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LES FILTRES DYADIQUES-SÉQUENTIELLES EN TRAITEMENT D'IMAGES

DYADIC SEQUENCY FILTERS IN IMAGE PROCESSING

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RESUME

Application, d'autres transformations que la transformation de Fourier,pour le traitement d'images ressortit de l'exigeance de diminnation de temps des calculs pour obtention un résultat desiré, ou de la exigeance d'obtention des meilleurs effets du traitement sans grand augmentation de temps des calculs.

La famille des transformations hybrides de Hadamard-Haar (HHHT) comme, entre autres, la transformation de Walsh-Hadamard et la transformation de Haar, donne les nouveax possibilities dans cette domaine.

Dans cette étude on a montré les divers modifications des versions hybrides de la transformation de Walsh-Hadamard et de la transformation de Haar en systématique naturelle. Factorisation des matrices transformantes des transformations hybrides - modifiées de Hadamard-Haar permet à obtention des algorithmes effectifs pour transformations rapides du type "in-place" (FMHHHT) .

On a presenté une nouvelle méthode de résolution du problème de la filtration rapide non-récoursive en domaine séquentielle. A la base des algorithmes pour les rapides transformations hybrides-modifiées de Hadamard--Haar on pent obtenir les filtres scalaires qui sont donne par les équations matricielles et qui sont présenté par les graphes de circulation pour les transformations rapides. Dans ce cas, une matrice filtrante présenté une caractéristique séquentielle avec les coefficients d'affaiblissement des composantes dans la groupe dyadique ont les mêmes valeurs.

Bidimensionelle filtration à l'aide de FMHHHT en réalite soit transformation des undimensionelles séries des lignes et colonnes du bidimensionelle ensemble. La mèthode présenté utilise seullement les operations d'addition et de soustraction,donc le nombre des operations demandées est moindre par rapport de la ordinaire filtration séquentielle avec les algorithmes de la transformation rapide de Walsh-Hadamard (FWHT) .

Les numeriques filtres rapides à la base des algorithmes FMHHHT font partie de la classe des filtres non-récursives (coupe--bande et multi-passe-bande) et nont utile pour la réalisation rapide du traitement d'images numeriquement.

SUMMARY

In image processing applications, the motivation for using transforms other than Fourier is either to reduce computation time for a given resolution or to increase the resolution without incurring the penalty of drastically enhanced computation time.

A family of Hadamard-Haar hybrid transforms (HHHT) including Walsh-Hadamard and Haar transforms have been used effectively to satisfy these requirements. The various modified hybrid versions of the Walsh-Hadamard and the Haar transform in natural order are defined and developed. Sparse matrix factoring of modified Hadamard-Haar hybrid transforms (MHHHT) is utilize in developing the efficient in-place algorithms for fast implementations (FMHHHT) .

A new generalized approach to fast non-recursive digital filtering in sequency domain is presented. Scalar digital filter structures are described by a matrix equation based on flow graphs which represent the fast modified Hadamard-Haar hybrid transform algorithms (FMHHHT) . The filter matrix in this case represents a filter weighting function in sequency domain with the same weighting coefficients in dyadic group. The 2-D filtering using FMHHHT is essentially an operation on the 1-D data series constructed from succesive subsets (rows and columns) of the 2-D data. The real number additions and subtractions are only required and the number of operations is reduced compared with the conventional sequency filtering which uses the fast Walsh-Hadamard transform algorithms (FWHT) . The fast digital filters based on the FMHHHT algorithms represent a class of non-recursive filters (bandstop and multiband) suggested for high speed implementation in digital image processing.

