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TECHNIQUES DE REDUCTION DE SPECKLE

SPECKLE REDUCTION TECHNIQUES

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RESUME

Le bruit de granularité cohérente, ou "speckle", se produit dans toutes les images cohérentes d'objets diffus. Soit f(x,y) exp iϕ(x,y) un objet diffus : ϕ est une phase aléatoire, seul le module A présente un intérêt lorsqu'il s'agit de former une image. La formation d'image cohérente, que ce soit dans les domaines radar ou ultra-sonore ou en éclairage laser, peut être décrite comme une convolution de l'objet diffusant par une réponse percussionnelle. Le speckle est dû à l'interférence des diffuseurs élémentaires dont les réponses percussionnelles se chevauchent et affecte la qualité et la résolution de l'image. Le problème de la réduction de speckle se présente donc comme l'estimation du module f à partir de l'image bruitée.

L'analyse des statistiques du speckle, et en particulier de la corrélation du speckle mène à des techniques de réductions adaptées aux différentes situations. Ces techniques sont étudiées ici dans le cadre d'un modèle d'image à moyenne et variance non stationnaires.

Dans le cas optique, seul l'éclairement de l'image est accessible ; si l'image est sous-échantillonnée, l'information de corrélation du speckle est perdue, et le bruit est alors blanc et multiplicatif. Nous utilisons dans ce cas, d'une part un algorithme de filtrage linéaire optimal au sens des moindres carrés locaux, d'autre part une estimation MAP, plus précise, qui prend en compte la ddp du bruit. Dans le cas d'un échantillonnage suffisant d'un speckle optique, nous employons un filtre de Kalman réduit qui constitue une approximation du filtrage linéaire optimal au sens des moindres carrés et allège la charge de calcul.

Dans le cas du speckle radar ou sonar, on accède à la phase aussi bien qu'au module de l'image bruitée. Une estimation MAP itéractive qui utilise l'information de phase comme l'information de module est possible.

Des résultats de simulations sont présentés pour les différents cas.

SUMMARY

Speckle noise occurs when a scattering object is imaged coherently. Let us describe a scattering object as f(x,y) exp iϕ(x,y), ϕ being a random phase due to object roughness : only the modulus A is of interest for imaging purposes. Coherent imaging, in the radar, sonar domains or in laser illumination, can be described as a convolution of the scattering object by an impulse response. Speckle arises from interference between the scatterers over the impulse response area and affects image quality and resolution. Speckle reduction therefore appears as the problem of estimating the modulus f from the noisy image.

An analytical investigation of speckle statistics, and in particular of speckle noise correlation leads to various speckle reduction techniques adapted to the different possible situations. They are developed here in the context of a non-stationary mean, non-stationary variance image model.

In optical speckle, only the image intensity is detected ; if the image is undersampled, so that no speckle correlation information is preserved, the noise is white, multiplicative. We develop both a local linear minimum mean square error algorithm and a more accurate MAP estimation taking into account the noise probability density. In the case of adequate sampling of optical speckle data, we approximate linear minimum mean square filtering by a reduced update mowing window Kalman filter to reduce computation.

For radar or sonar speckle, modulus and phase are available. An iterative MAP procedure that uses both amplitude and phase information appears to be tractable.

Simulation results are shown for the various cases.

TECHNIQUES DE REDUCTION DE SPECKLE - SPECKLE REDUCTION TECHNIQUES

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I - INTRODUCTION

Coherent imaging of scattering objects produces speckle. A scattering object may be described as a set of independent scatterers. When illuminated, the scatterers diffract wavelets with mutually independent phases. Speckle is a physical consequence of the ability of these wavelets to interfere when the illuminating beam is coherent. While early observations of speckle can be traced to the eighteenth century, recent developments promoted the investigation of the phenomenon. In particular, speckle is present in the images of laser illuminated scenes as well as in the radar and sonar domains. Speckle has been successfully applied to surface roughness analysis and metrology {1,2} ; in imaging situations however, speckle is detrimental to image quality and resolution : it has been estimated that a fully developed speckle reduces resolution by a factor at least 5 {3}. It is therefore a sensible goal to try and reduce speckle.

