



# Traitement, Synthèse, Technologie et Applications

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## MAPPING THE LOCAL INFORMATION CONTENT OF A SPATIAL IMAGE

(EXTRACTION DU CONTENU D'INFORMATION LOCALE D'UNE IMAGE SPATIALE)

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### RESUME

This paper defines precisely the spatial information content of a small segment of an image. Following Shannon's self information concept it is shown that the local information content of a segment of  $N$  adjacent pixels is carried out by the actual structure of the local histogram of the radiometries of that segment of image. Various transformations of a global image into their maps of local information are presented based on the simplex structure of the corresponding local histograms. Various illustrative examples and local information maps are also presented.

### SUMMARY

Cette communication définit en termes précis le contenu d'information spatiale que contient un petit segment d'image. En s'appuyant sur la définition de Shannon de l'information propre on montre comment le contenu d'informations d'un segment d'image comprenant  $N$  pixels se retrouve dans la structure de l'histogramme local des radiométries de ce segment. Plusieurs transformations d'images globales en leur cartes d'information locale sont présentées utilisant le principe de la décomposition barycentrique des simplexes de leurs histogrammes locaux.

**KEYWORDS :** LOCAL SPATIAL INFORMATION, SELF INFORMATION, TEXTURE ANALYSIS, LOCAL HISTOGRAMS, SIMPLEX STRUCTURE, CONTOUR INFORMATION, LOCAL AND SPATIAL PROPERTIES MAPS.

### INTRODUCTION

The segmentation of an image in homogeneous zones of texture is an important objective of image processing research. Though the field of texture synthesis has marked continuous progress in recent years, texture analysis and texture recognition do not seem to have progressed much. Perhaps, as noted by Harralick in 1982, this is due to the lack of a good and objective definition of texture for numerical images. Texture recognition is essentially a characteristic of the psycho-biological systems : Very little is known indeed on the mechanisms involved. It is however reasonable to assume that the biological processing produces a "GESTALT" type of perception in the cortex.

In a systemic approach to the solution of problem of texture recognition (what ever the biological solution might be) it seems logical to look for a measure of "emergence" or "synergy" that does occur when a small group of neighboring pixels are observed globally. In modern Physics the most commonly used measure of synergy of a group of interacting elements is Shannon's measure of information defined in a probabilistic context involving the actual states of realizations of the elements of the group. In order to avoid any inference to the biological perception of texture, we will call our emergence attribute the "local (spatial) information content" of that group of adjacent pixels, referenced, when necessary, to the central pixel  $(p, q)$  of the window



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delimiting that group. The approach is essentially non parametric : the underlying probability law is empirical, given by the actual (local) histogram of the radiometries of the group of pixels involved.

In order to better apprehend the approach we first enlarge the spatial information concept to the group of all the pixels of a global image comprising, say, 512 x 512 discrete radiometric values :  $N = 512 \times 512$ . These radiometric values  $x_i$ ,  $i = 1, \dots, r$  can range, say, over 256 integral realizations.

#### THE GLOBAL INFORMATION CONTENT OF AN IMAGE

The realizations  $x_i$  of a random variable  $X$  obey a probability  $p_i$  that can be computed from the histogram of that image:

$$p_i = p(x_i) = \frac{n_i}{N}$$

where  $n_i$  is the total count of the  $i$ -th radiometry in the range. Then Shannon's self information measure  $I(x_i)$  brought by the realisation  $x_i$  among  $r$  possible ones is given by :

$$I_B(x_i) = -\text{Log}_2 \{p(x_i)\} = \text{Log}_2(N) - \text{Log}_2(n_i)$$

Simpler notations result if the information measure is expressed in "Nats" rather than in "Bits". In this case the natural logarithms are used :

$$I_N(x_i) = \text{Ln}(N) - \text{Ln}(n_i)$$

Furthermore the information measure will be finite for  $n_i = 0$  if  $\text{Ln}(0)$  is defined by analytic continuation of  $\text{Ln}(x)$  by the tangent at  $x = 1$  : Convexity is maintained.

$$I_N(0) = \text{Ln}(N) + 1 \geq 0$$

It should be observed that  $I(x)$  is a monotonic and non negative function of  $n_i$ ,  $n_i$  is then a valid measure of  $I(x_i)$ . As  $i$  can range from 1 to  $r$ , i.e. on all possible radiometric values, it is quite clear that the histogram  $n_i, i=1, \dots, r$  contains all the spatial information, as defined, of the global image under analysis.