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1. Introduction

A modified version of the discrete Hadamard -Haar hybrid transforms (HHHT) described earlier [1] is now defined and developed. Several natural ordered modified Hadamard-Haar hybrid transform matrices are particularly useful for generating the discrete transforms such as Hadamard-Haar transform (HHT) [2], S-transform (ST) [3], modified Walsh--Hadamard transform (MWHT) [4] and the new ones all called MHHHT. The transform matrix of MHHHT is sparse compared to the Hadamard matrix and correspondingly, the matrix factors of the former are more sparse than those of the latter. Consequently, computation of the MHHHT requires fewer arithmetic operations. This paper is to demonstrate that algorithms for fast computation of the MHHHT requires fewer arithmetic operations for $N \cdot N$ image processing (only from $4N(N-1)$ to $2N^2 \log_2 N$ real additions and subtractions). As the concept of application of MHHHT algorithms in designing two-dimensional non-recursive digital filters without multiplications is the same as in [5], the more general class of the scalar sequency filters can be developed using both HHHT and MHHHT algorithms.

2. Modified Hadamard-Haar hybrid transforms

The forward orthogonal Hadamard-Haar hybrid transform (HHHT) or its modification (MHHHT) of $N_1 N_2$ image array $\{x(n,v)\}$ results in $N_1 \cdot N_2$ transformed image array $\{X(m,u)\}$ as dedefined [6] by

$$\underline{X}_D = \frac{1}{N_1 N_2} \underline{D}_{N_1} \underline{x} \underline{D}_{N_2}^T \tag{1}$$

where $\underline{D}_N$ is the HHHT or MHHHT matrix of order N. The superscript T denotes the transpose.

A reverse transformation provides a mapping from the transform domain to the image space as given by

$$\underline{x} = N_1 N_2 \underline{D}_{N_1}^{-1} \underline{X}_D \left(\underline{D}_{N_2}^{-1}\right)^T \tag{2}$$

where $\underline{D}_N^{-1}$ is the matrix inverse of $\underline{D}_N$.

Let $N=2^p$ where $p=0,1,2,\dots$ and

$m = 0,1,2,\dots,N-1$; $n = 0,1,2,\dots,N-1$;

$K = \{0,1,2,\dots,k,\dots,p-2\}$.

The Hadamard-Haar hybrid transform matrix $\underline{D}_N$ of order N can be generated on the following way:

Let $j \in K$ and $K_j = \{0,1,2,\dots,k,\dots,j\}$;

$L_k = \{k,\dots,p-2\}$.

For every $m = 0,1,2,\dots,N-1$

$$k_m = \begin{cases} k & \text{if } k \in K_j \\ p-1 & \text{otherwise} \end{cases}$$

where k is the index of the last significant non-zero value bit in the binary representation of m.

For every $k \in K_j$ let $l \in L_k$ and

$l = k, k+1, \dots, p-2$.

Let n_i and m_i denote the i-th bit in the binary representations of the integers n and m respectively; that is

$$\begin{aligned} (n)_{10} &= (n_{p-1}\, n_{p-2} \dots n_1\, n_0)_2 \\ (m)_{10} &= (m_{p-1}\, m_{p-2} \dots m_1\, m_0)_2 \end{aligned} \tag{3}$$

Then the elements $d(m,n)$ of $\underline{D}_N$ can be generated as follows [1] :

$$d(m,n) = \begin{cases} \left(\prod_{i=l+1}^{p-1} \delta_{m_i n_i}\right) \cdot \left(\prod_{i=0}^{l} (-1)^{m_i n_i}\right) & \text{for } k_m \neq p-1 \\ \prod_{i=1}^{p-1} (-1)^{m_i n_i} & \text{for } k_m = p-1 \end{cases} \tag{4}$$

where $\delta_{m_i n_i}$ is the Kronecker delta.

The equation (4) defines the elements of the Hadamard matrix (K_j and L_j are the empty sets), the elements of the natural ordered rationalized Haar matrix ($K_j, L_j = \{0,1,\dots,p-2\}$) and their hybrid versions which are also orthogonal matrices [1] .

A development of the modified Hadamard-Haar hybrid transform matrices is based on eq. 4 and bit-permuted notation of m and n .

