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LANDSCAPE SYNTHESIS BY FRACTAL APPROXIMATION
SYNTHESE DE PAYSAGE PAR APPROXIMATION FRACTALE

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RESUME

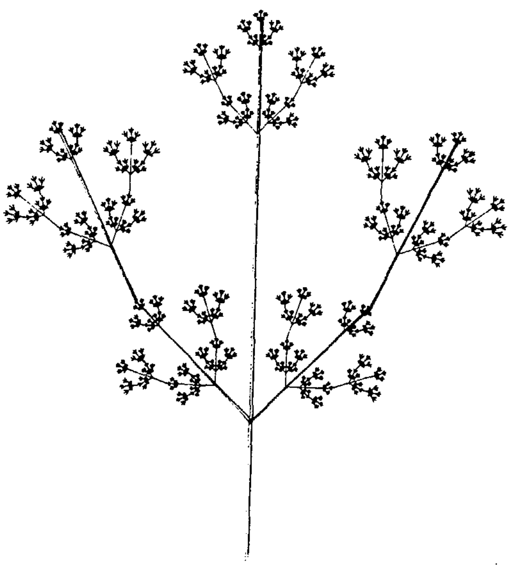
Les fractales ont démontré leur puissance de modelisation depuis leur application par Mandelbrot et Voss pour la génération des images ressemblantes à des paysages naturels. La méthode classique pour la construction des fractales est basée sur la génération de bruit blanc dont la distribution spectrale est modifié par un filtre dont le spectre de puissance est $1/f^k$. La série temporelle ainsi produite a les proprietes fractales fondementelles, l'autosimilarité et l'isotropie, tipiques des phenomenes naturels que nous voulons approximer. Dans cette méthode la possibilité de contrôler la forme globale de la scène construite est très faible parce que la méthode est basée sur le generateur de bruit blanc et sur le parametre k du filtre, qui, à son tour, contrôle la dimension fractale de l'approximation.

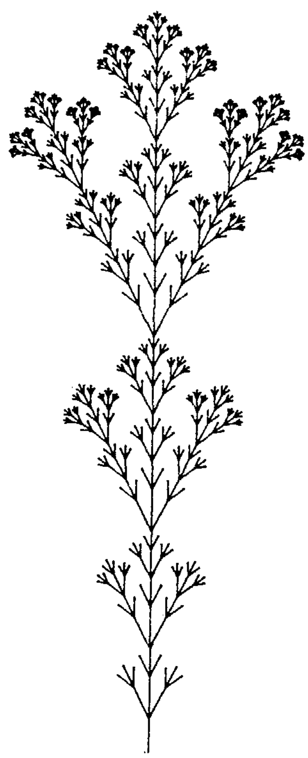
Dans notre travail nous présentons un réalisation de la méthode classique, basée sur FFT; en plus, nous considerons la possibilité de construir un système qui permèt de générer un bruit 1/f dans une façon contrôlée, à fin de synthétiser des paysages qui montrent un aspect particulier.

SUMMARY

Fractal revealed their modelling power since works by Mandelbrot and Voss applied this mathematical tool to the generation of images resembling natural scenes. The classical approach consists into the generation of a white noise whose spectral distribution is modified applying a $1/f^k$ filter, that produces a time series having the basic properties of fractals, self similarity and isotropy, that guarantee the natural appearance of approximation.
In this approach the possibility of controlling the overall shape of the generated scene is very low, since it is based only on the white noise generator and on the chosen exponent for the filter, which in turn, governs the fractal dimension of the approximation.

In the paper we present an implementation of the classical approach, based on FFT, and discuss moreover the possibility of constructing a system that allows to generate 1/f noise in a controlled fashion, in order to synthesize landscapes exhibiting a particular behaviour.





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1. Introduction

Random fractals, introduced by B.B. Mandelbrot in his essays, have revealed their modelling power in many different areas. The generation of surfaces, landscapes and free forms is beeing used in the entertainment industry, as Lucas Film, and as a way of creating artistic drawings with computers.

