



# Traitement, Synthèse, Technologie et Applications

BIARRITZ – Mai 1984 –

## APPLICATIONS DES PROCESSEURS OPTIQUES MATRICIELS AU TRAITEMENT D'IMAGES

### APPLICATIONS OF OPTICAL MATRIX PROCESSORS IN IMAGE PROCESSING

P. Das\*, John Guilford and R. M. Payton†

Electrical, Computer, and Systems Engineering Department, Rensselaer Polytechnic Institute, Troy, NY 12181

#### RESUME

Les matrices de vecteurs et la multiplication des matrices de dimensions larges peuvent être accomplies en utilisant un array d'ampoule et un acoustique-optique appareillage ou une matrice de transmission programmable comme le Light-MOD de Litton. Puisque les processeurs optique/matric (OMP) se contiennent un nombre élevé des opérations parallèles, les transformations des matrices à temps réel est possible. Consommation d'information dans la transformateur n'exige pas de digitalisation, de cette manière, la largeur des bandes de signaux sont plus facile qu'à diriger. Pour tant, les avantages de systèmes des transformations de temps discret sont tenables. Les applications de la OMP en discussion dans la littérature inclut: la décomposition des valeurs singulières, la détermination des valeurs "eigen", les solutions des matrices d'équations linéaires, les solutions des équations différentielles et le filtre de Kalman.

L'objet de cette papiers est d'examiner les applications de la OMP avec les problèmes de la restauration de l'image à temps réel et transmission des signaux. Spécifiquement, la utilisation de la OMP pour la restauration des images ultrasonore dégradé par les lentilles ultrasonique est examiner. La OMP peut aussi être utiliser pour améliorer la résolution de la fréquence de transformation "Fourier" à temps réel en utilisant l'algorithme de chirp.

#### SUMMARY

Matrix-vector and matrix-matrix multiplication of large dimensions can be performed using an array of light sources and an acousto-optic device or a programmable transmission matrix such as Litton's Light-MOD. Since Optical Matrix Processors (OMP) feature a high number of parallel operations, real time matrix processing is possible. Inputs to the processor do not require digitization so high bandwidth signals are easier to handle. However, the advantages of discrete time processing are maintained. Applications of the OMP discussed in the literature include: singular value decomposition; eigenvalue determination, solution of linear matrix equations, solution of differential equations, and Kalman filtering.

The purpose of this paper is to discuss application of the OMP to the real-time image restoration problem and signal processing. Specifically, the use of the OMP for restoration of ultrasonic images degraded by finite ultrasonic lens spot size is considered. OMP can also be used to improve the frequency resolution of a real time Fourier transformer using chirp algorithm.

\*Partially supported by NSF and ARO.

†Partially supported by Polaroid Corporation.



## 1. INTRODUCTION

Although all signals from a physical process are analog, for signal processing purposes it can be represented as analog, discrete or digital signals. For the case of discrete time signals, the time axis is discrete but the values at discrete times are analog. For the case of imaging, and multidimensional signals, the spatial coordinates are discrete. If the discrete signals are represented as a column matrix, the processing of signals can be identified as different matrix-operations, such as multiplication, inversion, etc. In this format of signal representation matrix-processors become synonymous with signal processors. Matrix-Processors can be implemented as digital array processing or analog matrix-processing. Recently there has been enormous activity in both the areas with significant improvements in the performance. For the digital case, the innovation of systolic array architecture [1] is a real breakthrough. For the analog case, the optical matrix-processor (OMP) either using acousto-optic interaction [2] or the programmable two-dimensional mask [3] promises to be capable of handling 1000 x 1000 matrix in parallel. For the digital processors one needs a fast highly accurate A/D converter for processing images. Excluding this disadvantage, the digital processors are well suited for image processing and have been extensively discussed in literature [4]. In this paper, we shall confine ourselves to the optical matrix processors which will be reviewed in the next section. The main objective of the paper is to apply this OMP for the restoration of ultrasonic images blurred by the finite aperture of the ultrasonic lenses used to focus in an ultrasonic transmission imaging system [5] or an acoustic microscope [6,7]. An architecture of how the experiment will be performed is proposed. Another objective of the paper is to use the same matrix processors to implement a "super-resolution" real time Fourier transformer using surface acoustic wave (SAW) chirp devices and chirp transform algorithm. This concept has recently been proposed [8] and our objective is to show a possible implementation.