Consider now the following pixel per pixel transformation of the input image : replace, on each pixel,  $(p,q)$  the original radiometric value  $x_i$  by the function  $I(x_i)$  precedently defined (or by its measure  $n_i$ ). One then obtains the "global information image" of the input image. An example of such transformation applied to a girl portrait will be found in Figure 2-B. Such a mapping of radiometric values into their probability (or measure) tends to enhance the feable count radiometries (or the high count ones).

Let us assume, as it is often the case for a large size image, that the underlying probability law tends to a gaussian shape, that is :

$$p = K \exp \left\{ -\frac{x^2}{2\sigma^2} \right\}$$

$$I(x) = \text{Ln}(N) + \frac{x^2}{2\sigma^2} - \text{Ln}(K)$$

$I(x)$  induces a quadratic norm on the  $x$ . For decorrelated images (K.L transform of a multispectral scene for instance), their transforms through  $x \rightarrow I(x)$

become strictly additive, pixel per pixel, and one can obtain an information image which is the information sum of the two.

In general, however, the empirical probability law  $p(x)$  can take any form and  $I(x)$  induces an empirical or "natural" metric on the the  $x$ 's that will permit addition if one relaxes the independance condition.

#### LOCAL INFORMATION CONTENT

Assume now that a global image (comprising, say, 512 x 512 pixels) is scanned by a window, moving a pixel at a time, centered presently on pixel  $(p,q)$  and comprising a total of  $N$  pixel ( $N=3 \times 3$  or  $8 \times 8 \dots$  or  $16 \times 16 \dots$ ). To each pixel  $(p,q)$  of the global image is virtually attached a  $N$  pixel neighborhood. Consider the local histogram of the  $N$  radiometric values delimited by that spatial window. This local histogram  $h(p,q)$  is a vector of  $\mathbb{R}^r$  (the Euclidean space defined on  $r$  counts), with components defined on non negative integral value less or equal to  $N^*$ . There is a linear relation between the  $r$  components :

$$\sum_{i=1}^r n_i = N,$$

as the sum of the  $r$  counts of this local histogram is precisely equal to the number  $N$  of pixels in the window.

For reasons developed in the preceding section, this local histogram maps the local information content of the group of the  $N$  radiometric values delimited by the window centered on the pixel  $(p,q)$  of the input image.

An interesting image transformation, that enhances the local information, consists of a mapping assigning to each pixel  $(p,q)$  (the center of the  $N$  pixel scanning window), the count of the histogram

\* : Such a vector is sometimes referred as a  $Z$  - module.

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corresponding to the original radiometry on (p,q). This map is called the local information image. It enhances contours as well as other features with a high information content. Examples of such maps are reproduced in this paper. This contour extraction method has the peculiarity of being effected by a summation operation : Counting.

The local histogram contains much more than contour information. When the input image is a portrait for instance, various type of caricatures can be obtained by various mapping of the local histogram components into the pixel (p,q) center of the scanning window.

THE SIMPLEX REPRESENTATION OF THE LOCAL HISTOGRAMS OF A GLOBAL IMAGE

The input global image is quantized in discrete radiometric levels. A local histogram  $h(p,q)$  is computed over a  $N$  pixel window centered on (p,q). The linear subspace of  $h$  in  $IR^r$  are the points with non negative integral components in the simplex<sup>3</sup> :  $\sum_{i=1}^r n_i = N$

where  $n_i$  is the count of the  $i$ -th radiometric value in  $h(p,q)$ . The structure of the local spatial information shares all the geometrical properties of that simplex, a linearly bounded convex body. For illustration purpose a 2-simplex has been drawn on an adjacent figure. It corresponds to  $r = 3$ . We assume that  $N = 9$ , i.e. a scanning window comprising  $3 \times 3$  pixels.