The most effective speckle reduction method is without doubt the incoherent superposition of a large number of statistically independent coherent images of the same object : this amounts in fact to incoherent imaging and may be called a priori speckle reduction. When only one or a limited number of such speckled images are available, a posteriori speckle reduction methods must be used. This is the case considered here.

Previous approaches to speckle reduction include low pass filtering {4} and smoothing of multiplicative noise {5-10}. Since the physical properties of speckle are now well understood and verified {11-12}, our approach has been to combine the physics of speckle statistics with a number of available effective noise smoothing methods ; this resulted in several algorithms adapted to the various situations in which speckle may occur. Part of this work has already been published {13-16}. In the present communication, we summarize and illustrate our work, which up to now has been restricted to the so-called "fully developed speckle".

In the next section, we introduce briefly the bases of our work : the statistics of speckle and a suitable image model. In sections III and IV, the cases of optical and of radar or ultrasound speckle for single images, will be examined and simultation results will be shown. In section V, the extension to several independent images will be mentioned. Section VI contains concluding remarks.

II - SPECKLE NOISE AND IMAGE MODEL

1) The statistics of speckle :

Figure 1 schematically depicts a typical situation of speckle formation : $f(m,n) \exp i\phi(m,n)$ is a scattering object : $\phi(m,n)$ is the random scattering phase and the modulus f_{mn} is the only quantity of

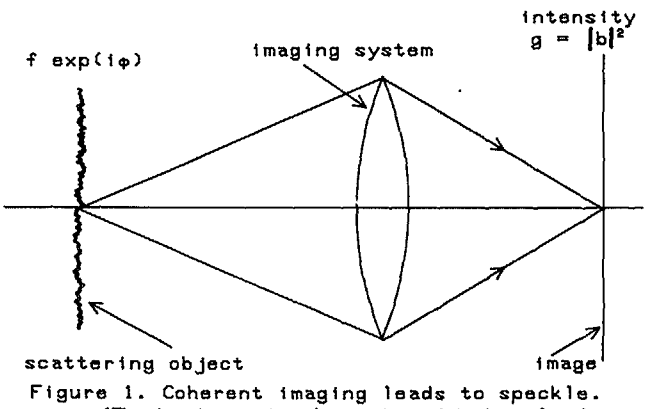


Figure 1. Coherent imaging leads to speckle.
(The imaging system does not need to be a lens).

interest in imaging. Due to diffraction by the system aperture, represented in figure 1 as a lens, the image $b(m,n)$ can be represented as the result of the convolution of the object by the imaging impulse response $h(m,n)$:

$$b(m,n) = \sum_{i,j} f(m-i,\ n-j) \exp i\,\phi\,(m-i,\ n-j)\ h(i,j) \quad (1)$$

In equation 1, the discrete double summation is extented over the entire support of the object and the impulse response.

The statistics of $b(m,n)$ given $f(m,n)$ may be derived from those of the phase $\phi(m,n)$. Of particular interest is the so-called "fully developped" speckle case : when each phase $\phi(m,n)$ is equally distributed over $0 - 2\pi$, and when moreover the correlation between neighbouring phases extends over much less than the impulse response area. Then each image pixel $b(m,n)$ obeys gaussian circular statistics. Moreover, the autocorrelation of $b(m,n)$ given $f(m,n)$ is

$$E\,[b(m,n)\ b^*(m,n)] = \sum_{i,j} h(m-i,\ n-j)\ h(m'-i,n'j)\ f^2(i,j) \quad (2)$$

Therefore, as opposed to many other types of noise in images, speckle noise is not only signal dependent, but also correlated.

Due to the quadratic nature of optical detectors, optical speckle data can only be illuminance data : the measured quantity at each pixel is the squared modulus of $b(m,n)$:

$$g(m,n) = |b(m,n)|^2 \quad (3)$$

It follows straightforwardly from equation (2) that the statistical mean of $g(m,n)$ is the incoherent image of the same object through the same instrument :

$$I(m,n) = E[g(m,n)] = \sum_{i,j} h^2\ (m-i,\ n-j)\ f^2\ (i,j) \quad (4)$$

In ultrasonic and radar imaging, the complex amplitude $b(m,n)$ itself can be measured.

More elaborate statiscical properties of fully developped speckle have been studied (see for example references 9 and 11) but will not be recalled here for the purpose of conciseness.