## 2. OPTICAL MATRIX PROCESSORS

For signal processing purposes using matrix-processors, we shall be dealing with the following matrix equation.

$$[G] = [H] [F] \quad (1)$$

Here  $[G]$  is a  $[n \times 1]$  matrix,  $H$  is a  $[n \times n]$  matrix and  $[F]$  is a  $[n \times 1]$  matrix. Often  $[G]$  and  $[F]$  will be identified as input and output of a system having the system represented by  $[H]$  matrix. In many applications of practical importance the system is linear and invariant. For this case  $[H]$  becomes a Toeplitz matrix. Also is the impulse response is finite,  $[H]$  is banded. The basic matrix processor should be able to perform the right hand side of eqn. (1). For the case of OMP this can be done in the two following ways: a) using acousto-optic interaction or b) using programmable mask.

There are many ways one can implement OMP using acousto-optic interaction. For our purpose, the suitable one is shown in fig. 1. It consists of a single light source whose output can be amplitude modulated in time. For example, an acousto-optic or electro-optic modulator can be used for this purpose or, if a junction laser is used, it can be directly modulated by modulating its current. Using the lens  $L_1$ , the output from the source illuminates an ultrasonic delay line (either bulk or SAW).

The delay time,  $T$ , of the delay line is given by

$$T = \frac{\ell}{v}$$

where  $\ell$  is the length of the delay line over which the acousto-optic interaction takes place and  $v$  is the velocity of the ultrasound in the delay line medium. If the bandwidth of the transducer is  $\Delta f$ , then the time bandwidth product (TB) is given by

$$TB = T \Delta f .$$

Note the typical values of  $T \sim 10 \mu\text{sec}$ ,  $\Delta f \sim 100 \text{ MHz}$  and  $TB \sim 1000$ , although  $\Delta f \sim 1 \text{ GHz}$  and  $TB \sim 10,000$  has been reported.

The diffracted light from the delay line is focussed by  $L_2$  on the linear array of photo-detectors or photo-sensitive charge coupled devices (CCD's) placed at the focal plane of  $L_2$ . The number of elements in the linear array is equal to the TB product. To perform the matrix operation, at one instance, one complete row of  $H$  matrix is frequency multiplexed and used as input to the delay line transducer. At the same time the appropriate  $[F]$  element is used to modulate the light source. The product elements like  $H_{ij} F_j$  are collected at the  $i$ th photo-detector. The process is repeated until all elements of  $[F]$  and all the rows of  $[H]$  are used up while the photo-detector integrates the output light. At the end of this operation, the output  $[G]$  can be obtained from the photo-detector outputs in parallel or serially if a CCD is used. For example, if  $T = 10 \mu\text{sec}$ ,  $\Delta f = 100 \text{ MHz} = 400\text{-}300 \text{ MHz}$  (250 MHz being the center frequency of the delay line) we need a 1000 element photo-detector linear array. The OMP can process 1000 x 1000 matrix in  $(1000 \times 10 \mu\text{sec}) = 10 \text{ msec}$  which is also the integration time for the detectors. The laser is modulated with  $[F]$  each  $F_j$  lasting for 10  $\mu\text{sec}$ .  $H_{ij}$  elements are used to modulate the center frequency  $f_j$  where  $f_j = 300 \text{ MHz} + i \times 1000 \text{ KHz}$ ,  $i = 1, \dots, 1000$ .