A sequence (3) in natural order can be rearranged in bit-permuted order as follows: For an integer expressed in binary notation, permute the binary form and transform to decimal notation, which is then called bit-permuted notation. For example, if an integer is represented by 3-bit binary number, then the bit permutation of m is defined by:

$$\begin{aligned} (m_2 m_1 m_0) = {} & (m_2' m_1' m_0') \cup (m_2' m_0' m_1') \dots \\ & \dots \cup (m_0' m_1' m_2') \end{aligned} \tag{5}$$

As an example, for N=8, one of the HHHT matrices and three of the MHHHT matrices result in the arrays shown in Fig. 1 .

$$\begin{bmatrix} 1&1&1&1&1&1&1&1 \\ 1&-1&0&0&0&0&0&0 \\ 1&1&-1&-1&1&1&-1&-1 \\ 0&0&1&-1&0&0&0&0 \\ 1&1&1&1&-1&-1&-1&-1 \\ 0&0&0&0&1&-1&0&0 \\ 1&1&-1&-1&-1&-1&1&1 \\ 0&0&0&0&0&0&1&-1 \end{bmatrix} \qquad \begin{bmatrix} 1&1&1&1&1&1&1&1 \\ 1&-1&1&-1&1&-1&1&-1 \\ 1&0&-1&0&0&0&0&0 \\ 0&1&0&-1&0&0&0&0 \\ 1&1&1&1&-1&-1&-1&-1 \\ 1&-1&1&-1&-1&1&-1&1 \\ 0&0&0&0&1&0&-1&0 \\ 0&0&0&0&0&1&0&-1 \end{bmatrix}$$

a) b)

$$\begin{bmatrix} 1&1&1&1&1&1&1&1 \\ 1&-1&0&0&0&0&0&0 \\ 1&1&-1&-1&1&1&-1&-1 \\ 0&0&1&-1&0&0&0&0 \\ 1&1&0&0&-1&-1&0&0 \\ 0&0&0&0&1&-1&0&0 \\ 0&0&1&1&0&0&-1&-1 \\ 0&0&0&0&0&0&1&-1 \end{bmatrix} \qquad \begin{bmatrix} 1&1&1&1&1&1&1&1 \\ 1&-1&1&-1&1&-1&1&-1 \\ 1&1&-1&-1&1&1&-1&-1 \\ 1&-1&-1&1&1&-1&-1&1 \\ 1&0&0&0&-1&0&0&0 \\ 0&1&0&0&0&-1&0&0 \\ 0&0&1&0&0&0&-1&0 \\ 0&0&0&1&0&0&0&-1 \end{bmatrix}$$

c) d)

Fig. 1 a) $K_j=L_j=\{0\}$, $(m_2 m_1 m_0)=(m_2' m_1' m_0')$ b) $K_j=L_j=\{0\}$, $(m_2 m_1 m_0)=(m_2' m_0' m_1')$ c) $K_j=L_j=\{0,1\}$, $(m_2 m_1 m_0)=(m_1' m_2' m_0')$ d) $K_j=L_j=\{0\}$, $(m_2 m_1 m_0)=(m_0' m_1' m_2')$

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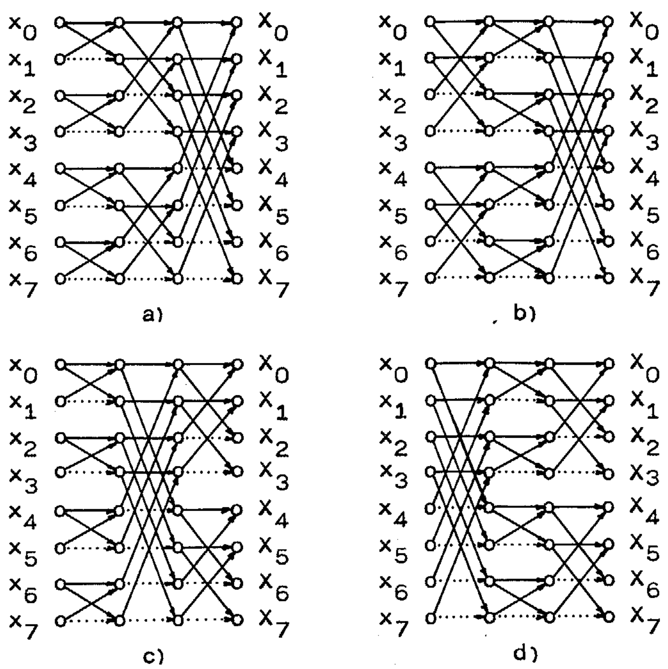
3. Fast modified Hadamard-Haar hybrid transforms

