REDUCED ORDER MODELING OF HARMONIC SIGNALS

Marina V. Dragošević*, Slobodan D. Jovanović* and Srdjan S. Stanković**

*"Boris Kidrič" Institute of Nuclear Sci., Computer System Design Lab., Vinča
P.O.Box 522, 11001 Belgrade, Yugoslavia

**Faculty of Electrical Engineering, University of Belgrade,
Bulevar revolucije 73, 11000 Belgrade, Yugoslavia

RESUME

Dans cet article on traîte le problème d'éstimation des fréquences des sinusoides dans le cas où la structure du modèle n'est pas connue a priori. Deux algorithmes sont proposés pour l'éstimation simultanée de la fréquence et de l'ordre du modèle. Le premier, applicable "off-line", est basé sur la métode des moindres carrés generalisée, tendis que le second est recursif, basé sur la méthode RML. Dans les deux cas on suppose que le modèle est dans la forme des "notch" filtres en cascades. L'éstimation de l'ordre est basé sur les tests directs de la puissance des signaux résudus. Les résultats de l'analyse experimentale donnent une illustration des qualités des méthodes proposées, qui représentent un outil simple, mais efficace, pour les applications pratiques.

SUMMARY

In this paper the problem of estimating frequencies of noisy sinusoids when the exact model structure is not a priori known is considered. Two algorithms are proposed for simultaneous frequency and model order estimation. The first one, applicable off-line, is based on the generalized least-squares method, while the second one is recursive, based on the RML method. The signal model is supposed to be in the cascaded notch form. Order estimation is based on direct residual power tests. Experimental results illustrate the characteristic properties of the methods which represent simple and reliable tools for practice.

1. INTRODUCTION

Estimation of frequencies of noisy sinusoids has attracted a great deal of attention of researchers from the field of signal processing. Numerous methods have been proposed starting from the general concepts developed within system identification [1-4]. Some of these methods are applicable off-line [1-5], while in some applications it is desirable to generate the estimates recursively [6-9]. The algorithms have mostly been constructed and verified starting from the supposition that the number of sinusoids is a priori known. A strategy based on the introduction of an additional recursive algorithm for amplitude and phase estimation has been introduced to deal with signals composed of an unknown number of noisy sinusoids, [4,8]. This procedure requires, however, finding the roots of high-order polynomials, together with an extra effort for amplitude estimation and the corresponding detection. Recently, recursive methods of the maximum likelihood type have been proposed for the signal models in the cascaded form consisting of second order notch blocks, so that the unknown frequencies are estimated directly one by one, without requiring polynomial rooting, [10-12]. Estimation of the number of sinusoids is done again by attaching an amplitude estimation algorithm to each block in the cascade, [10].

The problem of estimating frequencies of noisy sinusoids in the case when their number is not a priori known is addressed in this paper. It is assumed that the signal model is in the cascaded form of second order notch blocks with symmetric structure and contracted poles. First an off-line algorithm of the generalized least squares type is discussed. It is demonstrated that the sequential estimation of the frequencies (one by one) approaches the efficiency attained in the case when the model order is exactly known [3]. The experimental results also show that the number of sinusoids can be estimated efficiently using simple tests of residuals at the output of the notch blocks. In the second part a recursive algorithm derived from the same model is constructed, by using the methodology of [8]. It is shown that the algorithm provides an efficient tool for both reduced order estimation of harmonic signals and estimation of the number of sinusoids in a recursive way, by performing simple residual tests (without requiring any additional amplitude and phase estimation).