The second implementation of OMP is shown in figure 2 where the modulated light source is identical to that in fig. 1. However, for this case the ultrasonic delay line is replaced by a mask whose transmission coefficient can be changed. Again the output is obtained from the detectors after integration as shown in fig. 2a. The lens  $L_2$  after the mask images the mask on the photodetector array. Note that if a two-dimensional mask is available and an array of input laser sources can replace the single input light source then one can perform the whole matrix operation in parallel and in one iteration. For this case no integration of the output photo-detectors are needed as the integration is performed by the cylindrical lens  $L_2$  situated after the mask as shown in fig. 2(b). The lens system  $L_1$ , images the laser or light emitting diode (LED) array vertically while spreading the light from a single source horizontally. Lens combination  $L_2$  collects all light from a given column and focuses it on one element of the photodetector array. Note that for the OMP of fig. 1 and fig. 2(a) the  $[F]$  matrix is fed serially, whereas, it is fed in parallel for the implementation in fig. 2(b). The programmable masks are commercially available. The "Light-MOD" [9] for example, manufactured by Litton, is suitable for our purpose. It is electrically alterable, high speed, two dimensional, spatial light modulator and uses Bismuth substituted transparent iron garnet film grown on a non-magnetic substrate. Its typical characteristics are 512 x 512 elements where each line or pixel can be switched in approximately 1  $\mu\text{sec}$ .

We have focussed our attention on two particular OMP's which we believe are relevant to the application at hand. However, OMP is a subject of very strong interest and it has been applied for eigenvalue determination [10], singular value decomposition, solution of linear matrix equations [11], solution of differential equations [12], and Kalman filtering [2]. It has also been used as a systolic array processor [13].

3. APPLICATION OF OMP IN "SUPER RESOLUTION" REALTIME FOURIER TRANSFORMER

Realtime Fourier transformation of signals  $f(t)$  can be performed using the following chirp algorithm [14].

$$F(\omega = \frac{\beta t}{\pi}) = \left\{ f(t) e^{j\beta t^2} * e^{-j\beta t^2} e^{j\beta t^2} \right\} \quad (2)$$

where  $\beta$  is the chirp rate and  $F(\omega)$  denotes Fourier transform of  $f(t)$ . The symbol  $*$  denotes convolution. The SAW chirp devices have a limited time-bandwidth product. For example, the device might operate from 150 MHz to 250 MHz in 40  $\mu$ sec delay giving a TB product of 4000. Because of the finite TB product, the output Fourier transform can only be resolved approximately to (TB) number of frequencies. The ideal impulse response  $h(t,t')$  of the Fourier transformer if implemented using eqn. (2) is given by

$$h(t,t') = e^{j \frac{\beta t}{\pi} t'} \quad (3)$$

Because of the finite duration of the chirp waveforms, however, the impulse response of the Fourier transformer as denoted in Fig. 3 is given by

$$h(t,t') = e^{j \frac{\beta t}{\pi} t'} \times \text{rect} \left( \frac{T}{2} \right) \quad (4)$$

where  $\text{rect} \left( \frac{T}{2} \right)$  denotes the rectangular function of duration  $T$  centered at the origin. Then the output of the actual Fourier transformer for any input  $f(t)$  is given by

$$g'(t) = \left\{ \frac{\sin \frac{\beta t T}{\pi}}{\frac{\beta t T}{\pi}} \right\} F(\omega = \frac{\beta t}{\pi}) \quad (5)$$

$$\text{or } g'(t) = \int \frac{\sin \beta(t-t')T/\pi}{(t-t')T/\pi} F(\frac{\beta t'}{\pi}) dt' \quad (6)$$