2) Image statistics :

As has been known since the work of HUNT and CANNON |17|, image enhancement can considerably benefit from the use of an image model. Instead of a signal independent, stationary speckle reduction method, we therefore propose to incorporate adaptivity through an appropriate model describing the statistical properties of the object modulus $f(m,n)$. The complete statistical properties of the complex amplitude $b(m,n)$ can then straightforwardly be derived from those of $f(m,n)$ and from the conditional probabilities such as those of equations 2 and 4.

In order to allow nonstatonarity in the first and second order image statistics while keeping the computational load to a minimum, we introduced a nonstationary mean, nonstationary variance model which has been applied to images degraded by various types of noise (additive, multiplicative, Poisson) with or without a linear space invariant blur |13,18|. In the absence of blur and of noise correlation, the local linear minimum mean square estimator derived from our model is a point operator. In the following sections, we apply the model to the various cases of speckle reduction.

III - SPECKLE REDUCTION TECHNIQUES FOR INTENSITY SPECKLE IMAGES

As pointed out above, speckle noise is correlated and it is advisable to take the known correlation properties of the noise into account in the processing. Nevertheless, we shall begin in subsection 3.1 with the simpler case of uncorrelated speckle data ; this corresponds to undersampled data, where the sampling is so coarse that the noise correlation is effectively lost. The general case of correlated speckle will be examined in subsections 3.2 and 3.3, where two possible speckle reduction algorithms will be described.

3.1) Independent speckle samples :

In this subsection, we consider speckle reduction techniques for independent speckle samples. The adaptive noise smoothing filter for speckle reduction is shown to be the same as in the multiplicative noise case. A nonlinear MAP filter which considers the negative exponential distribution of speckle intensity is then derived. The MAP estimate is a real root of a cubic equation and can be easily calculated.

The gaussian circular character of the speckle noise amplitude b(m,n) (see section II) implies that the speckle noise intensity g(m,n) follows a negative exponential distribution with probability density function (p.d.f.) for a given object f(m,n)

$$P(g(m,n)/I) = \begin{cases} \frac{1}{I(m,n)} \exp \frac{-g(m,n)}{I(m,n)} & , g(m,n) \geqslant 0 \\ 0 & , g(m,n) < 0 \end{cases} \quad (5)$$

where the quantity I(m,n) is the variance and the mean of g(m,n) given by eq. (4). It depends on the object f(m,n). According to eq. (5), g(m,n) can be represented by a multiplicative noise model :

$$g(m,n) = u(m,n)\, I(m,n) \quad (6)$$

where u(m,n) obeys the p.d.f.

$$P(u) = \begin{cases} \exp(-u) & , u > 0 \\ 0 & , u < 0 \end{cases} \quad (7)$$

whence the conclusion :

<u>for uncorrelated intensity speckle data, the noise is multiplicative ; it has negative exponential p.d.f., and in particular, unit mean and variance.</u>

This justifies the use of multiplicative noise smoothing techniques for such data, as has been proposed in references 5-10. Using our nonstationary mean, nonstationary variance image model, we derived an adaptive noise smoothing filter which can be used in particular for multiplicative noise |18|. However, this adaptive noise smoothing filter only uses the local mean and local variance of speckle, and is the optimal MMSE filter for Gaussian statistics only. Since the speckle intensity g(m,n) has a negative exponential distribution that is very different from the Gaussian distribution, it is useful to consider a nonlinear MAP (maximum a posteriori probability in Bayes' sense)filter for better performance.

In our model, the conditional mean I(m,n) of the speckle intensity depends of the object f(m,n) ; since f(m,n) is considered random, so is I(m,n) and it obeys the Gaussian p.d.f.

$$P(I) = (2\pi\, \sigma_I^2)^{-1/2} \exp \frac{-(I-\bar{I})^2}{2\, \sigma_I^2} \quad (8)$$

Where $\bar{I}$ and σ_I are the nonstationary mean and variance of I(m,n). In this equation as in equations (9-10) the indices m,n have been dropped for conciseness. The two quantities $\bar{I}$ and σ_I may in practice be estimated from the local neighborhood of each speckle image point and thus require no additional a priori knowledge.