Besides such pleasing images there are wide potential applications of random fractals: data analysis and presentation, simulation of natural and artificial phenomena are the most interesting areas where this mathematical models can be of great help in research and application.

The study of diffraction in situation where geometrical optics can not be applied (Berry, 1978), or modeling of friction problems in smooth surfaces are examples of technical applications of random fields (Adler, 1981), of which fractals are a particular type. Such kind of structures are characterized by the erraticism of the underlying functions, or the property of frequent, unpredictable variation of their values. Therefore the general approach consists of the determination of the Hausdorff-Besicovitch D dimension of the set of points that constitutes the given fractal phenomenon, a value that characterizes the erraticism of the functions.

As a general methodology, the study of a phenomenon requires the development of a mathematical model, that, when validated, can be used as a knowledge basis from which one can deduce useful properties.

The kind of phenomena in our interest area are the modelling of underground water resources (Crosta-Marini, 1983, Marchioni-Marini-Sola, 1981), and the modelling of landscapes and other natural forms for creative imaging.

As Mandelbrot and other researchers have clearly shown, geomorphology cannot be easily modelled with deterministic approaches or with statistical methods, that do not take into account the self-similarity and erratic properties of such structures. The relationship between fractals and landscapes has been widely described in the works of Mandelbrot, Voss and other researchers, while modelling of underground structures is usually studied adopting classical approaches (at least as we at present know).

The modelling of underground phenomena is based on the knowledge of a small set of data types, deriving from information aquired during well drilling or carroting. In the case of the analysis of pollution problems of underground aquifers, we are faced with larger classes of data, which describe both the status of an aquifer and its contamination level. The types of data are, for instance, the level of the aquifer, the concentration of pollutants in particolar points. The possibility of developing control models for pollution problems in such cases depends essentialy upon the knowledge of the basic parameters governing flow equations (storativity, transmissivity etc.) and hydrodinamic dispersion and transport equation (porosity, dispersivity etc.). The purpose of our research is therefore to devise estimation methods for the determination of such parameters, in order to allow us to perform various kinds of simulations.

Assuming that typical underground parameters can be fractally modelled, we discuss the kind of approximation that can be used in order to estimate local values.

In the present paper, after recalling briefly the mathematical basis of random fractals, we present a system that has been developed in order to construct fractal surfaces. In subsequent paragraphs we discuss the problem of controlling the overall shape of a generated surface, and suggest some techniques that can be adopted in order to control also locally the visual characters of a fractal. Finally, possible solutions to the problem of fractal approximation are discussed.

2. A generator of fractal surfaces.

Many approaches have been proposed for the numerical approximation of fractal surfaces. The criteria that must be taken into account are, first of all, the ability of the method of preserving general properties of fractal surfaces, and, secondly, its efficiency in time and space.

The main properties of a natural landscape (that a fractal model must possess) are self-similarity, the absence of creases and persistency. The first idea is to model these properties by means of brownian motion which allows to simulate some of them, but persistency requires to perform a kind of filtering, consisting of a moving averege of the original function which produces the so called fractional Brownian motion (fBm).

A more precise description of the mathematical aspects of the general approach, recalls that brownian motion can be considered as the realization B(t) of a gaussian stochastic process having zero mean and a covariance given by:

$$R(s,t) = \tfrac{1}{2}\sigma\{||s|| + ||t|| - ||s-t||\}$$

The Hausdorff dimension of the graph of such function is D=3/2, given that $t \in R$. This function has other properties as statistical self-similarity. A first generalization would require to consider a function having arguments on a subset of R^2 , what Mandelbrot calls brownian plane-to-line function. A second generalization consists of the definition of the fBm $B_H(t)$, realization of a stochastic process whose covariance function differs from the former in the following fashion:

$$R_H(s,t) = \tfrac{1}{2}\sigma\{||s||^{2H} + ||t||^{2H} - ||s-t||^{2H}\}$$

where 0<= H < 1. If H=1/2 we have the case of brownian motion. From here on we consider t as a variable defined in a subset of R^2 , therefore producing realization in R^3 ; in such cases the dimension of the graph of the function is 3-H.

