# Pulse Compression with Periodical Binary Phased Signals

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## RESUME

Dans les applications en communication, navigation et radar on utilise des signaux en codage de phase binaire avec une fonction d'autocorrélation (AKF) impulsive /1, 2, 4/. L'approche basée sur l'analyse d'autocorrélation implique en effet un récepteur réalisé au moyen de la technique des filtres adaptés /3/. Du point de vue du traitement des signaux il n'est pas nécessaire de limiter le domaine des coefficients du filtre récepteur à des valeurs binaires.
En conséquence, dans ce travail on propose la réalisation d'un filtre presqu' adapté, dans lequel les coefficients du filtre récepteur sont considérés comme des paramètres à optimizer de façon que tous les lobes secondaires de la foction de corrélation mutuelle résultante (KKF) s'annulent. Dans ce travail on donne aussi des résultats pour signaux périodiques binaires aves les filtres presqu'adaptés rélatifs. Pour chaque longueur de mot de codage $N > 2$ ils existent plusieurs signaux et coefficients des filtres presqu'adaptés rélatifs, avec une fonction de corrélation mutuelle tout à fait impulsive; autrement dit, dans la fonction de corrélation mutuelle considerée les lobes secondaires manquent complétement.

## SUMMARY

In digital communication, navigation, and radar applications and for syncronisation and range measurement methods binary phase coded sequences are used with the property that their periodic or aperiodic autocorrelation function (acf) has an impulse-like form /1,2,4/. The concept of acf analysis implies a system receiver with matched-filter technique in practice /3/. In the case of binary sequences and from a signal processing point of view it is not necessary that the receiver filter coefficients are binary. Therefore, in this paper a mismatched-filter design is introduced where the receiver filter coefficients are used as free parameters which are optimized with the property that all sidelobes of the cross-correlation function (ccf) are zero. The present paper gives some results for binary sequences and the belonging mismatched-filters. For each length $N > 2$ exists at least one sequence and a belonging mismatched-filter which has no sidelobes at all in the output signal.

# 1 Introduction

Binary sequences have been succesfully employed over the past several years. The performance criterion due to this tasks in the field of code synthesis is always the acf or more precisely some properties of the acf, for example the maximum sidelobe. R.H. Barker considered in 1953 the question of the existence of finite binary sequences with the property that their aperiodic acf sidelobe coefficients should be restricted to -1, 0, +1.

Code synthesis problems with binary sequences lead almost always to large search tasks in a set M of $2^N$ different sequences $c_k$, where N describes the length or a single periode length of the considered sequence. In this consideration the binary signal coefficients (-1, +1) are the only and free parameters which should be optimized with the property that their acf sidelobe coefficients $a_k$, k=1,2,...,N-1 are as small as possible. The aperiodic acf coefficients $a_k$ and the periodic acf coefficients $p_k$ are calculated as follows, the second index modulo N.

$$a_i = \sum_{k=0}^{N-1-i} c_k\, c_{k+i} \qquad i = 0, 1, \cdots, N-1 \quad (1)$$

$$p_i = \sum_{k=0}^{N-1} c_k\, c_{k+i} \quad (2)$$

In the present paper a different binary code synthesis problem is discussed. Given a binary sequence $c_k$ the question arises whether a receiver mismatched-filter with coefficients $w_k$ exists which has the property that all ccf sidelobe coefficients $\varrho(i)$ are zero. It is important to note, that in this paper the meaning of cross-correlation is the correlation between the given sequence $c_k$ and the filter coefficients $w_k$ which is described by the following equation and contains some differences to the conventional meaning of cross-correlation, where two different sequences are correlated.

$$\varrho(i) = \sum_{k=0}^{N-1} c_{k+i}\, w_k = \begin{cases} 1 & if \quad i = 0 \quad mod(N) \\ 0 & else \end{cases} \quad (3)$$

It is an interesting result that a large number of sequences $c_k$ and belonging mismatched-filter coefficients $w_k$ exist which fulfil the property (3) above.

