Maximum angular accuracy (incoherent detection, fluctuating target model: Swerling II)

$$(19)\qquad \frac{g(\theta_1-\hat{\theta}_t)}{g(\theta_2-\hat{\theta}_t)} = \frac{(\sum_{i=1}^{n_1} v_{i1})^2/n_1^2}{(\sum_{i=1}^{n_2} v_{i2})^2/n_2^2} = \frac{\hat{\gamma}_1}{\hat{\gamma}_2} .$$

Notice that $\hat{\gamma}_j=(\sum_{i=1}^{n_j} v_{ij})^2/n_j^2$ is the MLE of γ_j.

In the case of Rayleigh fading target detection we obtain by a similar procedure:

$$(20)\qquad \frac{g(\theta_1-\hat{\theta}_t)}{g(\theta_2-\hat{\theta}_t)} = \frac{(\sum_{i=1}^{n_1} v_{i1}^2/n_1)-2}{(\sum_{i=1}^{n_2} v_{i2}^2/n_2)-2} = \frac{\hat{\gamma}_1}{\hat{\gamma}_2} ,$$

where again $\hat{\gamma}_j=(\sum_{i=1}^{n_j} v_{ij}^2/n_j)-2$ represents the MLE of γ_j. Notice that eqs. (19) and (20) coincide with equation (1), which was suggested by intuitive considerations.

Eqs. (19) and (20) show that $\hat{\gamma}_j$ are obtained from all the data $\{v_{ij}\}$ observed before deciding but they may be profitably extracted from the integrated value of the logarithmic likelihood ratios Z_{n_1} , Z_{n_2} when the SPRT's are stopped. In fact, we have:

$$(21)\qquad \sum_{i=1}^{n_j} v_{ij}/n_j = Z_{n_j}/(2n_j\sqrt{\gamma_s}) + \sqrt{\gamma_s}/2 ,$$

$$(22)\qquad \sum_{i=1}^{n_j} v_{ij}^2/n_j = (1+1/\gamma_s)[Z_{n_j}/n_j + \ln(1+\gamma_s)] .$$

For a gaussian traverse power pattern we have both in coherent and in Rayleigh fading target detection:

$$(23)\qquad \hat{\theta}_t=(\theta_1+\theta_2)/2 + [\alpha^2/5.54(\theta_1-\theta_2)]\ \ln(\hat{\gamma}_1/\hat{\gamma}_2).$$

The accuracy of maximum likelihood estimate $\hat{\theta}_t$ can be evaluated by its mean $E\{\hat{\theta}_t\}$ and the error variance σ_ε^2. Since these parameters depend on the statistical behaviour of the SPRT, numerical computation needs. It will be carried out through the following steps:

- compute the stopping bounds A and B to obtain the required P_D and P_{FA} for the standard signal-to-noise ratio γ_s (eqs.(3-5));
- compute the probability that the SPRT ends at the n-th stage (eq. 2);
- compute $E\{\hat{\gamma}_j\}$ and $\sigma^2_{\hat{\gamma}_j}$ through the conditional expected values:

$$E\{\hat{\gamma}_j\} = \sum_{n=1}^{n_j} E\{\hat{\gamma}_j|N=n\}P(N=n) ,$$

$$\sigma^2_{\hat{\gamma}_j} = \sum_{n=1}^{n_j} \sigma^2\{\hat{\gamma}_j|N=n\}P(N=n)$$

We point out that azimuth estimation takes place under the condition $p(H_1)<1$ (the signal is not surely known to be present); thus detection-directed estimation is assumed (Fig. 3).

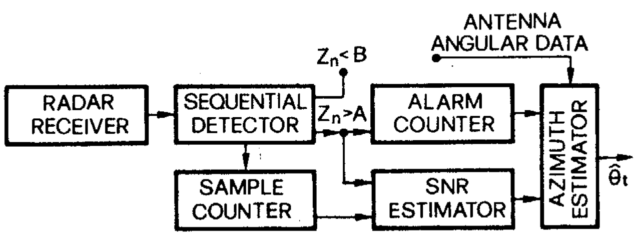

Fig. 3: Block diagram of the angle estimator

The results, summarized in Figs. 4, 5 and 6, suggest the following remarks:

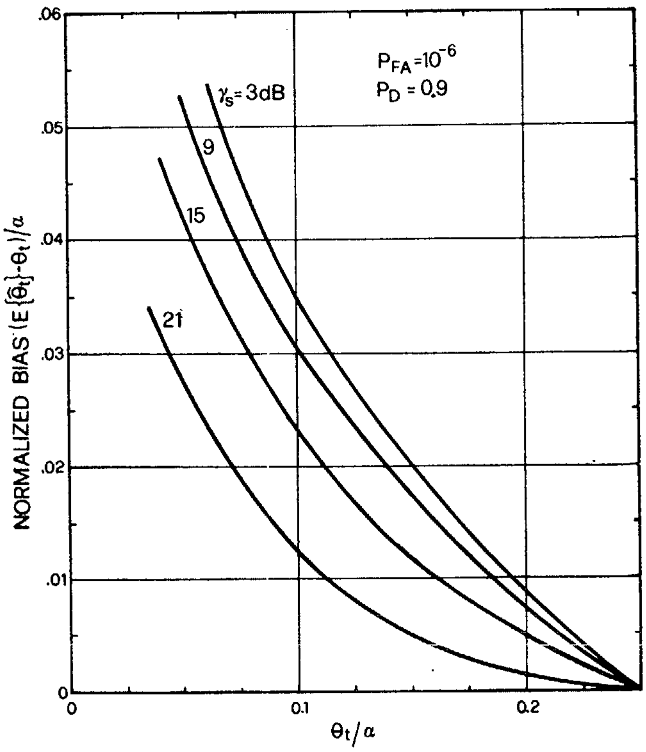

Fig. 4: Normalized bias of the angular estimate (coherent detection)