The MAP estimate of I(m,n) is obtained by maximizing with respect to the variable I the a posteriori p.d.f. :

$$P(I/g) = \frac{P(g/I)\ P(I)}{P(g)} \quad (9)$$

It is important to note at this point that since both the conditional mean I(m,n) and the speckle noise g(m,n) are uncorrelated with the present assumptions, equation (9) does not need to be considered globally over the image but rather is valid for each pixel individually. This is not true for correlated speckle data (see the following subsections) or for a correlated image model.From eq (6) - (9), it is easy to show that the MAP estimate I (m,n) of I(m,n) is the only root of the cubic equation :

$$\frac{g}{I^2} - \frac{1}{I} \cdot \frac{(I-\bar{I})}{\sigma_I^2} = 0 \quad (10)$$

whose value is between $\bar{I}$ and g. Let us note parenthetically that the maximum likelihood estimate of I(m,n), obtained by maximizing over I P(g/I) rather than P(I/g), is the speckle image g(m,n) itself and therefore is of no interest for speckle reduction : here, the image model plays a crucial role for the smoothing and therefore a proper estimation of the local mean and variance $\bar{I}$ and σ_I is of major importance.

We now present some simultation results for this case. Figures 2 a-g correspond to uncorrelated intensity speckle data. 2a is the original image, 2b the uncorrelated intensity speckle image, 2c the 7 x 7 local neighborhood image used to obtain a first estimate of the local image mean and variance, knowing the mean and variance of the speckle noise. The two filtering methods described above have been tested on these data. Figure 2d is the adaptive noise smoothing filter, which is the minimum mean squared error for gaussian noise. If 2d is used to obtain an improved estimate of the local image mean and variance, the filtering can be iterated, yielding the improved image 2e. Further iteration shows little improvement. Figure 2f is, similarly, the first MAP estimate and figure 2g is the second iterated MAP estimate. Comparing this picture with figure 2e, we note that the MAP estimate seems to put a number of black dots on the sharp transition region of the image while the adaptive noise smoothing estimate puts white dots. This difference is because the MAP filter considers the negative exponential distribution of speckle intensity and tends to "guess" on the low intensity side if the local variance estimate is large.

3.2) Correlated speckle samples

If a speckle intensity image is adequately sampled so that the covariance structure of speckle is preserved, in principle, this information can be used to further reduce speckle noise.

Like in the previous section, we could try to derive the optimal MAP filter. In the case of correlated samples however, only the two-point joint p.d.f. of speckle intensity can be expressed analytically |13|, and even then, the maximization of the MAP is a non trivial problem. We therefore restrict the present work to the slightly less efficient approach of linear minimum mean square filtering ; let us stress once more that with our image model,the mean squared error is minimi-

