DETECTION USING CROSS-TERMS IN THE WIGNER-VILLE DISTRIBUTION

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RESUME

L'utilisation de la distribution de Wigner-Ville (DWV) pour l'analyse temps-fréquence des signaux non-stationnaires présente, en comparaison avec d'autres méthodes, plusieurs avantages, mais en même temps elle possede un inconvénient essentiel: génération des éléments interférentiels entre de différentes composantes du signal. Très souvent ce défaut rend difficile l'interprétation du spectre. L'objectif de l'article est la mise en évidence du fait que cet inconvénient essentiel de la DWV est très avantageux du point de rue dela détection des signaux.

L'article rappelle brièvement le méchanisme de la génération des interferences réciproques dans la DWV, il recapitule la formule temps-fréquence de la détection optimale à l'aide de DWV et propose l'idée de la nouvelle méthode de détection qui exploite drectement des interferences en DWV. Dans la méthode proposée le signal de test est additionné au signal analisé et on compte le spectre réciproque entre le signal de test et le signal d'entrée modifié. Parceque les interferences sont toujours symétriques dans l'espace temps-fréquence entre les objects qui les génèrent, si l'on connait la localisation temps-fréquence du signal de test et de toutes ses interferences avec des composantes non-connues du signal analisé, on peut trouver les principes temps-fréquence de la modulation de ces composantes, en appliquant de simples règles géometriques.

En comparaison avec des méthodes standard, l'idée proposée réalise la détection des composantes du signal de la manière "indirecte". Dans l'article cette méthode est decrite avec plus de détails et on présente des examples experimentoux de son application.

La méthode est la plus utile dans la verification adjointe de l'hypothèse.

SUMMARY

Using the Wigner-Ville distribution (WVD) for general purpose time-frequency (TF) analysis of nonstationary signals has a lot of advantages over the other methods but has also one very important drawback: generation of cross-terms between different signal components which makes the interpretation of the spectrum difficult in many cases. A goal of this paper is to emphasize the fact that the main weakness of the WVD is profitable in detection applications.

The paper briefly reviews the cross-terms generation mechanism of the WVD, recapitulates the TF formulation of optimum detection by means of the WVD and puts forward an idea of new detection scheme that directly take advantage of the cross-terms generation in the WVD. In the proposed method a test signal is added to the analyzed one and a cross WV-spectrum between the test and the modified input signal is computed. Since the cross-terms always lies symmetrically in the mixed TF plane between objects whih generate them, knowing the TF localization of the test signal and its interferences with all unknown components of analyzed signal we can find TF mixed time-frequency signal representations (MTFRs) modulation laws of these components using simple geometrical rules.

In comparison with standard techniques the presented idea realizes the detection of signal components in indirect way. In the paper the method is described in a more detailed way and initial experimental examples of its using are given.

The proposed method can be especially useful for additional verification of hypothesis.

1. INTRODUCTION

The Wigner-Ville distribution (WVD) has been established recently as one of the basic tools in time-varying signal analysis and signal processing [1-3]. It is due to its many advantages over the other mixed time-frequency signal representations (MTFRs) from the class of Cohen. But because of its bilinearity the WVD possesses one feature which is rather unpleasant for a common user: generation of the so-called cross-terms [4-7]. They make interpretation of the WV-spectrum very difficult in many cases. The goal of this paper is to show that the main weakness of the WVD is profitable in detection applications.

The structure of the paper is as follows: in section 2 the WVD is introduced and a short description of the cross-terms generation is made, in section 3 standard methods of the WVD using for

detection are briefly reviewed and in section 4 a presentation of the proposed indirect detection method is made.