The underlying principle for the efficient implementation of the transformations presented here is a high degree of redundancy in the transform matrix description. Good [7] has shown that for matrices which have a certain degree of redundancy and which have resolution equal to a highly composite number, they can be factored into a product of matrices which allow vector-matrix multiplication on the order of $rN\log_r N$ operations as compared to N^2 operations where $N=r^p$. Good's paper has laid the foundation for the fast discrete transforms [8] such as Fourier, Walsh-Hadamard and Haar transforms.
The Walsh-Hadamard transform is particularly suited to digital computation because the basis functions take only the values +1 and -1. The rationalized Haar transform takes the values +1,-1 and 0 plus scaling of transform coefficients and is similarly suited to digital computation. Other discrete transforms [9], such as the Hadamard-Haar hybrid transforms also have fast algorithms [1].

Computation for the fast transform algorithms can be described conveniently by means of the signal flow graphs. They will be used to depict the organization of the FMHHHT algorithms for N=8. To illustrate the presented idea let us consider the example shown in Fig. 2 ; the 6 natural ordered FWHT algorithms which exist for N=8. The solid lines in the flow graph indicate that the calculated value is to be carried forward to the addition at the next node with the same sign, and a dotted line indicates multiplication by -1 before addition takes place. The flow graphs depicted in Fig. 2 have the in-place structure. The memory or storage requirements are therefore considerably reduced. The number of real additions and subtractions required to implement the 1-D WHT is $N\log_2 N$.



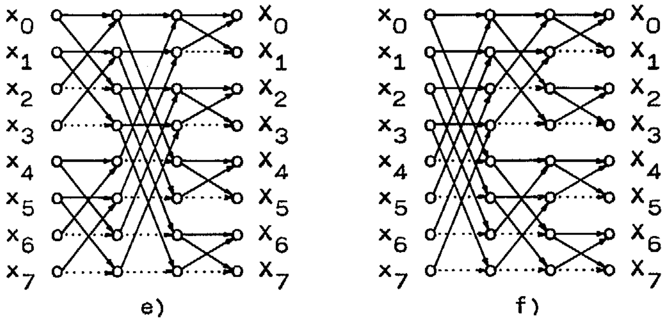


Fig. 2 $K_j=L_j=\{\emptyset\}$, $(m_2m_1m_0)$ =
a) $(m_2'm_1'm_0')$ b) $(m_2'm_0'm_1')$ c) $(m_1'm_2'm_0')$
d) $(m_1'm_0'm_2')$ e) $(m_0'm_2'm_1')$ f) $(m_0'm_1'm_2')$

The fast Hadamard-Haar hybrid transform algorithms for image processing have been developed earlier [1][5] . The object of this paper is to define and develop the modified versions of these transforms abbreviated as FMHHHT. The transform matrices of these modified versions based on eq. 4 and bit-permuted notation of m and n have a number of zeros as their elements , and consequently their matrix factors are more sparse. This results in flow subgraphs of the flow graphs presented in Fig. 2 . For example, the flow subgraphs based on the sparse matrix factors of the transform matrices Fig. 1 are shown in Fig. 3 . The notation and structure used in Fig. 3 are the same as those in Fig. 2 .

