



IMAGE RECONSTRUCTION FROM CODED EDGE DATA

Reconstruction d'images à partir des données codées des contours

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RESUME

Les techniques traditionnelles pour le codage des images posent des problèmes en introduisant des structures fausses et en perdant de l'information. Dans cet article nous présentons une méthode alternative, fondée sur le principe que les propriétés fondamentales, comme les contours, doivent être représentées avec exactitude. Utilisant une combinaison de l'opérateur de Marr-Hildreth et des opérateurs de gradient nous dérivons les contours essentiels. On transmet les contours et l'intensité le long des contours. La reconstruction est obtenue comme la solution d'un problème d'optimisation contrainte: nous cherchons l'image ayant maximum d'égalité et satisfaisant les contraintes sur les contours. Une solution itérative est obtenue. Si tous les contours importants sont dérivés et transmis nos expériences montrent qu'on peut reconstruire les images avec une bonne qualité subjective.

SUMMARY

The problems with traditional image coding methods at high compression ratios like introduction of spurious features or loss of important information motivates a search for alternatives. This paper is a preliminary investigation of an image coding algorithm based on the principles that important features like contours should be coded and reproduced as exactly as possible and no spurious features should be introduced in the image reconstruction process. Using a combination of the Marr-Hildreth operator and gradient operators, we extract the essential contours of the image. Together with the geometrical contour information the image intensity along the contour is transmitted. The reconstructed image is obtained as the solution to a constrained optimisation problem where we seek the maximally smooth image consistent with the given contour information. The optimisation problem is solved iteratively and can be considered as a repeated low-pass filtering applied on the received data. Provided all essential contours are extracted and transmitted, reconstructed images are subjectively of acceptable quality and we believe this is a promising approach to a low bit rate image coding algorithm.



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Coding principles

When trying to increase the compression ratio with traditional image coding methods such as predictive or transform coding, one eventually encounters problems with unacceptable image degradation. Typical degradations are introduction of features like block boundaries in the transform coder and granularity noise in the predictive coder, or the loss of important features like smoothing of edges. These degradations are especially severe considering the fact that the human visual system is very sensitive to edges and contours (1). In designing a high compression ratio image coding algorithm it therefore seems important that the following requirements should be fulfilled:

1. Essential features should be coded and reproduced as exactly as possible.
2. The reconstruction process should not introduce any non-existing features in the image.

Typical essential features are of course edges and contours. A natural first step is therefore to extract the contours in the image and describe these as compactly as possible, i.e. as curves in the plane. Together with the geometrical contour information the image intensity along the contour should be encoded. The idea of paying special attention to the contours in image coding is not a new one (2). The procedure presented here differs from earlier ones however, in the respect that we will not try to reproduce the image intensity like traditional waveform coders. The goal we have in mind is instead just to find a reconstructed image that is compatible with the extracted and transmitted contour information.

Contour extraction

The purpose of the contour extraction is to get a one-dimensional representation of the essential contours in the image. This will make the contour coding efficient and methods from graphics coding like chain coding or function approximation can be used.

The conventional method of contour finding is to apply some gradient operator at each image point and declare an edge where the output of the operator is above some threshold. In order to avoid spurious edges due to noise, the operator should be made sufficiently large. If we want a one-dimensional representation of the contours however, standard gradient operators will not be enough, since they in general give rise to contours that are several pixels wide. For the one-dimensional representation we have to define the position of the contour. For the estimation of the position of the contour one should look for the maximum of the gradient operator since this in general corresponds to the point of steepest change. The maximum of the output of a 1:st derivative gradient operator coincides with a zero crossing of a second derivative operator. This is the idea behind the Marr-Hildreth operator, which detects zero-crossings of the output after the application of the laplacian:

$$\begin{array}{ccc} & -1 & \\ -1 & 4 & -1 \\ & -1 & \end{array}$$

in an image which has been smoothed with a gaussian filter. The size of the smoothing gaussian is a compromise between noise suppression and position accuracy of the contour.