Equation (6) can be written for our purpose in terms of a matrix equation given by

$$G' = H' F \quad (7)$$

Here  $G'$  is the corrupted result written in terms of a column vector and  $F$  the desired value.  $H'$  for this is a Toeplitz matrix. It is also banded because although theoretically sinc function goes from  $-\infty$  to  $+\infty$ , we shall consider it zero whenever its value goes below the noise floor. To obtain  $F$  from eqn. (7) we need to invert  $H'$  and use

$$F = H G' \quad (8)$$

where  $(H')^{-1} = H \quad (9)$

It is well-known that eqn. (9) is ill-conditioned especially in the presence of noise. However, it is also known that using methods such as singular value decomposition [15] and linear programming [8] with known constraints on the elements of  $H'$ ,  $H$  can

be obtained. Note that for a particular realtime Fourier transformer this matrix inversion has to be performed only once. Thus in actual implementation, although OMP can be used for this matrix inversion, it is better to use the conventional numerical technique using digital computers. For this particular case, OMP's discussed in sec. 2 become directly applicable. For the acousto-optic OMP the  $H$ -matrix is known previously and is generated electronically by frequency multiplexing in sequence row after row. The output of the Fourier transformer can then be directly applied to the light modulator. Let us consider a simple example where the output of the Fourier transformer can be resolved to 10 points only if no OMP is used. For this device  $\Delta f = 1$  MHz and  $T = 10$   $\mu$ sec. If we sample the output for 100 points at an interval of 0.1  $\mu$ sec, we get 100 values for  $G$ . Using actual experimental values or numerically we generate  $H'$  and then  $H$ . We note that for using an acousto-optic OMP, with  $T = 0.1$   $\mu$ sec we need  $\Delta f = 1000$  MHz which might require quite an expensive delay line. However, a better solution might be to change the OMP to one with  $T = 10$   $\mu$ sec and  $\Delta f = 10$  MHz. For this case, to slow down the sample values,  $G$ , one can use a 100 element CCD whose clock rate is switched by a factor 100 once the  $G$  values are loaded into the CCD. This implementation is shown in fig. 4. An implementation for parallel processing is shown in fig. 5. Note that for this case, the mask is more or less constant and needs to be changed only when the noise characteristics change significantly. Note that for this implementation, one needs to use a serial in-parallel out CCD.

In some applications, where the noise characteristics are changing frequently one can use the iterative scheme of matrix inversion discussed in reference 11.

It is to be noted that, in the example used, the use of OMP has enabled one to use very low TB chirp devices to obtain ten times better resolution.

4. APPLICATION OF OMP TO ACOUSTIC IMAGING

Ultrasonic imaging is of great interest because it can image through materials where X-ray fails or is not desirable. Many forms of acoustic imaging systems and acoustic microscope exist [5-7]. However we shall concentrate on the focussed transducer transmission imaging system shown in fig. 6. The procedure discussed here is applicable to other systems also if proper modifications are made. The focussing lenses have a finite aperture of radius " $r_0$ ". For uniform ultrasound of wavelength  $\lambda$  incident on the lens of focal length  $f$ , the radiation pattern at the focal plane is given by

$$u(x,y,f) \propto \left\{ \frac{J_1 \left( \frac{2\pi r_0 r}{\lambda f} \right)}{\frac{2\pi r_0 r}{\lambda f}} \right\} \quad (10)$$

where  $r = (x^2 + y^2)^{1/2}$  and  $J_1$  is the Bessel function of the first kind. If the actual transmission function of the object, is given by  $f(x,y)$ , the corrupted image,  $g(x,y)$  obtained will be given by

$$g'(x,y) = u(x,y,f) * f(x,y) \quad (11)$$

If eqn. (11) is converted to matrix equation like the one given by eqn. (8), we note that for a 256 point x 256 point image the dimension of the  $G$  and  $F$  matrix is 65576 x 1 and  $H'$  matrix is given by a (65576 x 65576) matrix. This is a formidable task for an OMP although it is known that most of the elements of  $H'$  matrix are