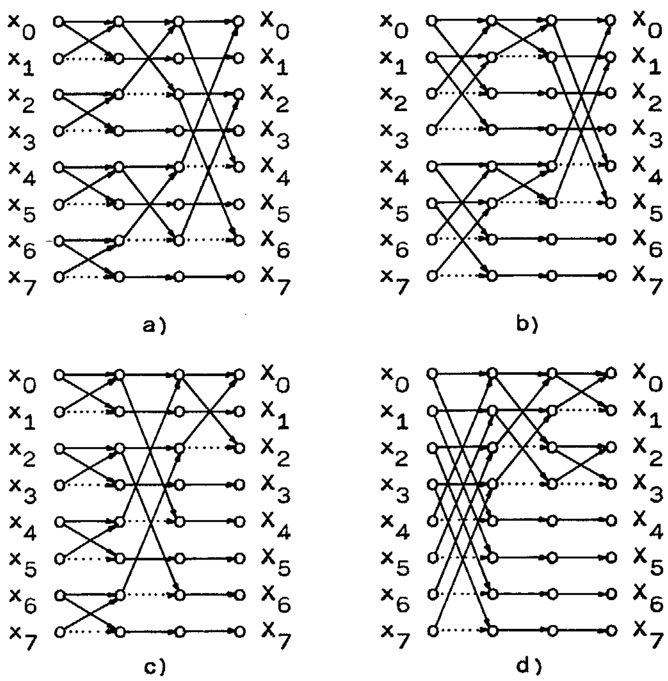


Fig. 3 a) $K_j=L_j=\{0\}$, $(m_2m_1m_0)=(m_2'm_1'm_0')$
b) $K_j=L_j=\{0\}$, $(m_2m_1m_0)=(m_2'm_0'm_1')$
c) $K_j=L_j=\{0,1\}$, $(m_2m_1m_0)=(m_1'm_2'm_0')$
d) $K_j=L_j=\{0\}$, $(m_2m_1m_0)=(m_0'm_1'm_2')$

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An inspection of the flow graphs Fig. 2 and the flow subgraphs Fig. 3 shows that the FHHHT algorithms are described by the flow subgraphs of the flow graph in Fig.2a, and the FMHHHT algorithms are presented by the flow subgraphs of the permuted flow graphs in Fig. 2b÷e . The number of real additions and subtractions required for fast implementation of MHHHT in N·N image processing is then from $4N(N-1)$ to $2N^2\log_2 N$.

4. Dyadic non-recursive digital filters

Non-recursive digital filtering is defined [10] as linear combination of the entire transform coefficients to produce a modified transform of $N_1 \cdot N_2$ points and is given by

$$\underline{y} = \underline{D}_{N_1}^{-1}\, \underline{G}_{N_1}^{D}\, \underline{D}_{N_1}\, \underline{x}\, \underline{D}_{N_2}^{T} \left(\underline{G}_{N_2}^{D}\right)^{T} \left(\underline{D}_{N_2}^{-1}\right)^{T} \qquad (6)$$

where $\underline{x}$ and $\underline{y}$ are the input and output data respectively; $\underline{G}_{N_1}^{D}$, $\underline{G}_{N_2}^{D}$ are the filter matrices and $\underline{D}_{N_1}$, $\underline{D}_{N_2}$ are the HHHT or MHHHT matrices of order N_1 and N_2.

Let the non-recursive digital filtering be defined by

$$\underline{y} = \frac{1}{N_1 N_2}\, \underline{D}_{N_1}^{T}\, \underline{D}_{N_1}\, \underline{x}\, \underline{D}_{N_2}^{T}\, \underline{D}_{N_2} \qquad (7)$$

It can be shown [1] that

$$\underline{D}_{N}^{T} = N\, \underline{D}_{N}^{-1} \left(\underline{\mathcal{N}}_{N}^{-1}\right)^{2} \qquad (8)$$

where $\underline{\mathcal{N}}_N$ is the diagonal matrix containing weighting factors consisting of multiplies of $\sqrt{2}$ and

$$\underline{\mathfrak{D}}_N = \underline{\mathcal{N}}_N\, \underline{D}_N \qquad (9)$$

where $\underline{D}_N$ is the orthogonal transform matrix consisting only of +1,-1 and 0, and $\underline{\mathfrak{D}}_N$ is the orthonormal transform matrix.
Then

$$\underline{y} = \underline{D}_{N_1}^{-1} \left(\underline{\mathcal{N}}_{N_1}^{-1}\right)^{2} \underline{D}_{N_1}\, \underline{x}\, \underline{D}_{N_2}^{T} \left(\underline{\mathcal{N}}_{N_2}^{-1}\right)^{2} \left(\underline{D}_{N_2}^{-1}\right)^{T} \qquad (10)$$