The approach we have adopted is based on a combination of the Marr-Hildreth operator and a standard gradient operator. The flow-chart of fig.1 describes the contour extraction process. Vertical and horizontal zero-crossings of the output of a Marr-Hildreth operator are detected. These zero-crossings constitute potential contour points. A definite contour-point is declared after the application of a horizontal or vertical gradient operator at the potential contour points. The horizontal and vertical gradient operators are shown in fig.2 This process works quite well for most edge orientations but if the edge just slightly deviates from the horizontal or vertical direction there will be vertical and horizontal gaps respectively in the detected contours due to the mismatch between the gradient ope-

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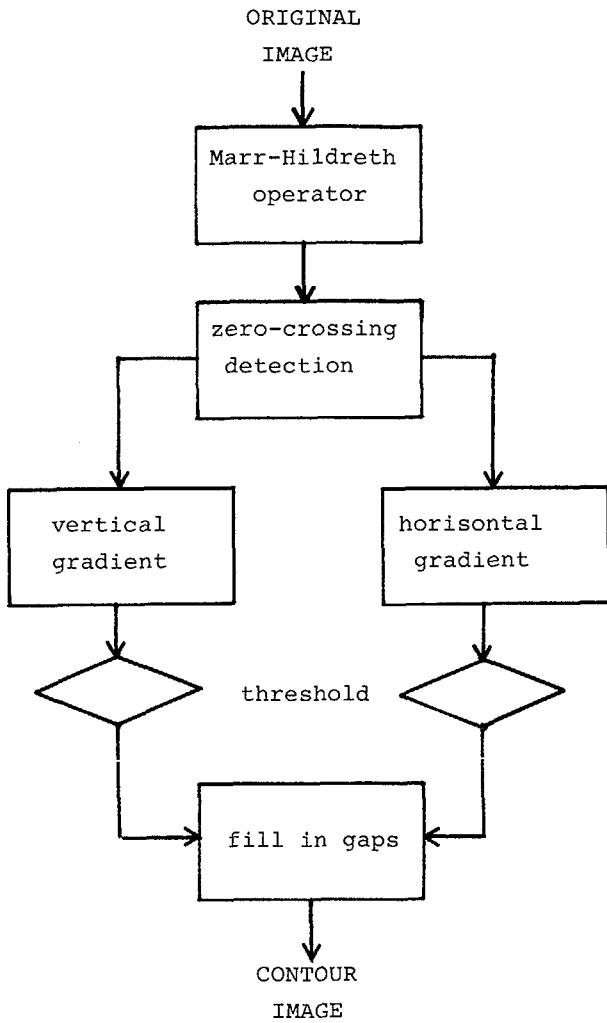
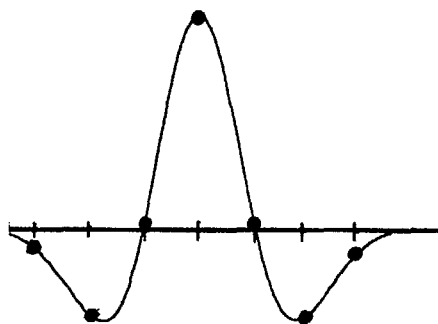


Fig.1 Contour extraction algorithm



1	1	1					
1	1	1		-1	-1	1	1
-1	-1	-1		-1	-1	1	1
-1	-1	-1		-1	-1	1	1
	Vert.				Hor.		

Fig. 2 Cross section of Marr-Hildreth operator and vertical and horizontal gradient operators

rator and edge direction. A final process is therefore added to the contour extraction where we fill in such gaps in order to get more connected contours. Fig.3 shows the result of the contour extraction applied to the central part of the image in fig.5.

After having extracted the contours we insert the intensity values of the original image at pixels on both sides of the contours, fig 4. For efficient coding the redundancy of the intensity values along the contours should be exploited. In this preliminary investigation we will however not use any coding of intensity. For the image reconstruction the geometrical contour information together with the image intensity along the contour will be the only information used.

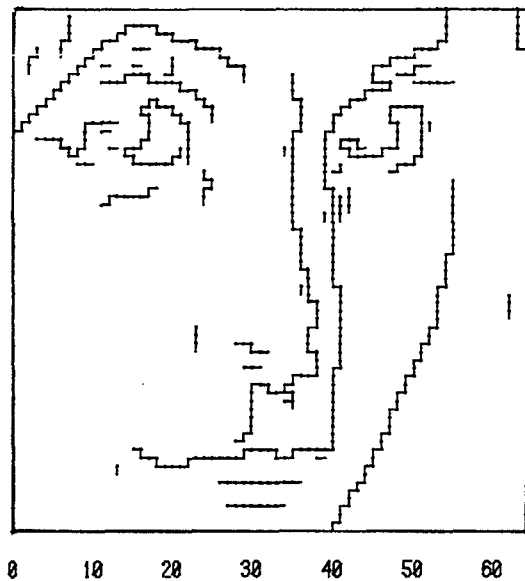


Fig.3 Contours of central part of the image in fig.5

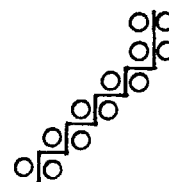


Fig.4 Intensity values along contour



IMAGE RECONSTRUCTION FROM CODED EDGE DATA
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Image reconstruction