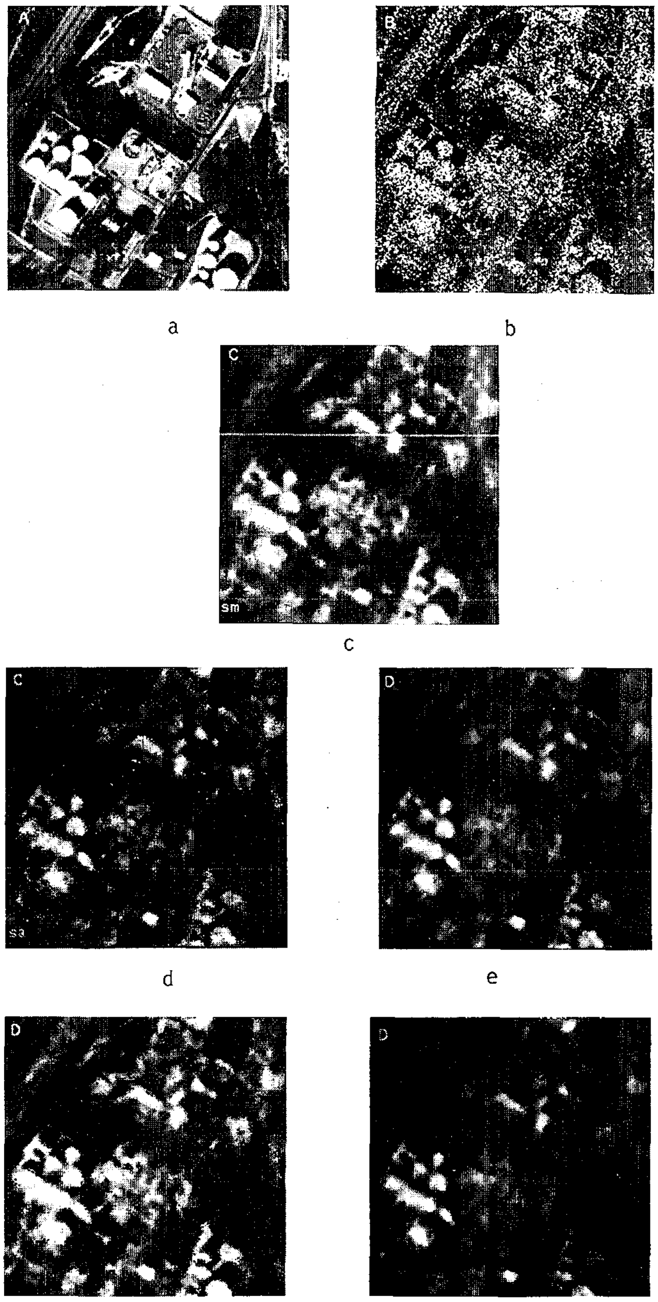


Figure 2 : Speckle reduction, simulation results : uncorrelated intensity data. a) Original ; b) Speckle image ; c) 7x7 local mean of b ; d) adaptive noise smoothing filter, first iteration ; e) second iteration ; f) MAP filter, first iteration; g) second iteration.

see figure 3 at the end of the paper.

zed locally, not globally, thus making the whole process adaptive to the scene. We refer to it as the local linear minimum mean squared error (LLMMSE) filter. Underlining the notations for lexicographically ordered image data, we can express the estimate $\hat{\underline{f}}$ of the object $\underline{f}$ under the usual form |19|

$$\hat{\underline{f}} = \overline{\underline{f}} + C_{fg}\, C_g^{-1}\, (\underline{g} - \overline{\underline{g}}) \qquad (11)$$

Where $\overline{\underline{f}}$ and $\overline{\underline{g}}$ denote the ensemble mean of $\underline{f}$ and $\underline{g}$ and C_{fg} and C_g the cross-correlation and auto-correlation matrices. From the knowledge of the speckle statistics and from our image model, all the quantities needed can be obtained directly from the speckled image; however, the filtering requires the inversion of matrix C_g, which is of size $N^2 \times N^2$, where N^2 is the number of pixels in the image. The computational load is therefore rather demanding.

Figure 3a shows the same original image as figure 2a ; figure 3b shows the simultated speckled image generated with a point spread function of triangular shape, separable in x and y and extending over a 5 x 5 pixel area ; the bandpass is therefore roughly 5 times smaller in this image than in figure 2c and the pixels in any neighborhood are strongly correlated. Figure 3d is the 7 x 7 local mean of 3c. Figure 3e shows the processed image obtained by applying the LLMMSE filter to the image ; to make the computation tractable, the image was sectioned into squares of size 12 x 12 ; the squares were processed separately and overlapped to avoid boundary effects. For the 256 x 256 pixel image, the processing time on a DEC KL 10 was 4 hours. However, it is apparent that the processed image is at least as good as those of figure 2 d,e,f,g, although the speckle is much coarser in the present case ; this illustrates the usefulness of taking speckle correlation into account in the speckle reduction algorithm.

The LLMMSE filter just introduced is nonrecursive and computationally demanding even if a sectioning method is used. It is therefore worth while to consider a recursive implementation as an approximation of this filter both for fast computation and better adaptation to local processing. We therefore developed a speckle reduction approach in the forma lism of recursive filtering and implemented it using a reduced update algorithm similar to Woods' |20| where only correlated points are taken into account in the gain vectors. It will not be detailed here because of space limitations, but the quality of the results obtained is very similar to that of figure 3, and the computation time is only a linear function of the number of pixels N^2 |14|.