Comparison of (10) with (6) reveals that the filter matrices are the diagonal matrices whose nonzero elements are negative integer powers of 2 :

$$\underline{G}_{N_1}^{D} = \left(\underline{\mathcal{N}}_{N_1}^{-1}\right)^{2} \quad ; \quad \underline{G}_{N_2}^{D} = \left(\underline{\mathcal{N}}_{N_2}^{-1}\right)^{2} \qquad (11)$$

It means that every matrix $\underline{D}_N$ "produces" the matrix $\underline{\mathcal{N}}_N$ which defines 1-D scalar non-recursive digital filter.Thus 2-D digital linear filter is described by two diagonal matrices $\underline{\mathcal{N}}_{N_1}$ and $\underline{\mathcal{N}}_{N_2}$. The set of these filters defined by eq. 7 and based on FHHHT and FMHHHT algorithms can be treated as a new class of the scalar sequency filters [11] using

$$\underline{G}_{N}^{W} = \underline{W}_N\, \underline{D}_{N}^{-1}\, \underline{G}_{N}^{D}\, \underline{D}_N\, \underline{W}_{N}^{-1} \qquad (12)$$

where $\underline{W}_N$ and $\underline{W}_N^{-1}$ are the direct and inverse Walsh transform [12] and a filter matrix $\underline{G}_N^{W}$ represents a filter weighting function in sequency domain.

In this case, the filter matrix $\underline{G}_N^{W}$ represents scalar sequency digital filter with discrete coefficient values selected from the negative powers-of-two coefficient space. It can be shown [1] that $\underline{G}_N^{W}$ is a discrete filter weighting function in sequency domain with the same weighting coefficients in dyadic group - dyadic sequency filter.

The dyadic sequency filters based on FHHHT algorithms are the lowpass filters [5] . Using FMHHHT algorithms bandstop and multiband sequency filters have been obtained. For example, the lowpass, bandstop and two multiband sequency filters described by matrices $\underline{G}_8^{W}$ which correspond to the four subgraphs in Fig. 3 are represented as shown in Fig. 4 .

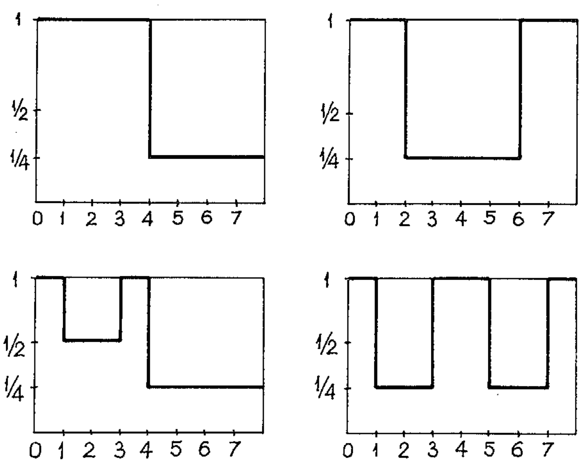

Fig. 4

In the case of conventional scalar sequency filtering operation on N·N image array a maximum of $4N^2\log_2 N$ additions and subtractions plus $2N^2$ real products would be required. The calculation involving the $2N^2$ real products will generally be the dominant factor in determining the speed of filtering operation [13] and can be reduced by a process (7) using FHHHT or FMHHHT in-place algorithms.

5. Conclusions

The various modified versions of the discrete Hadamard-Haar hybrid transforms in natural order are defined and developed.Sparse matrix factoring of modified Hadamard-Haar hybrid transforms(MHHHT) is utilize in developing the efficient in-place algorithms for fast implementations (FMHHHT) .

A generalized approach to fast non-recursive digital filtering (lowpass, bandstop and multiband) in sequency domain is presented. Dyadic digital filter structures are described by a matrix equation based on flow graphs which represent the FHHHT or FMHHHT algorithms. The real number additions and subtractions are only required and the number of operations is reduced compared with

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the conventional sequency filtering which uses the fast Walsh-Hadamard transform (FWHT) algorithms. Presented method is particularly useful for designing digital non-recursive filters with the powers-of-two coefficient grid, suggested for high speed implementation in digital image processing.

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