The generalization from brownian motion to fractional brownian motion can be performed via a fractional integration yielding therefore a function possessing the desired property of persistency. Imposing persistency on a stochastic process is an operation equivalent to the elimination of high frequency in a long run time series, obtaining smoother results. This kind of filtering is usually performed by means of moving averages, therefore applying a convolution to the given function. As Mandelbrot recalls, this convolution can be performed by the Riemann-Liouville fractional integration, obtaining the fBm:

$$B_H(t) = [\Gamma(H+1/2)]^{-1} \int_{-\infty}^{t} (t-s)^{H-1/2}\, dB(s)$$

The integral does not converge, but the increments $B_H(t) - B_H(0)$ converge.

The theoretical description of the process of fractal synthesis must be revised in order to find an approximation method, necessary for numerical calculation with a computer.

Mandelbrot (1969, 1971) proposed various approaches to the approximation of fBm; an evaluation based on computational complexity criteria shows that the best methods require O(NlogN) operations. An analysis of the properties of fBm show that power spectrum of this kind of processes is ruled by f^{-K} , that guarantees statistical self-similarity. A solution to the numerical approximation of fBm therefore can be obtained trying to construct from a white noise a function having f^{-K} power spectrum. This basic idea has been developed by Voss (1983), and has been adopted in our system.

Normally for processes like fBm the power spectrum is not defined, given their non stationarity; anyway one can generalize classical Fourier analysis to this kind of processes considering only finite intervals, and considering only frequency values strictly larger then zero.

The process of fractal synthesis therefore can be thought as a linear system, whose input is a white noise, the transfer function has the form $f^{-\beta}$, and the output is a time series possessing the desired power spectrum f^{-K} . The construction of the fractal generator then proceeds applying fast discrete Fourier transform to a white noise $w(t_n)$, where $t_n = n\Delta t$, and n = 0,..,N-1. Discrete FFT produces the spectrum $W(k_n)$ with $k_n = n/(N\Delta t)$, that is multiplied by the transfer function

$$T(k_n) = 1/(k_n^{H+1/2}).$$

This transfer function is defined for n > 0; in the origin its value is arbitrary chosen, giving an offset to the values of the function $z(t_n)$ generated through inverse discrete FFT.

We underline that the input is a real function whose spectrum is therefore Hermitian:

$$W^*(-k_n) = W(k_n).$$

Since also the output function must posses the property of beeing real, in order to preserve the hermitianity of the ouput function spectrum the transfer function must be even.

This method has been generalized to the case of a white noise surface, $w(\underline{x})$, $\underline{x} \in R$; in order to preserve isotropy of the output function z(x), the transfer function must depend from the modulus k of spatial frequency k. The transfer function is:

$$T(k) = 1/k^{H+1}.$$

3. Approximation and interpolation problems

The interest in the interpolation and approximation, as we said before, is based on the possibility of using fractals as a modelling tool for many different processes and phenomena. The model to be constructed must be "close" to the original (partly) unknown system, therefore we need tools that allow us to control the generation of fractal surfaces, in such a way that they conform to a desired behaviour.

The existing and experimented control parameters of a fractal surface are: the seed used for the generation of the white noise surface, the exponent H of the transfer function, a possibly multiplicative coefficient c of the same transfer function, and the value to be chosen for the simulation of sea level. A different seed (and, of course, different pseudorandom number generator) produces different surfaces in an unpredictable way, therefore this control parameter is useful for creative purposes only. The effect of the exponent H can be observed in many examples, and consists of a modification of the Hausdorff-Besicovitch dimension D of the fractal, producing therefore surfaces that show higher erratic behaviour at larger values of D; D and H are related by: D = 3 - H, therefore smaller H's produce erratic surfaces, larger H's give rise to surfaces resembling natural ones.