IV - SPECKLE REDUCTION TECHNIQUES FOR COMPLEX AMPLITUDE SPECKLE IMAGE

In section III, we discussed various speckle reduction techniques for intensity speckle images where only the speckle intensity is recorded and the phase information is lost through the recording process. The phase information is usually lost for laser speckle and for synthetic aperture radar (SAR) images that are processed with a coherent optical system. The MAP speckle reduction filter for the correlated speckle samples in this case was shown to be analytically difficult to derive and implement.

In digitally processed SAR images and in sonar images both the amplitude and phase of the speckle image are preserved. It will be shown in this section that with the extra phase information in the complex amplitude correlated speckle image, the optimal nonlinear MAP filter can be derived easily compared with the intensity speckle case and speckle reduction can be improved.

Using equations (1) and (2) and the complex gaussian character of the fully developped speckle amplitude b(m,n), the MAP estimate of the squared modulus $f^2(m,n)$ given the noisy observation b(m,n) and our image model can be derived. If we use the lexicographic notation, the MAP equation for point i = (m-1) N + n, m = 1 to N, n = 1 to N can be written in the following form |13|

$$-\underline{h}_i^T C_b^{-1} \underline{h}_i + \underline{h}_i^T C_b^{-1} \underline{b}\,\underline{b}^* C_b^{-1} \underline{h}_i - \frac{f_i^2 - \overline{f_i^2}}{\sigma^2_{f^2}} = 0 \qquad (12)$$

In this equation, $\underline{h}_i$ is the i th column of the impulse response matrix ; T denotes transposition ; $\overline{f_i^2}$ and $\sigma^2_{f_i}$, are the mean and variance of our nonstationary mean, nonstationary varaince image model ; and C_b is the conditional covariance matrix of the speckle amplitude b given modulus f. Equation (12) is an equation with unknown $\underline{f}^2$, i.e. the N^2 components $f_i^2 = f^2(m,n)$ of the colum vector $\underline{f}^2$. Let us note that if the last term of (12) is omitted, i.e. the object variance is assumed very large, the remaining terms constitute the ML equation for the same problem. The N^2 equations (12) for all values of i form a system with N^2 unknowns f_i. These equations can be solved, in principle, using iterative methods such as the Newton-Raphson or Picard method. However, in each iteration, we must numerically invert the matrix $C_b(f)$, which has dimension $N^2 \times N^2$ where N^2 is the number of pixels. Even though we can use sectioning techniques to reduce the dimensionality of $C_b(f)$, the procedure is still very computationally demanding. We also need to find the optimal direction for updating the estimate in each iteration. This approach is still impractical even with fast computers.

It is nevertheless possible to solve the equations (12) at least to a good approximation, using the following approach.

Instead of estimating $\underline{f}^2$ from b, we can estimate in a first step the complex modulus $\underline{f \exp i \phi}$ from $\underline{b}$

This is an image restoration problem on complex amplitude data.

The LLMMSE estimate of $\underline{f \exp i \phi}$ can be expressed by an equation analogous to equation (11) and our recursive method for approximately solving the LLMMSE equation can be used on that equation as well as on the intensity data considered in section III. This yields a complex estimate for $\underline{f \exp i \phi}$; let us call $\underline{f}_o^2$ the modulus squared of this estimate. Let us point out that the LLMMSE filter uses our image model and therefore requires a first knowledge of the local mean and variance of f^2. Now, we can use the intermediate estimate $\underline{f}_o^2$ to solve the MAP equation (12). For that purpose, we also need the covariance matrix C_e of the complex LLMMSE error

$$\underline{e} = \underline{f \exp i \phi} - \text{LLMMSE estimate of } \underline{f \exp i \phi} \qquad (13)$$

$$C_e = \text{ensemble mean of } (\underline{e}^T \ \underline{e}^*) \qquad (14)$$

A calculation which will not be reproduced here allows one to reformulate the MAP equations (12) using only the intermediate estimate $\underline{f}_o^2$ and the error covariance matrix C_e:

$$\frac{C_e(i,i) + f_{oi}^2}{(f_i^2)^2} - \frac{1}{f_i^2} - \frac{(f_i^2 - \overline{f_i^2})}{\sigma^2_{f_i^2}} = 0 \qquad (15)$$

In this equation, $C_e(i,i)$ is the ith diagonal element of matrix C_e. Equation (15) has exactly the same form as (10) ; it reduces to a scalar equation with one unknown f_i^2; the speckle intensity g is replaced by the quantity $C_e(i,i) + f_{oi}^2$ derived from the complex LLMMSE estimation and the incoherent image intensity I is replaced by the squared modulus f^2 of the object itself.