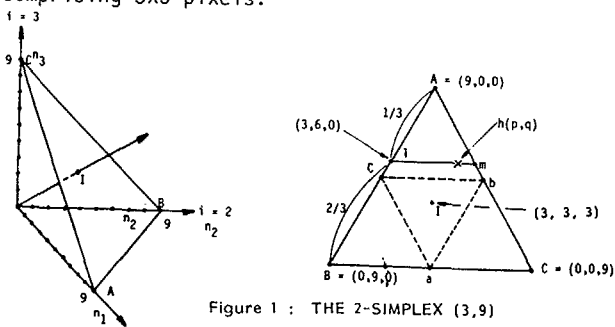


Figure 1 : THE 2-SIMPLEX (3,9)

This particular simplex is the equilateral triangle ABC in  $IR^3$ . When the pixel (p,q) scans the global image, the vector  $h$  falls somewhere inside this triangle ABC. The points A, B, C, are the apexes, they correspond to the particular local histograms of homogeneous zones within the window. They are the vectors (9,0,0), (0,9,0) and (0,0,9). The center I of the simplex corresponds to the unic "flat" histogram (3,3,3).

An interesting and simple metric to choose for  $IR^3$  is the norm sup metric: One chooses as a scalar attribute for the vector  $h$  the largest component.

In the (N,r) simplex the apexes A,B,C share the highest measure, the center I the smallest one :

For any truncation parallel to a face (such as  $I_m$  in the figure) the histograms contained in it share the same norm sup (the norm sup of I or m). For the truncation illustrated (1/3, 2/3) this norm sup will be 6. All the histograms on  $I_m$  form an equivalence class and, after the choice of the norm sup metric for the information content of  $h$ , we say that their local information content is 6. Consider the mid-points abc of the 3 faces. The triangle abc partitions the simplex in 3 "external" sub-simplices, each one having its own apex A,B or C and 1 "internal" sub-simplex containing the center I. We have effected the so call classical "barycentric" partitioning of the simplex. Note that on the segments ab, bc and ca the norm sup is constant (equal to 4.5), as the norm sup is constant on  $I_m$  (equal to 6): The triangle abc is then a "circle".

We can differentiate the local histograms falling in the 3 external sub-simplices - though they may have the same norm, i.e. the same local information content - by their "phase" or "state" : They share the state of the apex they contain. The state of A is 1, i.e. the index of the radiometry which assumes the norm sup. The state of B is 2, the state of C is 3. The state of an external histogram  $h$  is then the index of the dominant radiometry in the scanning window. As far as the internal sub-simplex abc the situation is different, it contains the central the central point I where the norm sup is assumed on all the counts. (3,3,3). We say that the state of an internal histogram is not (well) defined. In order to rank or classify the various classes of equivalence of internal local histograms (i.e. those with a norm sup less than  $N/2$ ) we rely only on their norm, or, if more convenient, their "module" :

$$\text{Mod}(h) = \|h\|_{NS} - \|I\|_{NS} = \|h - I\|_{NS}$$

The module is by definition the distance between  $h$  and the center I.

These very simple geometrical notions permit an effective segmentation of the input image in zones of similar local information content. When the dynamic range of the input image comprises  $r$  discrete levels, the  $r$  external classes of local histograms are coded by the index of their apex, for instance they are assigned one of  $r$  saturated colors (except black or white) the internal class of local histogram will be coded with grey levels



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according to the value of their norm sup, from black assigned to  $N/r$ , the norm sup value of  $I$ , to  $N/2$  the norm sup of the faces of the internal sub-simplex. The reverse coding can also be used : one codes with colors the various levels of information content, and, in grey levels, the various zone of quasi homogeneous radiometric content. The later coding have been used on the local histogram analysis of the image of Roumare presented in this paper and on the image of Lapali : Figures 3 and 4.

We obtained very spectacular maps of local information content that way.

The internal sub-simplex abc does contain all the "busy" local zones of the image, contours and active textural patterns. The external sub-simplexes are zones with a dominant radiometry, they are quasi-homogeneous with very low textural content.

#### STRUCTURAL ANALYSIS OF LOCAL INFORMATION CONTENT

In the preceeding section one has used only the geometrical properties of the simplex representation of the local histogram  $(N,r)$  of a global image. A further analysis can be performed on the actual structure of the cloud of points of the  $h(p,q)$ . This cloud can be handled as the histogram of the local histograms and supervised or un-supervised classification can be effected. In order to reduce the dimensionality  $r$  of the simplex representation one can operate on two reduced maps : the maps of modules of the local histogram  $h$ , and the map of states, that is a map of the radiometric index of the dominant radiometry. The respective dynamic range of these maps are  $N(1-\frac{1}{r})$  and  $r$ . In our own experimentation we have used values of  $N$  equal to 9, 25, 64, 256, 441 and values of  $r$  equal to 8 and 16. We are presently experimenting contour extraction, texture segmentation and form recognition by these methods. We hope be able to present results shortly in a latter paper.