The compression gain, that means the increase in signal-to-noise ratio is defined as follows

$$I_{MMF}(N) = \frac{\left(\sum_{k=0}^{N-1} c_k w_k\right)^2}{\sum_{k=0}^{N-1} w_k^2} \quad (4)$$

In opposite to eq. (4) the increase in signal-to-noise ratio for a matched-filter (MF) and arbitrary sequence $c_k$ of length N is a constant value $I_{MF}(N) = N$.

The quotient

$$I = I_{MMF}(N)/I_{MF}(N) \quad (5)$$

is used in this paper as a normalized performance parameter which is called efficiency or normalized efficiency. In the case of mismatched-filter design the synthesis task "reduces" to the problem of finding that sequence $c_k$ which shows the maximum value in eq. (4).

# 2 Calculation of Mismatched-filter Coefficients

An arbitrary binary periodical sequence $c_k$ is considered where the coefficients $c_k$ of a single period form a vector c and the mismatched-filter coefficients $w_k$ form a vector w of length N.

$$\begin{aligned} c &= (c_0, c_1, \ldots, c_{N-1})^T \\ w &= (w_0, w_1, \ldots, w_{N-1})^T \end{aligned}$$

The mismatched-filter coefficients $w_k$ (if they exist) shoul solve the following equation (5), which shows the ccf between signal $c_i$ and normalized receiver filter coefficients $w_k$.

$$\varrho_{cw}(i) = \sum_{k=0}^{N-1} c_{k+i} w_k = \begin{cases} 1 & if \quad i = 0 \quad mod(N) \\ 0 & else \end{cases} \quad (6)$$

If the ccf has the property of eq. (5) it is called a "perfect periodical ccf". Equation (5) can be written in matrix form

$$S \cdot w = g \quad (7)$$

where the signal matrix S has a cyclic Toeplitz form

$$\mathbf{S} = \begin{pmatrix} c_0 & c_1 & c_2 \cdots & c_{N-1} \\ c_{N-1} & c_0 & c_1 \cdots & c_{N-2} \\ c_{N-2} & c_{N-1} & c_0 \cdots & c_{N-3} \\ \vdots & \vdots & \vdots & \vdots \\ c_1 & c_2 & c_3 \cdots & c_0 \end{pmatrix} \quad (8)$$

and the vector $g = (1, 0, ..., 0)^T$ is a unit vector.
The mismatched-filter coefficients $w_k$ are calculated by a matrix product between the inverse signal matrix S (if it exists) and the unit vector $g = (1, 0, ..., 0)^T$ .

$$w = S^{-1} \cdot g \quad (9)$$

Different search methods were used and some results are given in table 1 above for code lengths N up to N=16. The following Figure 1 shows the respevtive maximum relative efficiency values for the analysed sequences up to N=100. The mismatched-filter losses against the matched-filter technique are in many cases smaller than 0.5 dB signal-to-noise ratio.

| length N | sequence $c_k$ | filter coefficients $w_k$ | efficiency $I_{MMF}(N)$ | relative efficiency |
|---|---|---|---|---|
| 3 | 1 1 -1 | 1 1 0 | 2.0 | 66.67% |
| 4 | 1 1 1 -1 | 1 1 1 -1 | 4.0 | 100.00% |
| 5 | 1 1 1 1 -1 | 1 1 1 1 -2 | 4.5 | 90.00% |
| 6 | 1 1 1 1 1 -1 | 1 1 1 1 1 -3 | 4.57 | 76.19% |
| 7 | 1 1 1 1 1 1 -1 | 1 1 1 1 1 1 -4 | 4.55 | 64.93% |
| 8 | 1 1 1 1 -1 -1 1 -1 | 1 1 3 1 -1 -1 3 -1 | 6.0 | 75.00% |
| 9 | 1 1 1 1 1 -1 1 1<br>-1 | 1 1 6 1 1 -4 1 1<br>-4 | 5.41 | 60.11% |
| 10 | 1 1 1 1 1 -1 1 -1<br>-1 -1 | 9 19 1 7 5 -9 25 -1<br>-7 -5 | 5.97 | 59.66% |
| 11 | 1 1 1 1 1 -1 1 1<br>-1 -1 -1 | 71 20 47 17 80 -79 68 53<br>-49 5 -55 | 8.52 | 77.47% |
| 12 | 1 1 1 1 1 -1 -1 1<br>1 -1 1 -1 | 2 1 1 2 1 -1 -2 1<br>1 -2 1 -1 | 10.67 | 88.92% |
| 13 | 1 1 1 1 1 -1 -1 1<br>1 -1 1 -1 1 | 2 2 2 2 2 -3 -3 2<br>2 -3 2 -3 2 | 12.5 | 96.15% |
| 14 | 1 1 1 1 1 1 -1 -1<br>1 1 -1 1 -1 1 | 1 4 4 1 4 1 -5 -5<br>4 4 -5 4 -5 1 | 11.52 | 82.29% |
| 15 | 1 1 1 1 1 1 -1 1<br>-1 -1 1 1 -1 -1 -1 | " | 13.02 | 86.81% |
| 16 | 1 1 1 1 1 1 -1 1<br>-1 1 -1 -1 1 1 -1 -1 | 3 21 45 39 27 5 -27 39<br>-45 21 -3 -25 43 37 -43 -25 | 12.71 | 79.43% |