## APPLICATIONS DES PROCESSEURS OPTIQUES MARTRICIELS AU TRAITEMENT D'IMAGES

## APPLICATIONS OF OPTICAL MATRIX PROCESSORS IN IMAGE PROCESSING

P. Das, John Guilford and R. M. Payton

zero. To make the problem manageable, we note that although Bessel function theoretically extends from  $-\infty$  to  $+\infty$  in  $r = (x^2 + y^2)^{1/2}$ , due to noise and limitation of the amplifier at the output transducer, one should only consider values up to a finite radius, as shown in fig. 7 for a typical case. Thus we note that, as shown in fig. 7, for the object point P at the center of the circle only the points within the circle interact with it in eqn. (10) through non-zero elements of the H matrix. Note that the radius of the circle is dependent on  $(\frac{\lambda f}{r})$  and the number of sidelobes of eqn. (10) one should include before it is swamped by noise. Thus if we can relabel our G values in the matrix using line delays and element delays as shown in fig. 8, the matrix equation applicable for this case is given by

$$G = H' F \quad (12)$$

where the matrices  $\hat{G}$ , F and  $H'$  are of much smaller dimensions. Again for a particular system,  $H'$  is in general fixed and it can be inverted numerically using stable algorithms. Note that in an actual case, if  $H'$  is measured using a 'point' object, it can include the effect of defocussing and noise also.

Once  $H = (H')^{-1}$  is obtained one can use the OMP's as discussed in sec. 3. A typical case is illustrated in fig. 9.

## 5. DISCUSSION

OMP's have enormous potential in different fields of signal processing where a large array of data is to be manipulated. Its major strength is the parallel processing it can perform using analog signals. However, there are many questions that need to be answered before it becomes viable. For example, for a 1000 element case, the dynamic range of the detectors has to be at least 60 db or more. The electronics involved with matrices of large dimensions can be quite complex and should be given careful consideration. For the image restoration case, eventually the output of OMP must be in a format so that it can be displayed on a conventional electronic display. Digital processing of signals is quite expensive. However, with the coming of the systolic array processing, dramatic improvement in the cost factor is expected.

One important topic we have not considered in depth is that if the noise in measurement system is quite large, then we may not be able to obtain the optimum [F] without performing the iterative solution of the following equation

$$G = H F + N$$

where N represents noise. The linear programming method [8] proposed for this problem can be directly implemented using OMP and electronic feedback to perform iterative solution [11]. However these methods are more complex than the ones discussed here. Initially our goal is to experimentally implement these simpler ones and study their performance.