2. THE WIGNER-VILLE DISTRIBUTION AND CROSS-TERMS GENERATION

The WVD is a pseudo power spectral density function that describes a signal by the distribution of its power density in the mixed time-frequency (TF) plane. The auto WVD (AWD) of a signal $x(t)$ is defined by the form [1,3]

$$AWD^x(t,\omega)=\int_{-\infty}^{+\infty} x(t+\frac{\tau}{2})x^*(t-\frac{\tau}{2})e^{-j\omega\tau}d\tau \qquad (1)$$

and is a function of time and frequency. The cross WVD (CWD) of two signals $x(t)$ and $y(t)$ is given by the formula [1,3]

$$CWD^{x,y}(t,\omega)=\int_{-\infty}^{+\infty} x(t+\frac{\tau}{2})y^*(t-\frac{\tau}{2})e^{-j\omega\tau}d\tau \ . \qquad (2)$$

As it can be seen the auto WV-transform (1) is not a linear operation on signal. It can be interpreted as the Fourier transform performed over variable τ on the result of multiplication of the signal $x(t+\frac{\tau}{2})$ and its reversed and conjugated form $x^*(t-\frac{\tau}{2})$ [3]. In this situation a power spectral density function is estimated and the multiplication is performed on the signal before the Fourier transform on the contrary to the spectrogram, another MTFR, where it is performed after it on the already computed standard Fourier spectrum. The consequences of this fact are tremendous for the features of the WVD.

The definition of the auto WV-spectrum (1) gives a lot of advantages for the WVD in comparison with the other MTFRs (a better and separable resolution in time and frequency, concentration of signal power near the instantaneous frequency for monocomponent signals, etc.) but at the same time due to its bilinearity it causes the generation of the so called cross-terms in the resultant spectrum of the signal [4-7] which make its interpretation cumbersome in many cases [9-11].

Let $x(t)$ be a multicomponent signal

$$x(t)=\sum_{i=1}^{n} s_i(t). \qquad (3)$$

Then the WVD of the signal $x(t)$ is given by the following formula [6]

$$AWD^x(t,\omega)=\sum_{i=1}^{n} AWD^{s_i}(t,\omega)+\sum_{\substack{i=1 \\ i>j}}^{n}\sum_{j=1}^{n} 2Re\{CWD^{s_i,s_j}(t,\omega)\} \qquad (4)$$

It can be seen that the resultant auto spectrum of $x(t)$ is not simply equal to the sum of auto spectra of $s_i(t)$, $i=1,..,n$, but there also exist in it cross spectra between its every two different components $s_i(t)$. These spectra are called cross-terms, they have an oscillatory nature and no physical significance.

For the two component signal $x(t)=s_1(t)+s_2(t)$, (4) is simplified to the form

$$AWD^{s_1+s_2}(t,\omega)=AWD^{s_1}(t,\omega)+AWD^{s_2}(t,\omega)+$$
$$+2Re\{CWD^{s_1,s_2}(t,\omega)\} \qquad (5)$$

Separate computation of the cross-WVD of signals $s_1(t)$ and $s_2(t)$ with $x(t)$ gives the following results

$$CWD^{s_1,s_1+s_2}(t,\omega)=AWD^{s_1}(t,\omega)+CWD^{s_1,s_2}(t,\omega) \qquad (6)$$

$$CWD^{s_2,s_1+s_2}(t,\omega)=AWD^{s_2}(t,\omega)+CWD^{s_2,s_1}(t,\omega) \qquad (7)$$

It is obvious that

$$AWD^x(t,\omega)=Re\{CWD^{s_1,x}(t,\omega)+CWD^{s_2,x}(t,\omega)\}. \qquad (8)$$

Fig.1 exemplifies the cross-terms mechanism of the WVD. It visualizes cross WV-spectra (2) for different signals (fig.a÷d -time series $x(t)$ and $y(t)$, fig.e÷h -real parts of the CWDs, fig.i÷l - appropriate imaginary parts of the CWDs). All the presented spectra were computed for the analytic signals.

The origin of cross-terms is explained in detail in [4-6] and methods for its reduction are discussed in [7-11].

3. STANDARD DETECTION METHODS USING THE WVD

The addressed detection problem is the following [12]:

$$\left.\begin{array}{l} H_0: x(t)=w(t) \\ H_1: x(t)=w(t)+s(t) \end{array}\right\} \quad t\in(T)$$

where the observed (complex) signal $x(t)$ is known on the time interval (T), $w(t)$ is zero mean complex white Gaussian noise (whose real and imaginary parts are independent and of equal power spectral densities) such that

$$E[w(t)]=0; \ E[w(t)w^*(u)]=N_0\delta(t-u),$$

and $s(t)$ is the (complex) nonstationary Gaussian signal to be detected, and characterized by

$$E[s(t)]=m(t); \ E[(s(t)-m(t))(s(u)-m(u))]=K_s(t,u).$$

(E denotes the expectation operator and the star the complex conjugation).