In fig. 1 and 2 examples are given of the degenerate cases where H = 1 and H = 0 respectively. In fig. 3 it is shown a case obtained modifying the same white noise as in fig.1 but rescaling the filtered values z by a factor 5. This corresponds to a value 5 assigned to the third parameter c in the transfer function. The meaning of this parameter can be understood if we think at the surface as a view of a mountain considered in a square area whose side is ten times the maximum height; the aspect of the generated image depends of course from chosen resolution. In our examples we generate functions so defined:

$$z(x,y) : [0,1]x[0,1] \rightarrow [0,0.1].$$

In fig. 5 to 8 there is a sequence of fractal surfaces differing only in the value of the sea level. This choice can be made after performing the Fourier transform, but the resulting function preserves desired spectral properties above the sea. This observation does not contradict intuition, since this approach intends to model different phenomena: terrain and sea. More accurately one should determine the fractal dimension of sea surface model, that should be larger then that of terrain.

As one can see, the controllability of a fractal surface is very poor, leaving large space to randomness. In order to reduce randomness one can choose different approaches, but at present none seems reliable and capable to preserve the general properties of a fBm.

Fournier, Fussel and Carpenter (1982) proposed an approach motivated mainly by computational complexity reasons, that could be used to interpolate fractally a given set of points. Unfortunately their method cannot be considered a fractal generator, due to the lack of self-similarity and other properties (cfr. Mandelbrot 1982)

The approximation and interpolation problem can be stated in the following way: given a finite set of points $\{t_i\}$ randomly distributed in a real domain Ω, they can be considered the values assumed by a real function z(t). The problem is to find a function f(t) passing through the points $\{t_i\}$ (interpolation) or such that a minimum criterium is satisfied (approximation), and possessing all the properties of a fractal function.

The advantage of considering fractal functions as interpolator or approximator can be shown by the following quotation from Matheron (1969):

> "Nous nous proposons,.., de formuler en terme de fonctions aleatoires non stationnaires le probleme de l'estimation de derives, ... ajuster un polynome par un méthode de moindre carres, donnent un sentiment de malaises; on a l'impression, en effet, de faire violence a la nature, en lui imposant de force une expression polynomiale, qui n'a aucune raison a priori de presenter le moindre rapport avec la structure réelle du phenomene que l'on veut representer."

An idea for the solution of the interpolation problem can be that of superposing on a given process (the data to be interpolated) a f^{-k} noise. In order to do that we need a mathematical description of the noise suitable for superposition with other processes. The best candidate to this task seems Weierstrass function that unfortunately has numerous drawbacks. In particular, beeing its components non orthogonal, it is necessary more research in order to find a suitable version for our purposes.

4. Conclusion

The approximation of natural structures requires us to accept their intrinsic non stationary character. More research is required in order to find a satisfying solution to the problem, the possibility of controlling in a pointwise fashion the generation of fractals remaining an open problem. At the moment fractional Brownian motion are limited in their applicability as emulators of real phenomena, because there is no immediate possibility of imposing them a trend.

Anyway fractals remain an extraordinary tool for representing and illustrating natural phenomena: not only fBm but also recursive Koch function, that allow to emulate even bothanical shapes.

As a concluding remark, some notes on the implementation. We used an HP 9836 graphic system with a monochromatic display of 512x390 pixels; this low resolution did not prevent us from experimenting different visualization techniques of fractal surfaces, as that shown in fig.4, where the shaded effect had been obtained operating on a grid of 4x4 pixels for each point of the generated surface.

We implemented also a system which allows to define freely generalized Koch curves, like those shown in the first page of our paper.