#### CONCLUSION

The local information content of an image segment has been precisely defined and various methods of extraction have been outlined. According to the given definition, the local spatial information is contained in a local histogram of radiometries. The structure of all the local histograms of a global image has been shown to be a simplex. Some of the classical properties of the simplex have been used to build up maps of the local information

content of a given spatial image. Examples of such maps have been presented. The local histogram analysis of an image is very simple because only counts of identical label objects are involved, the computing cost can be brought down to a minimum.

The specific users will appreciate if the defined attributes and the suggested image segmentations can be of any use, specially as tools for image interpretation. The users will also appreciate if this type of spatial analysis yields acceptable results in texture segmentation. We do believe it does.

#### ACKNOWLEDGEMENTS

The author wishes to acknowledge the early contribution of George LEBOUCHER as far as the concept of an information map of a global image is concerned.

However the author wants to stress the lack of enthusiasm of many colleagues and scientific friends about the possible use of local histograms to map usefull spatial information, in particular texture.

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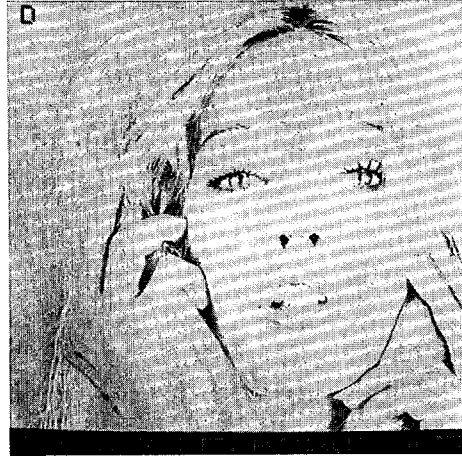




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original



global information



local information



map of modules

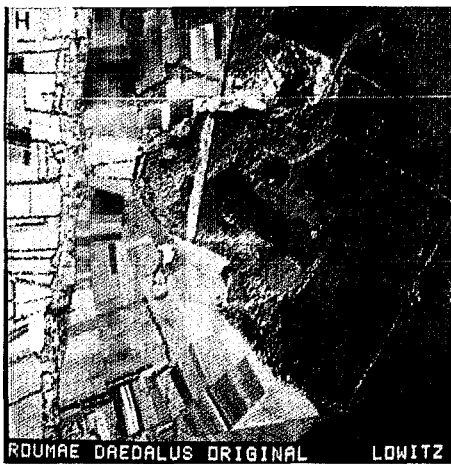


map of states

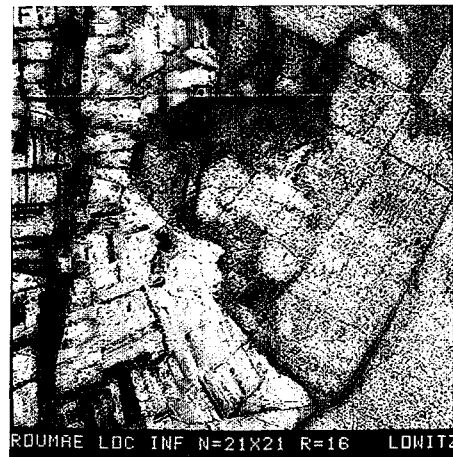
FIGURE 2, GIRL PORTRAIT



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original



map of local information  $N = 21 \times 21$ ,  $r = 16$



barycentric partitioning  $N = 21 \times 21$ ,  $r = 16$ .

FIGURE 3, ROUMARE DAEDALUS



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original

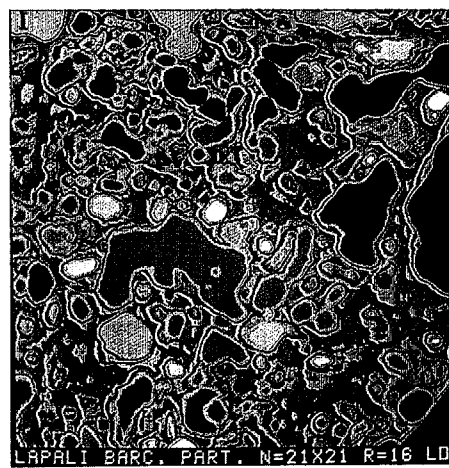


Sobel contours



map of local information

$N = 21 \times 21, r = 16$



barycentric partitioning

FIGURE 4, ARIES IR IMAGE