## REFERENCES

- [1] H. T. Kung, "Why Systolic Structures?", Computer, Vol. 15, pp. 37-46, 1982.
- [2] D. Casasent, J. Jackson and C. Neuman, "Frequency Multiplexed and Pipelined Iterative Optical Systolic Array Processors", Applied Optics, Vol. 22, pp. 115-124, 1983.
- [3] J. W. Goodman, A. R. Dias and L. M. Woody, "Fully Parallel, High Speed Incoherent Optical Method for Performing Discrete Fourier Transforms," Optical Letters, Vol. 2, pp. 1-3, 1978.
- [4] Per-Erik Danielsson and Stefano Levialdi, "Computer Architectures for Pictorial Information Systems", Computer, Vol. 14, pp. 53-67, 1981.
- [5] P. Das and R. Werner, "Digital Enhancement of Ultrasonic Images and Its Application to Non-Destructive Testing of Composite Materials", Acoustic Imaging, Vol. 11, Edited by John P. Powers, Plenum, pp. 263-276, 1982.
- [6] R. A. Lemons and C. F. Quate, "Acoustic Microscopy Scanning Version", Appl. Phys. Letters, Vol. 24, pp. 163-165, 1974.
- [7] L. W. Kessler and D. E. Yuhas, "Acoustic Microscopy - A Tutorial Review", Acoustic Imaging, Vol. 9, Edited by K. W. Wang, pp. 275-299, 1979.
- [8] R. J. Mammone, O. McKee and D. Schilling, "Frequency Resolution Enhancement of a Compressive Receiver by Spectral Estimation", Proceedings of 1983 IEEE Military Communications Conference, IEEE Publication No. 83 CH1909-1, pp. 713-719, 1983.
- [9] W. E. Ross, K. M. Snapp and R. H. Anderson, "Fundamental Characteristics of the Litton Iron Garnet Magneto-Optic Spatial Light Modulator", SPIE Proceedings, Vol. 388, pp. 1-10, 1983.
- [10] H. J. Caulfield, D. Dvon, J. W. Goodman and W. Rhodes, "Eigenvector Determination by Noncoherent Optical Methods", Applied Optics, Vol. 20, pp. 2263-2265, 1981.
- [11] D. Casasent, "Acousto-Optic Transducers in Iterative Vector-Matrix Processors", Applied Optics, Vol. 21, pp. 1859-1865, 1982.
- [12] J. W. Goodman and M. S. Song, "Performance Limitations of an Analog Method for Solving Simultaneous Linear Equations", Applied Optics, Vol. 21, pp. 502-506, 1982.
- [13] H. J. Caulfield, W. T. Rhodes, M. J. Foster and S. Horvitz, "Optical Implementation of Systolic Array Processing", Optics Communications, Vol. 40, pp. 86-90, 1981.
- [14] D. R. Arsenault and P. Das, "SAW Fresnel Transform Devices and Their Applications", IEEE Ultrasonics Symposium Proceedings, IEEE Publication No. 77 CH1264-15U, pp. 969-973, 1978.
- [15] W. K. Pratt, "Digital Image Processing", Wiley, p. 402, 1978.



APPLICATIONS DES PROCESSEURS OPTIQUES MATRICIELS AU TRAITEMENT D'IMAGES  
 APPLICATIONS OF OPTICAL MATRIX PROCESSORS IN IMAGE PROCESSING  
 P. Das, John Guilford and R. M. Payton

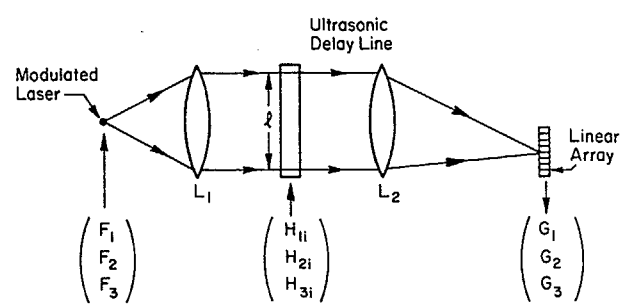


Fig. 1 Optical Matrix Processor using acousto-optic interaction. All the elements of a single row (e.g.  $H_{1j}$ ) are fed to the transducer simultaneously using frequency multiplexing. Output,  $G_j$  is obtained after integration.