**Table 1:**
In order to solve the synthesis problem, different construction methods were used. The above binary sequences $c_k$ were found to have the highest efficiency values for each given lenght N.

# 3 Radar applications

For practical applications there is only a small modification necessary in real receiver systems. The binary matched-filter coefficients are changed to longer integer numbers (mismatched-filter coefficients $w_k$). Figure 2 shows the two possible system design methods with matched and mismatched-filters in the system receiver and the resulting acf and ccf properties respectively.

Practical applications for periodical binary sequences are given in the field of radar range measurement, synchronazation, code multiplex, spread spectrum communication, system measuring and identification for example. The basic reason for the developement of binary sequences from an application point of view is that they have anti-jamming and low probabilty of intercept (LPI) capabilities. This feature is of importance especially in radar /3,5/ and spread spectrum communication applications. A waveform design for pulse and continuous-wave (CW) radars is presented using pseudorandom coded biphase modulation to achieve unambiguous measurements of range and Dopplerfrequency.

In pulse radars with high pulse repetition frequencies (HPRF) a phase-coded waveform design can be used where the signal phase is changed from pulse to pulse. Figure 3 shows a conventional HPRF waveform (a) and a very high PRF waveform design (b). The discussed mismatched-filter technique used in this radar receiver is a special pulse compression method. Using this mismatched-filter coefficients in the radar receiver no range sidelobes will be observed at all. At the same time the losses in signal-to-noise ratio are small when compared with the matched filter technique. The system has no blind speeds in the target Dopplerfrequency, and the maximum unambiguous range can be made out to any desired distance.

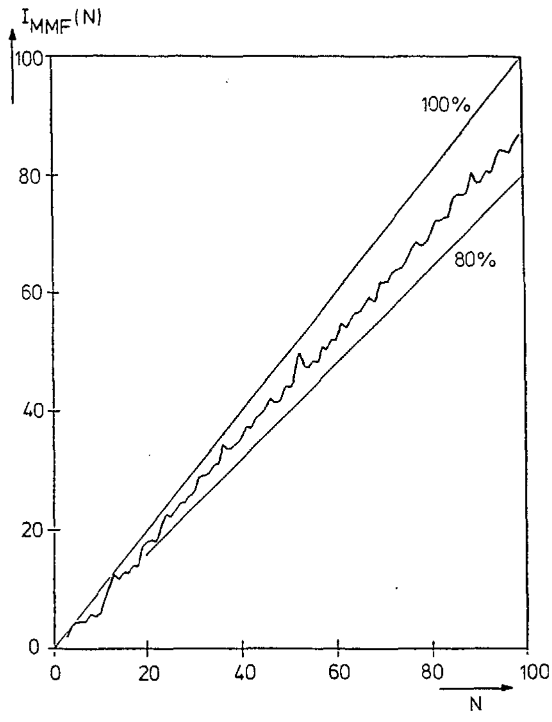


**Figure 1:**
Largest efficiency values for codeword lengths $N \leq 100$ found with different synthesis methods. The codewords $c_k$ and the belonging mismatched-filter coefficients $w_k$ are given in Table 1 for $N \leq 16$ .