It is therefore helpful to view the function of the LLMMSE filter operating on the complex amplitude speckle observations as decorrelating the data. The outputs of the LLMMSE filter are combined to form an intensity image and processed by a one-point MAP filter that is derived by assuming that the speckle intensity samples are statistically independent. With this in mind, a simple iterative algorithm is developed. In each iteration, we apply a nonstationary 2-D recursive filter on the complex amplitude speckle image. The filtered estimate and the diagonal element of the filtered covariance matrix are combined to form a new intensity speckle image. The one-point MAP filter is then applied to this new intensity speckle image and we have a MAP estimate of the original object intensity. This new estimate is used as the local variance of the 2-D recursive filter and starts the next iteration. The block diagram of this algorithm is illustrated in figure 4. The one-point MAP

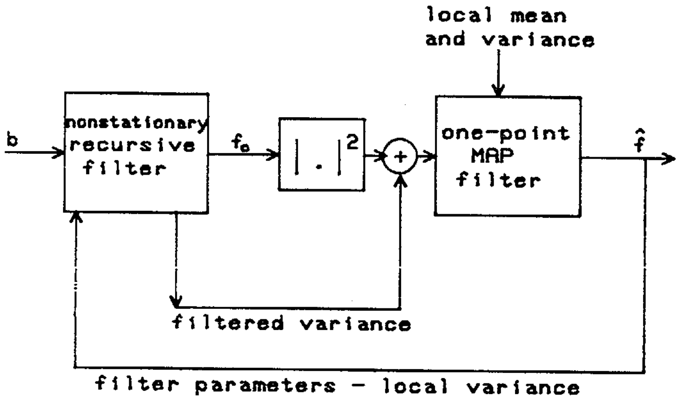


Figure 4. Block diagram of MAP estimation for complex amplitude speckle.

estimate is the real solution of a cubic equation whose value is between the local estimate and the speckle intensity, and this prevents the possibility of divergence in the iteration.

Figure 5 presents some simulation results. The original imageis the same as in figures 2a and 3a. The

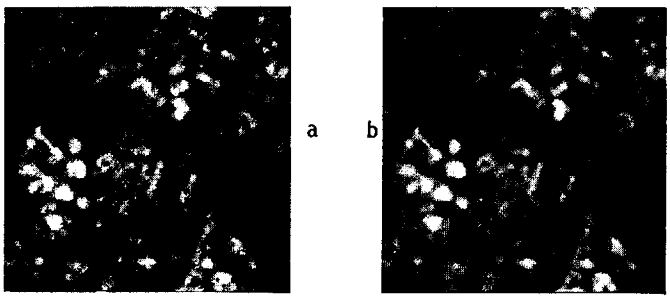


Figure 5 : Speckle reduction, simulation results : correlated complex amplitude data, same example as figures 3a and 3b ; a) processed MAP image, first iteration ; b) second iteration.

speckle intensity is that of figure 3b, but this time phase information is preserved. The first iteration MAP estimate is shown in figure 5a and the seconde estimate in figure 5b. Some improvement with respect to figure 3 can be observed.

V - MULTIPLE FRAMES CASE

Up to now, only single speckle images have been considered. It is quite usual to superimpose the intensities of several independent speckle images wherever possible, for example in SAR imagery. This produces a first speckle reduction effect although phase information is lost. All the techniques discussed in section III can be applied to such "multiple frames" speckle images to further reduce speckle noise. The only basic difference is that the p.d.f. of speckle intensity is modified and the appropriate statistics must be used. The simultation results are encouraging |13|. Space restrictions do not allow to show them here.

VI - CONCLUDING REMARKS

In this work, we attempted to take advantage of the exact statistics of fully developed speckle noise to develop suitable speckle reduction techniques. In the framework of a nonstationary mean, nonstationary variance image model, a one point MAP filter for intensity speckle data was first derived. A useful refinement is to take advantage of the known correlation properties of speckle noise to further reduce it. Finally, a speckle reudction technique involving speckle correlation prperties and operating on complex speckle data has been developed. Further work is still needed to simplify if possible these algorithms, to assess quantitatively the improvenent obtained by our methods and to attack the case of non-fully developed speckle.