Aknowledgements

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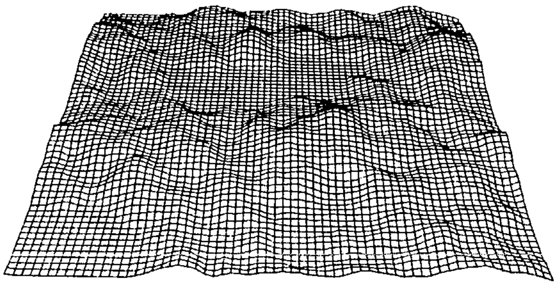

Fig. 1

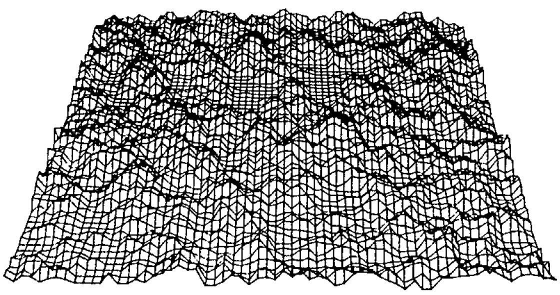

Fig. 2

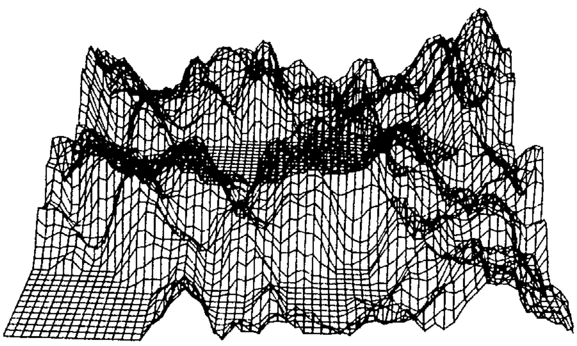

Fig. 3

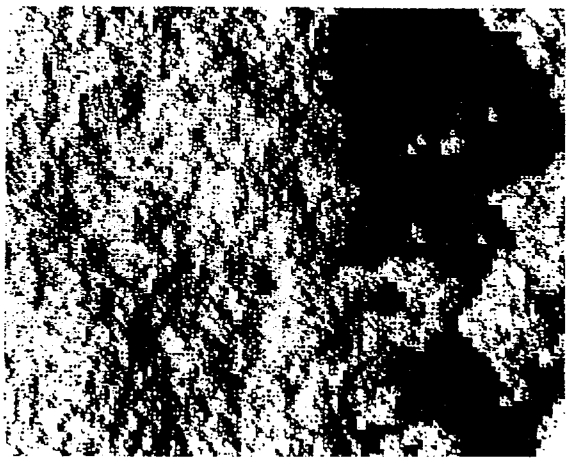

Fig. 4

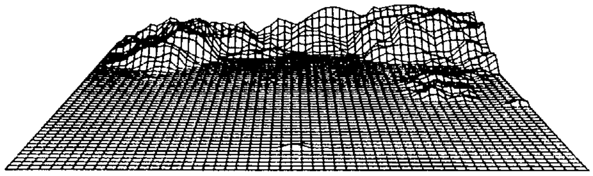

Fig. 5

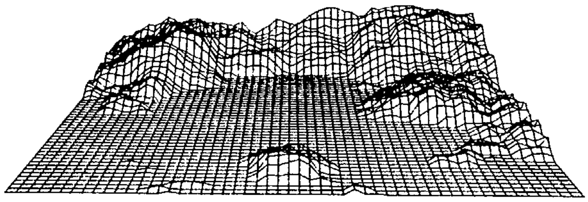

Fig. 6

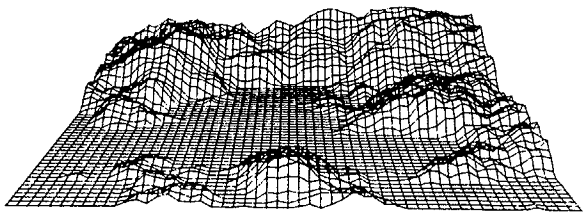

Fig. 7

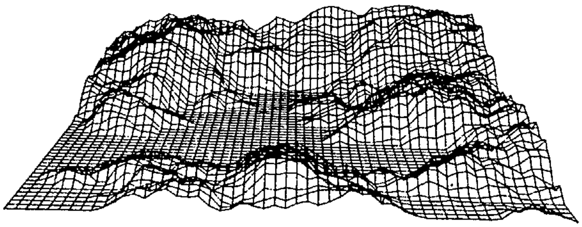

Fig. 8