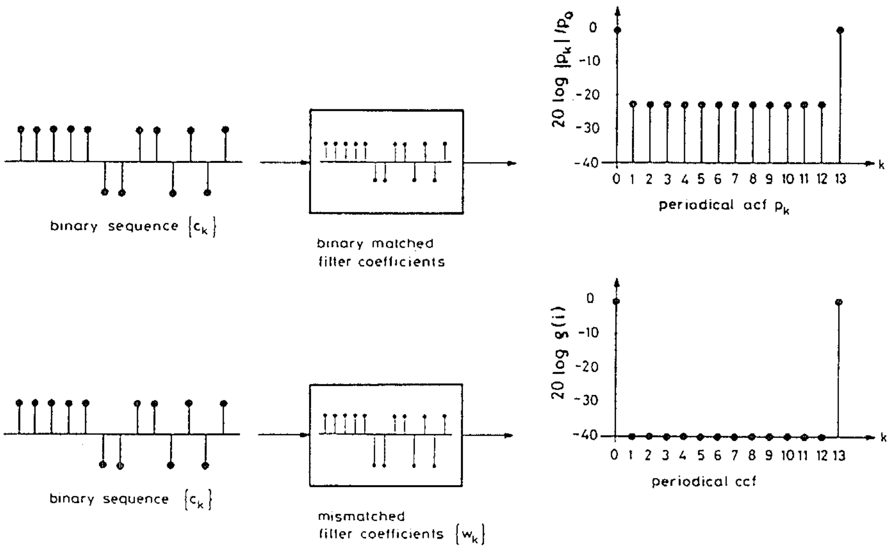


**Figure 2:**
Radar receiver concepts with matched- and mismatched-filter design and the resulting output signal.

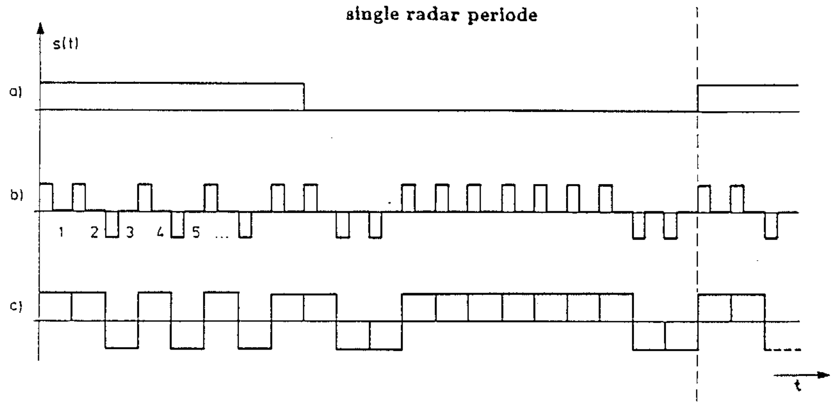


**Figure 3:**
Waveform design for radar applications.
a) conventional HPRF radar waveform
b) very high PRF waveform } in connection with the
c) CW radar application } mismatched-filter receiver technique

Figure 3c) shows a waveform design for a continuous-wave (CW) radar using a coded binary phase modulation in the transmitted signal /5/. Both the binary phase signal and the mismatched-filter coefficients in the radar receiver can be implemented as described in Table 1. These radar waveform design methods allow a simultaneous range and Doppler frequency measurement even with a HPRF radar. The wave form parameters can be chosen in a way that the resolution in both coordinates is sufficiently high. Occuring range ambiguities can be removed by staggering the code clock frequency or changing the codeword lengths during the time on target.

The discussed mismatched-filter design is an alternative signal processing and pulse compression technique for radar applications, communication techniques (spread spectrum) and gives furthermore an interesting theoretical insight into properties of binary and periodical sequences. It is obvious that the mismatched-filter design can be extended to polyphased signals.

## 4 References


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