REFERENCES

1) J.C. DAINTY, ed., Laser speckle and related phenomena, (Springer, Berlin 1975).

2) Journal of the Optical Society of America, special issue on speckle in optics, 66, 1145-1316 (1976).

3) A. KOZMA, C.R. CHRISTENSEN, ref. 2, pp 1257-1261.

4) B.D. GUENTHER, C.R. CHRISTENSEN & A. JAIN, IEEE conf on pattern recognition and image proc., Chicago, 1978.

5) L.J. PORCELLO, N.G. MASSEY, R.B. INNES & J.M. MARKS, ref 2, pp 1306-1311.

6) J.S. LIM & H. NAWAB, Opt. Engin., 20, 472-480(1981).

7) J.S. LEE, Comput. Graph. and Image Proc., 17, 24-32 (1981).

8) H.H. ARSENAULT & G. APRIL, ref 2, pp 1160 - 1163.

9) V. FROST, J.A. STILES, K.S. SHANMUGAN & J.C. HOLTZMAN, IEEE trans. Pattern Anal. and Mach Intel, PAMI 4, 157-166 (1982).

10) J.S. LEE, IEEE trans Syst., Man and Cyber, SMC 13, 85-89 (1983).

11) J.W. GOODMAN, Chap. I of ref 1 and pp 1145 - 1149 of ref 2.

12) J.C. DAINTY, in Progess in Optics, volume 14 (North Holland, Amsterdam), E. WOLF, ed., pp 1-46 (1976).

13) D.T. KUAN, Ph D, University of Southern California report USCIPI 1060 (1982).

14) D.T. KUAN, A.A. SAWCHUK, T.C. STRAND & P. CHAVEL, Proceedings IEEE int. Conf. on acoustics, speech and sig. proc., Paris, 1982, IEEE catalog n° 82 CH 1746-7, pp 1561-1564.

15) D.T. KUAN, A.A. SAWCHUK, T.C. STRAND & P. CHAVEL, SPIE Proc., vol 359, pp 28-38 (1982).

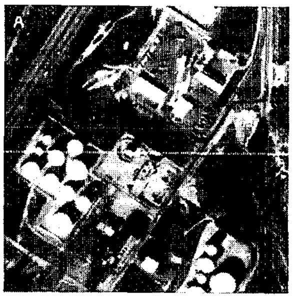
16) D.T. KUAN, A.A. SAWCHUK, T.C. STRAND & P. CHAVEL, Proceedings IEEE int conf on pattern recognition and image processing Las Vegas, 1982, pp 58 - 63.

17) B.R. HUNT & T.M. CANNON, IEEE trans. Syst.,Man and Cyber, SMC 6, 876-881 (1976).

18) D.T. KUAN, A.A. SAWCHUK, T.C. STRAND & P. CHAVEL, to be published in IEEE trans. Pattern Analy. and Mach. Intel., may 1984.

19) A.P. SAGE & J.L. MELSA, Estimation theory with applications to communications and control, (Mc Graw Hill,New York , 1971).

20) J.W. WOODS & C.H. RADEWAN, IEEE trans Infor Theory, IT23, 473-482 (1977).



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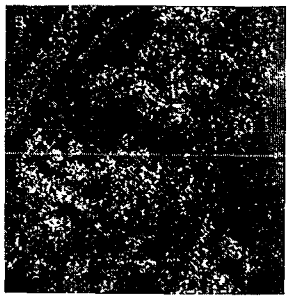
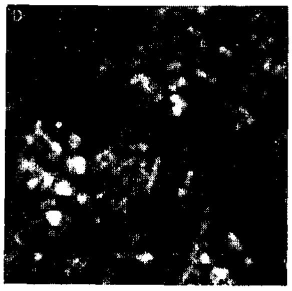


b

c



d

Figure 3 : Speckle reduction, simulation results : correlated intensity data.
a) Original (same as 2a) ; b) Speckle image with same sampling as 2b but bandpass 5 times smaller ;
c) 7x7 local mean of b ; d) Local linear minimum mean square error estimate.