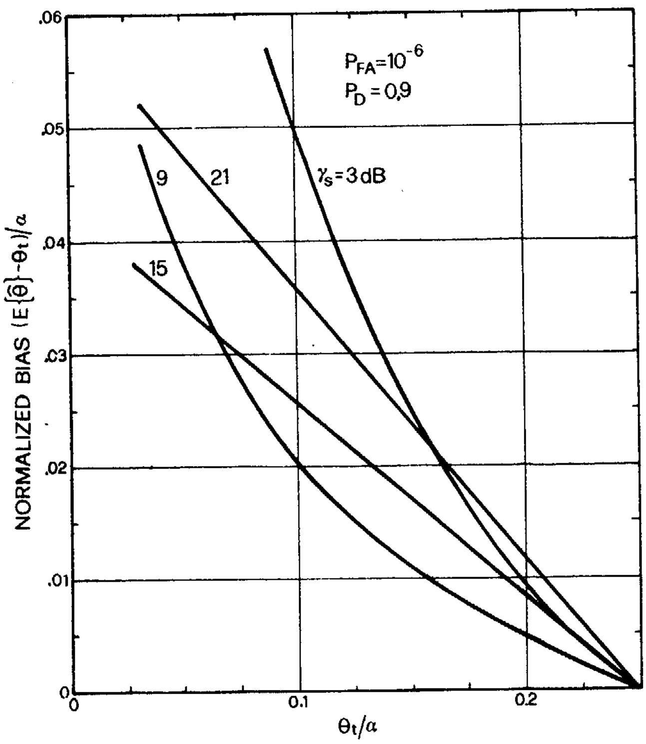

Fig. 5: Normalized bias of the angular estimate (incoherent detection)

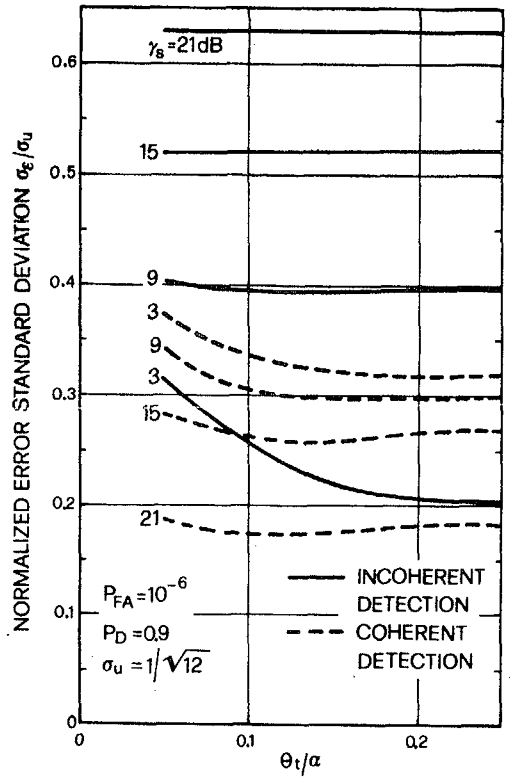

Fig. 6: Standard deviation of the MLE error, normalized with respect to the median estimate

- the MLE of $\hat{\theta}_t$ is biased except for $\theta_t/\alpha=0.25$; the bias level, which is an odd function of θ_t, for $0<\theta_t<0.5\alpha$, is caused by different bias levels in the estimates of γ_1 and γ_2. The bias effect derives from the estimation procedure used; in fact, the estimate is obtained under the condition that the target has been detected;
- the bias level depends on γ_s and $\bar{N}$ for fixed θ_t; the conflicting effects of the decrease in the SNR and of the increase in the average number of observations as θ_t/α approaches zero cause the anomalous behaviour of the curves in Fig. 5;
- the comparison between Fig. 6 and Figs. 1, 2 shows that, for SNR values of practical interest, the error of MLE has a standard deviation slightly lower than the Cramer-Rao bound, because of the bias.

It is of interest to compare the MLE with the median estimate $(\theta_1+\theta_2)/2$ obtained from eq. (23) by setting the correction term equal to zero. The error variance σ_u^2 of the median estimate has been computed by supposing θ_t uniformly distributed in the interval $(0,\alpha)$. The result is shown in Fig. 6: in the case of incoherent detection it is apparent that the improvement attainable by the MLE decreases as γ_s increases. We have no significant inprovement for high SNR's; this means that the correction term in eq. (23) gives little information on target bearing owing to the poor estimates of γ_1 and γ_2.

5. CONCLUSIONS

The angular estimation of radar targets detected by the sequential probability ratio test has been analysed. The algorithm for the maximum likelihood estimation has been derived, assuming a scanning step equal to the half power beamwidth. The results obtained by numerical computation lead to the following main conclusions:

- the maximum likelihood estimate of the target bearing is biased by a low level with-respect to the angular mainbeam aperture. Thus the accuracy of the estimate approaches the Cramer-Rao bound for low signal-to-noise ratios, which are of particular interest in many practical situations;
- an angular estimate with an error standard deviation of the order of one tenth of the antenna aperture may be obtained with a signal processing which makes use of the final values of the statistic of the detection test; thus it may be easily implemented.

AKNOWLEDGMENTS

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