The optimum detector can be written as [12]

$$l_R+l_D \mathop{\substack{H_1 \\ \lessgtr \\ H_2}} \gamma \qquad (9)$$

where γ is a threshold and l_R and l_D will be defined later.

When $s(t)=f(t)$ and $f(t)$ is deterministic and known, the values l_R and l_D in (9) are as follows [12]:

$$l_R=0,$$

$$l_D=A\int_{-\infty}^{+\infty}\int_{(T)} Re\{CWD^{x,f}(t,\omega)\}AWD^f(t,\omega)dt\frac{d\omega}{dt} \ , \qquad (10)$$

where A is a constant.

In the case $s(t)=bf(t)$, where $f(t)$ is deterministic and known and b is a zero-mean complex Gaussian variable, we have [12]:

$$l_R=B\int_{-\infty}^{+\infty}\int_{(T)} AWD^x(t,\omega)AWD^f(t,\omega)dt\frac{d\omega}{dt}, \qquad (11)$$

$$l_D=0.$$

where B ia a constant.

As it can be seen from (10)(11) the detection procedures based on the WVD perform masking in the TF plane of the $Re\{CWD^{x,f}\}$ (10) and AWD^x (11) by the AWD^f of the reference $f(t)$ and then calculate its energy contained in the "shadow" of the AWD^f. In the special case of chirp signals this operation is simplified to the line integration [13]. When the $s(t)$ is present in $x(t)$, spectra of the $Re\{CWD^{x,f}\}$ and AWD^x lying in the "shadows" of the AWD^f are approximately (noise component) equal to the AWD^f and do not have oscillatory cross-terms.

Figure 1 illustrates this situation. It presents some cross WV-spectra of two signals: $r_1(t)$ and $x(t)=s_1(t)+s_2(t)$ where $r_i(t)$, $s_i(t)$ are sinusoids with Gaussian envelopes and $r_1(t)$ is a reference of $s_1(t)$. When the reference signal $r_1(t)$ takes the same time position as $s_1(t)$ in $x(t)$, in the resultant cross WV-spectrum it is received:
- in the real part of it the cross WVD of the signals $r_1(t)$ and $s_1(t)$ (which is equal to the auto WVD of the signal $r_1(t)=s_1(t)$) and a half (as a value) of the oscillatory cross-terms between $r_1(t)$ and $s_2(t)$ in comparison with the auto WVD of $x(t)$ (5)(6)(7)(8);
- only cross-terms, in the imaginary part, which take the same place in the TF plane as the cross-terms in the real part but they are shifted in phase about 90° in comparison with them.
When $r_1(t)$ does not coincide in time with the $s_1(t)$ the both parts of the cross spectrum have entirely oscillatory nature since in this case the cross WVD between signals $r_1(t)$ and $s_1(t)$ can not be simplified by the auto WVD of one of them.

The cross-terms generation plays a crucial role in the "matching window" analysis technique of Jones and Parks [14].
Additional information concerning using the WVD for detection can be found in [12-17].

4. INDIRECT DETECTION METHOD

Let add a test signal $r(t)$ to the analyzed data $x(t)$ (3) and compute a cross WVD between $r(t)$ and $x(t)+r(t)$

$$CWD^{r,x+r}(t,\omega)=AWD^{r}(t,\omega)+\sum_{i=1}^{n}CWD^{r,s_i}(t,\omega) \qquad (12)$$