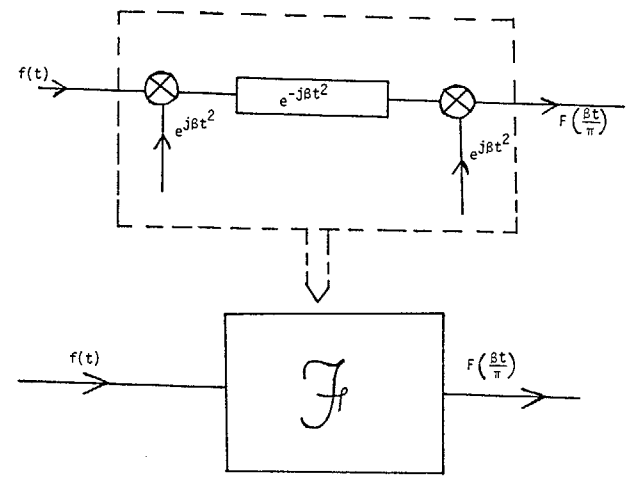
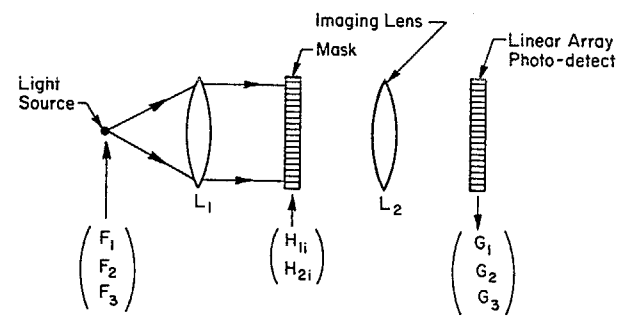
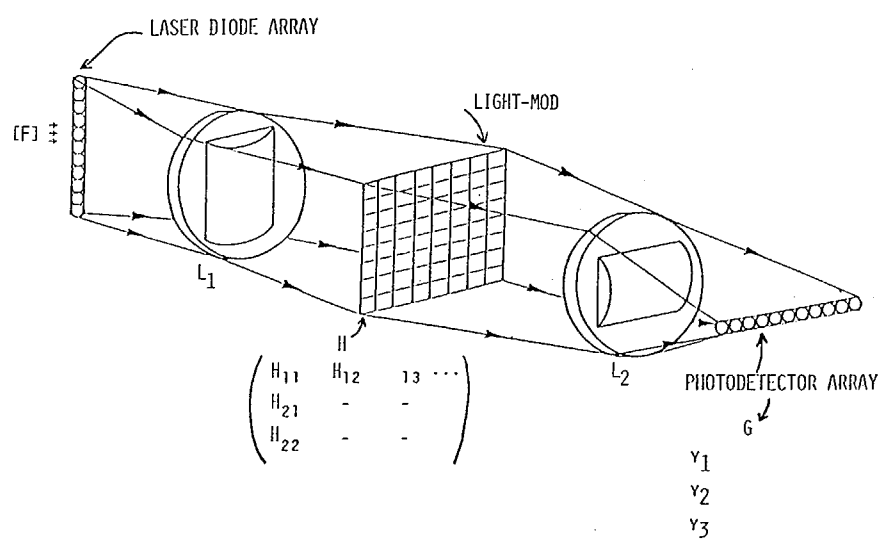


Fig. 3 The chirp transform algorithm to obtain real-time Fourier Transformation.



(a)



(b)

Fig. 2 Optical matrix processor using programmable masks.  
 a) Programmable mask with linear array elements and integration of photodetectors are needed.  
 b) A two-dimensional fixed mask and no detector integration is needed for this implementation.



APPLICATIONS DES PROCESSEURS OPTIQUES MARTRICIELS AU TRAITEMENT D'IMAGES  
 APPLICATIONS OF OPTICAL MATRIX PROCESSORS IN IMAGE PROCESSING  
 P. Das, John Guilford and R. M. Payton