2. PROBLEM FORMULATION AND ESTIMATION ALGORITHMS

We shall consider the problem of estimating frequencies of n sinusoidal components in the signal

$$y(t)=\sum_{i=1}^{n} A_i \sin(\omega_i t+\theta_i)+\varepsilon(t) \qquad (1)$$

where $\{\varepsilon(t)\}$ is a zero mean white noise process. Frequency estimation algorithms constructed on the basis of system identification methods are usually related to the dynamic model

$$A(q^{-1})y(t)=A(q^{-1})\varepsilon(t) \qquad (2)$$

where $A(q^{-1})=a_0+a_1q^{-1}+\dots+a_{2n}q^{-2n}$; $a_j=a_{2n-j}$, $j=0,1,\dots,n$, $a_0=1$, [1,3]. As $A(z)$ has its zeros on the unit circle, we shall introduce the pole contraction factor α and assume that the signal model for identification purposes is $A(q^{-1})y(t)=A(\alpha q^{-1})\varepsilon(t)$, $0<\alpha\le 1$. Factorization of $A(q^{-1})$ and $A(\alpha q^{-1})$ leads to the following approximate model of (1)

$$\varepsilon(t)=\prod_{i=1}^{n} H_i(q^{-1})y(t)=\prod_{i=1}^{n}\frac{1+a_iq^{-1}+q^{-2}}{1+a_i\alpha_iq^{-1}+\alpha_i^2q^{-2}}\,y(t) \qquad (3)$$

where $a_i=2\cos\omega_i$ and α_i can vary from block to block; $H_i(q^{-1})$ (i=1,...,n) are second order notch filters for the n frequencies in the signal y(t). Model (3) has been used as a starting point for constructing adaptive notch filters in cascaded form in [10-12].

2.1. Off line algorithm

An off line algorithm for estimating unknown frequencies $\omega_1,\dots,\omega_n$ on the basis of N signal samples $Y_N^T=[\,y(1)\dots y(N)]$ can be directly derived from (3) by using the generalized least-squares (GLS) methodology proposed in [3]. The algorithm consists of the following steps:

1. Let i=1;
2. Assume the second order model $H_i(q^{-1})$ and get the estimate $\hat{\omega}_i$ by using the GLS algorithm of [3];
3. Filter data through $H_i(q^{-1})$;
4. Terminate if all frequencies are estimated; otherwise increase i by one, replace the original data by the filtered data and go to step 2.

The GLS algorithm applied in step 2. is described in detail in [3]; it resembles the methods described in [4,5], but it is better adapted to the model structure in (3). It consists essentially of an iterative application of the LS algorithm to data obtained by filtering measurements through $1/A(\alpha_i q^{-1})$, where the contraction factor α_i can be made data adaptive [3]. The step 4. contains, essentially, a detection procedure, indicating the presence of sinusoids in the signal obtained after multiple filtering. The structure of (3) indicates that simple tests of decrease in the filtered signal power (during the successive passes through steps 2. to 4.) can lead to a reliable estimate of the number of sinusoids. This approach is straightforward in the case of sinusoids with nearly eqaual powers, but the situation may be somewhat different if weak sinusoidal components exist in the signal along with the strong ones. Then the rejection of the strong sinusoids, performed by the leading sections of the described algorithm, may be insufficient. Weak sinusoidal signals can be detected by increasing the number of notch blocks and continuing the procedure in order to find frequencies differing from the preceding ones. The algorithm is applied until a treshold of the decrease in the residual power is reached and it depends on the required sensitivity to weak harmonic signals.

2.2. Recursive algorithm

A recursive algorithm for estimating parameters in the cascade (3) consists of a set of recursions, each estimating one of the unknown frequencies, i.e. a single parameter a_i in (3). Each recursion has the form of the adaptive minimal parameter notch filter proposed in [8], which represents a version of the recursive maximum likelihood (RML) algorithm adapted to the specific model structure. According to the idea presented in [8] the first block utilizes the row measurement data, while the data sets for all other notch blocks consist of residual (a posteriori prediction error) sequencies coming out from the preceding notch block. For example, the second section is fed by

$$\hat{\varepsilon}_1(t)=\hat{H}_1(q^{-1},t)y(i)$$

where $\hat{H}_1(q^{-1},t)$ represents a time varying filter obtained by using the corresponding parameter estimates

$$\hat{H}_1(q^{-1},t)=\frac{1+\hat{a}_1(t)q^{-1}+q^{-2}}{1+\hat{a}_1(t)\alpha_1q^{-1}+\alpha_1^2q^{-2}}$$