As a result we receive in the real part of the spectrum (12) the auto WVD of $r(t)$ and its interferences with all unknown components of $x(t)$ and in the imaginary part we get only the interferences. Because oscillatory cross-terms always emerge symmetrically between every two components of multicomponent signal and they are characterized by a very regular geometrical pattern [6], when the test signal and its cross-terms with unknown signal components are known approximate time-frequency laws of these components can be found with the use of geometrical rules [6].
The proposed idea is demonstrated in fig.2 and fig.3. Fig.2a presents the auto WV-spectrum of analyzed exemplary signal $x(t)$ which consists of two different sinusoids with Gaussian time envelopes $(x(t)=s_1(t)+s_2(t))$ and figures 2b and 2c show the real parts of the cross WV-spectra between the test signal $r(t)$ (also a sinusoid with Gaussian envelope) and a composite signal $x(t)+r(t)$ for two different time positions of the test signal $r(t)$. Figures 2d and 2e present the same spectra as figures 2b and 2c but with other contour values. Modulation laws of the components $s_i(t)$ are found approximately with the use of simple geometrical rules and they are marked in figures 2b and 2c. Simple example of detection of the chirp signal with Gaussian time envelope is demonstrated in figures 3a and 3b.

Special classes of test signals should be chosen for detection of different signals. To assure higher accuracy of the method at least two test signals or one in two different time positions should be used for precise localization of each signal component.
The presented indirect detection technique does not require a time-consuming computing of many WV-spectra as the standard ones (10)(11), but instead of this it should use comprehensive pattern recognition methods when used for unsupervised detection. It can be used separately or as an additional tool for verification of a hypothesis. In the second case, when the localization of the

cross-terms is known approximately, the application of the method is easier, more appropriate and more effective and probably it can improve the detection resolution.

5. CONCLUSIONS

It was shown that in the case of signal detection the cross-terms generation in the Wigner-Ville distribution plays a very important role.
The proposed new detection idea based on the symmetry feature of cross-terms generation was initially tested. It looks attractive but requires an additional development of tools for an unsupervised detection.
The presented method can be especially useful for additional verification of hypothesis.