Given only geometrical contour information and image intensity along the contours the problem in the receiver is to reconstruct the image as exactly as possible from a subjective point of view, i.e taking account of the properties of the human visual system. The given contour information does of course not uniquely specify the image. There are several images that are compatible with the given contour information. The common property of these images is that they should not contain any contours in the areas between the given contours. In order to find a unique image we will therefore demand that the image intensity between the contours should be maximally smooth, i.e minimising the probability of introducing any non-existing contours in the reconstructed image. The reconstruction problem can then be formulated as a constrained optimisation problem in the following way: Given a local measure $F(f_x, f_y, f_{xy}, \dots)$ of image intensity variation at the image coordinate x, y involving the derivatives of the intensity with respect to the coordinates, the reconstructed image $f(x, y)$ is obtained by minimising the global variation:

$$\min_f \iint F(f_x, f_y, f_{xy}, \dots) dx dy$$

subject to the conditions given about f along the contours. A simple measure of variation is the squared magnitude of the gradient:

$$F = f_x^2 + f_y^2$$

Using this criteria and discretising the problem, i.e replacing the integral with a sum and the derivatives with differences, the problem is transformed to the minimisation of the quadratic form:

$$Q = \sum_m \sum_n (f_{m,n} - f_{m,n-1})^2 + (f_{m,n} - f_{m-1,n})^2$$

where $f_{m,n}$ is the intensity at the discretised coordinate m, n . The minimisation is carried out over all values $f_{m,n}$ except at the contour points where we use the given values of f .

A necessary and sufficient condition for minimum is:

$$\frac{\partial Q}{\partial f_{m,n}} = 0$$

Defining the laplacian:

$$\Delta f_{m,n} = f_{m-1,n} + f_{m,n-1} + f_{m+1,n} + f_{m,n+1} - 4f_{m,n}$$

this implies:

$$\Delta f_{m,n} = 0$$

at all points except the contour points. This is a finite difference equation and in order to solve it we will use the method of successive overrelaxation (4) where the solution f is computed iteratively:

$$f_{m,n}^{(i+1)} = f_{m,n}^{(i)} + \frac{w}{4} \Delta f_{m,n}^{(i)}$$

where $\Delta f_{m,n}^{(i)}$ is defined as:

$$\Delta f_{m,n}^{(i)} = f_{m-1,n}^{(i+1)} + f_{m,n-1}^{(i+1)} + f_{m+1,n}^{(i)} + f_{m,n+1}^{(i)} - 4f_{m,n}^{(i)}$$

w is a relaxation parameter determining the stability and speed of convergence.

The iterations are carried out at all points except the contour points, where we use the given values of f . For the initial approximation $f^{(0)}$ we use the given values at the contour points and 0 at all other points when intensity is in the range $-128, +128$.

Noting that the laplacian is the gradient of Q with respect to f we see that the iterative algorithm above can be considered as a steepest descent algorithm for the minimisation of Q with respect to f , where at each step we move in the direction of the negative gradient.

A central question is of course how many iterations will be needed in order to get an acceptable reconstructed image. In order to study the convergence rate we shall consider the error:



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$$e_{m,n}^{(i)} = f_{m,n}^{(i)} - \hat{f}_{m,n}$$

where $\hat{f}_{m,n}$ is the maximally smooth reconstruction. Using the equation for the iteration we get:

$$e_{m,n}^{(i+1)} = e_{m,n}^{(i)} + \frac{w}{4} \Delta e_{m,n}^{(i)}$$

i.e the same equation as for the intensity. The iterative reconstruction can be considered as a repeated application of a recursive low-pass filter on the given data. An approximate evaluation of the error behaviour in the frequency domain can be obtained by disregarding the fact that the filter is not shift invariant, i.e it is not applied at the contour points. Consider the two-dimensional fourier transform of the error $e^{(i)}$. We then get:

$$\tilde{e}^{(i+1)}(\omega_1, \omega_2) = H(\omega_1, \omega_2) \tilde{e}^{(i)}(\omega_1, \omega_2)$$

where $H(\omega_1, \omega_2)$ is the transfer function of the low-pass filter and can be evaluated to:

$$H(\omega_1, \omega_2) = \frac{4(1-w) + w e^{i\omega_1} + w e^{i\omega_2}}{4 - w e^{-i\omega_1} - w e^{-i\omega_2}}$$

for the error after the k:th iteration we have

$$\tilde{e}^{(k)}(\omega_1, \omega_2) = H^k(\omega_1, \omega_2) \tilde{e}^{(0)}(\omega_1, \omega_2)$$

In fig. 6 is plotted the function $|H(\omega_1, 0)|$ for values of the relaxation parameter $w=1.0$ and 1.8 respectively. We see that the effect of the iterations is primarily to attenuate the high frequency components of the error. Low frequency components are attenuated more slowly. The choice of a larger value of w implies that low frequency components are attenuated somewhat faster at the expense of high frequency error attenuation.