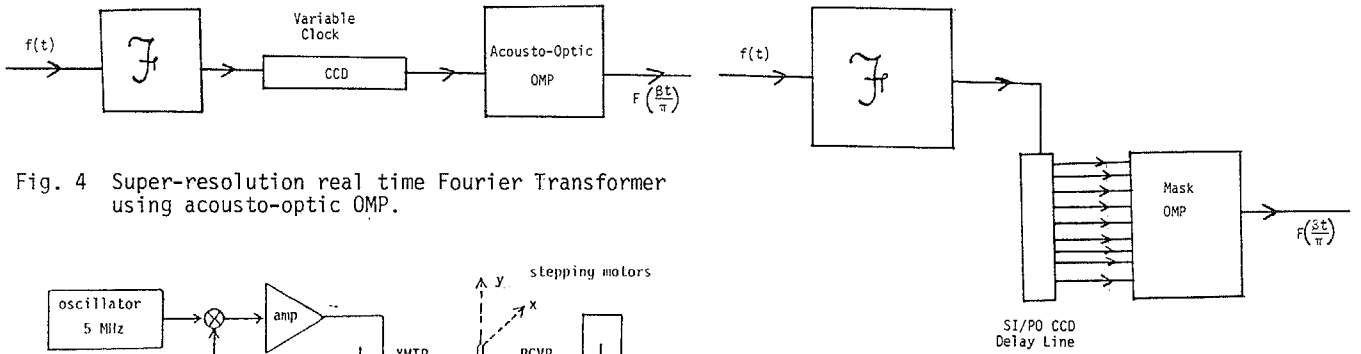


Fig. 4 Super-resolution real time Fourier Transformer using acousto-optic OMP.

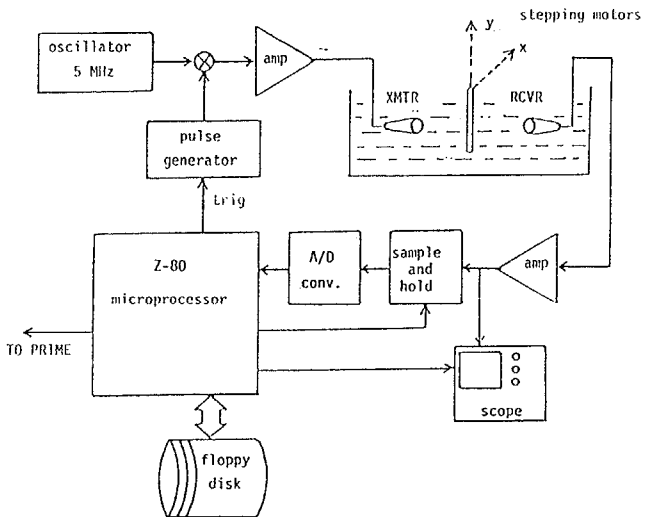


Fig. 6 Ultrasonic transmission imaging system using focussed transducer [5].

Fig. 5 Super resolution real time Fourier Transformer using Mask OMP.

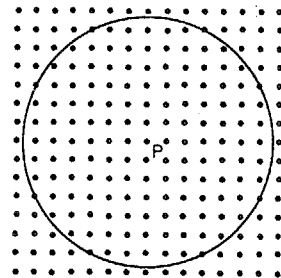


Fig. 7 Figure showing the elements which interact with the element P through eqn. (10).

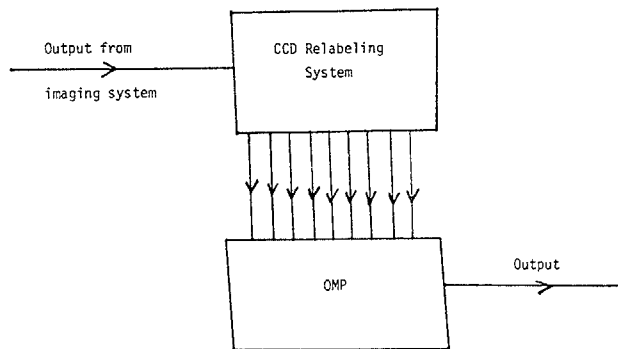


Fig. 9 A block diagram for the image restoration scheme using OMP.

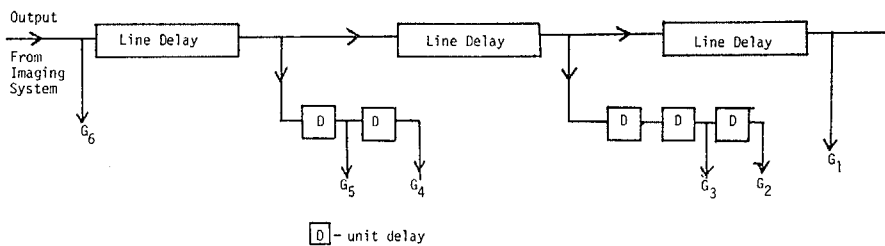


Fig. 8 An illustration of relabeling scheme for the G vector.