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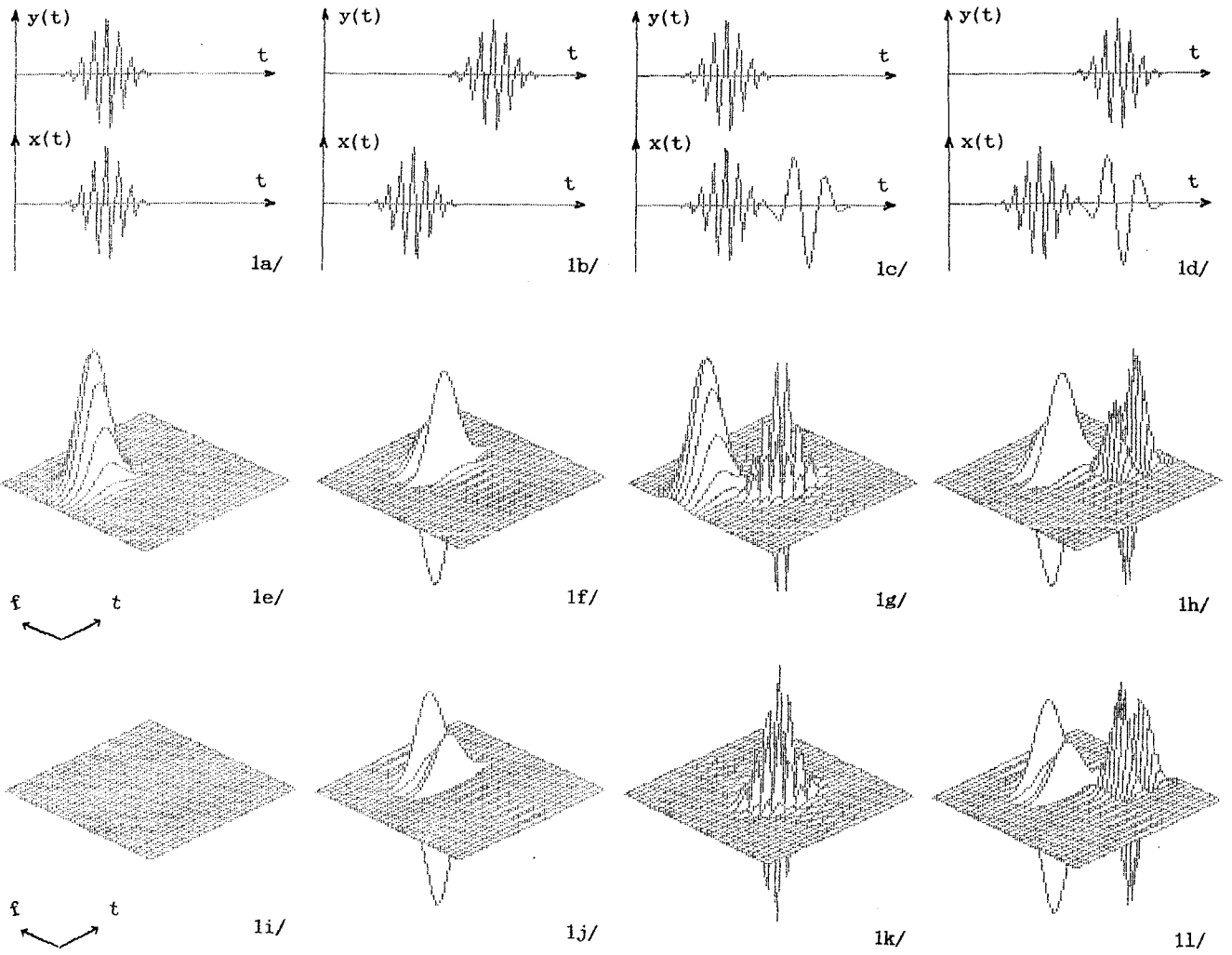[17] Mullick, S. K. et V. A. Topkar. A Wigner distribution based receiver. Signal Processing, 14, 1988, No.2, 185-196.
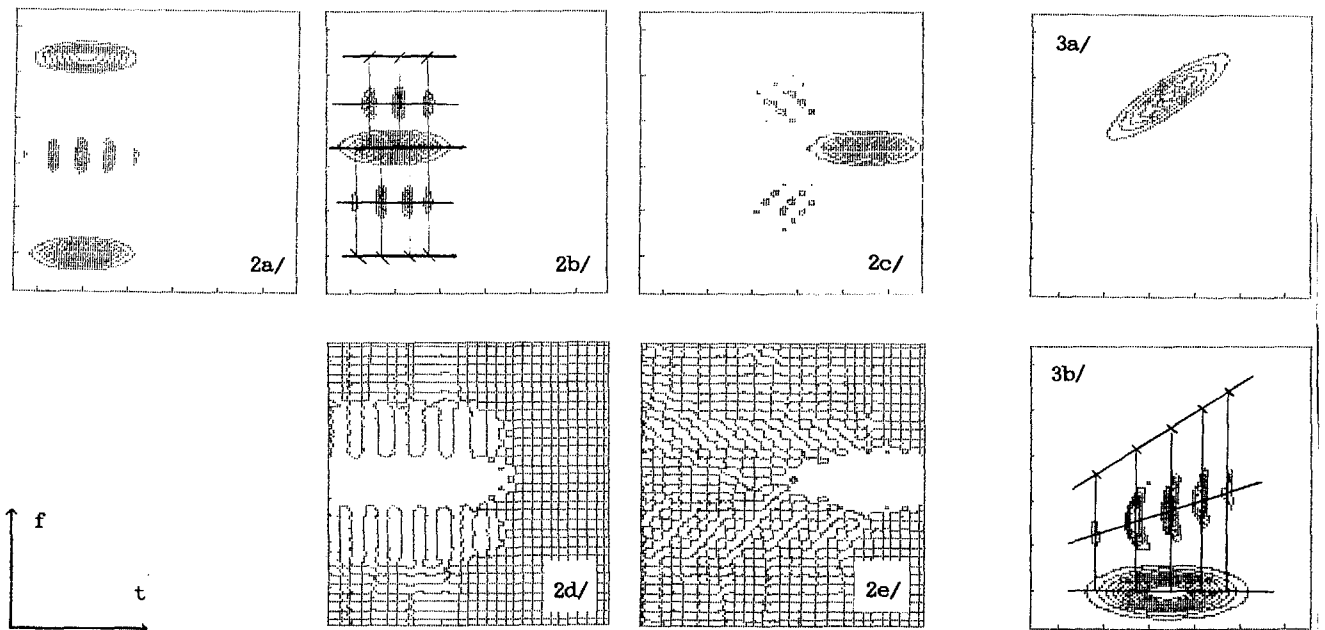
FIGURE 1

FIGURE 2

FIGURE 3