The convergence properties of the reconstruction algorithm are very interesting considering the modulation transfer function of the human visual system (1), which has a low re-

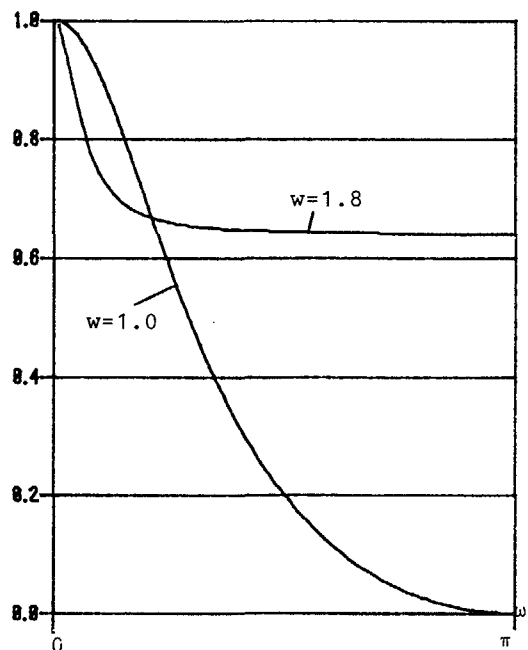


Fig.6. Transfer function $|H(\omega_1, 0)|$

sponse at low spatial frequencies. The effect of the iterations is therefore to reduce the error at the frequencies where the human visual system is most sensitive. Stopping the iterations after a limited number therefore in general gives a subjectively acceptable reconstructed image.

The choice of the first image in the iterations that better approximates the low frequency properties of the original would probably also speed up the reconstruction. For image sequences the natural choice of a start image is the previous reconstructed frame possibly with motion compensation.

Results and conclusions

Computer simulations of the contour extraction and the reconstruction algorithm has been carried out using the picture in fig.5. Contour images with inserted intensity values and reconstructed images for different contour detection thresholds are shown in fig.7. The reconstructed images are the result after 30 iterations with the relaxation parameter $w = 1.8$, which was found to give a reasonable trade off between convergence rate and error suppression.



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From the pictures in fig. 7 it can be seen that the criteria of maximum smoothness gives a subjectively acceptable reconstructed image provided the threshold for the contour extraction is sufficiently low. The effect of a higher threshold is that important contours are not detected and results in diffusion of image intensity across boundaries. The diffusion problem seems however not to be so severe for small gaps in the contours. Isolated contour points tend to result in unacceptable blobs in otherwise homogenous areas, and should probably be deleted.

For high compression ratio coding one would like as few contours as possible. Highly textured areas with rapid intensity variation tend to give unnecessarily many contours. For efficient contour coding the thresholding in the contour detection should therefore not be made on single pixels but on a more global basis considering the strength of the whole contour and the contour density in the surround.

The main purpose of this work has been to investigate to which extent contour information only can be used to represent an image. Judging from the results we believe this is a fruitful approach to a high compression ratio coding algorithm.

References

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Fig.5 Original image 256 x 256 pels

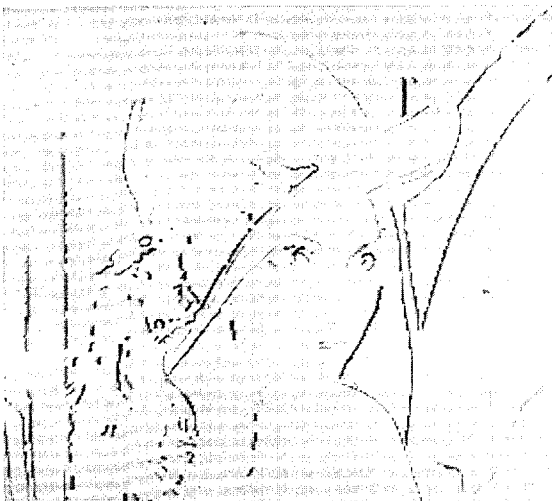


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Fig.7 Contour images with inserted intensity values and reconstructed images for different contour detection thresholds. (30 iterations)