The crucial theoretical problem is to demonstrate that the reduced order model of a specific structure, which represents one block in the cascade (3), can pick up successfully one of the frequencies when applied to raw data. The analysis of criterion extrema of the corresponding criterion function shows that if the algorithm converges it will converge to one of the existing unknown frequencies. However, it is difficult to derive conditions for the global convergence. The undertaken simulation studies show that the algorithm behaves very well in practice provided a good care is taken of the relevant parameters in the algorithm influencing convergence properties. The first among these parameters is the contraction (or debiasing, notch) parameter α_i in (3), which is taken to be time varying and genereted recursively by

$$\alpha_i(t)=\alpha_{io}\alpha_i(t-1)+(1-\alpha_{io})\alpha_{if} \qquad (4)$$

where $\alpha_i(0)$ and α_{if} represent the initial and the final values of $\alpha_i(t)$, respectively, while α_{io} defines the rate of change in $\alpha_i(t)$. The initial low values enlarge the notch filter bandwidths, while the high final values (close to 1) sharpen the frequency responses and enable both debiasing and preparation of data for the subsequent recursions. The second important parameter is the forgetting factor in the algorithm [8], which is also taken to be time varying and generated recursively by

$$\rho_i(t)=\rho_{io}\rho_i(t-1)+(1-\rho_{io})\rho_{if} \qquad (5)$$

where $\rho_i(0)$ and ρ_{if} are the initial and final values of $\rho_i(t)$, respectively, and ρ_{io} determines the rate of change in $\rho_i(t)$. The experimental results presented in the next paragraph will show that with $\alpha_i(0)$ low enough, α_{if} close to 1, $\rho_i(t)$ decreasing with t and $\rho_{if}=1$ it is possible to obtain good convergence properties even in the case of a large number of sinusoids and low SNR's.

Frequency estimation methodology, based on the signal model in the cascaded form, is particularly convenient for the recursive estimation of the number of sinusoids present in the signal. The main advantage here is the decoupling effect of the estimation blocks allowing for simple power testing at the notch outputs, which can directly be related to the magnitude of the removed sinusoid. Thus the polynomial factorization required in [8] is avoided, as well as the additional LS amplitude estimation suggested in [13]. The proposition of this paper is to introduce a sufficiently large number of cascades, m>n, and to recursively estimate the powers of the original signal and all the residuals (for i=0,...,m; $\hat{\varepsilon}_o(t)\equiv y(t)$) by

$$p_i(t)=p_i(t-1)+\frac{1}{t-t_o}\left(\hat{\varepsilon}_i(t)^2-p_i(t-1)\right) \quad \text{for } t>t_o \qquad (6)$$

where t_o is an initial time required for the algorithm to stick to the line components in the signal spectrum. A decrease from $p_{i-1}(t)$ to $p_i(t)$ serves as an indication whether the corresponding section provides a frequency estimate of a truely existing sinusoid or not. If the ratio $r_i(t)$ between $p_i(t)$ and $p_{i-1}(t)$ falls below a predefined treshold, the decision is made that the frequency estimate does not correspond to a sinusoid. The number of blocks for which $r_i(t)$ exceeds the given treshold represents an estimate of the number of sinusoids at the instant t.

3. EXPERIMENTAL STUDY

Properties of the off-line algorithm are examined both on long and short data sequencies; in the first case asymptotic properties can be seen, while in the second case the practical efficiency can be tested. The signal is composed of two sinusoids with frequencies $\omega_1=0.4\pi$ and $\omega_2=0.6\pi$ in white noise with partial SNR's of 0 dB. The Monte Carlo simulation results (rms of the frequency estimation error) are presented in Table 1 for the data length N=1000, and in Table 2 for N=50. The algorithm based on the reduced order model cascaded sections and the sequential removal of the sinusoids from the signal (1) is only slightly inferior to the GLS algorithm applied to the model (2) incorporating both sinusoids (columns 1 and 4 vs. columns 2 and 5). Columns 3 and 6 describe the estimates in the case when only single sinusoids are present in the measured signal. The number of sinusoids has been assumed known.

$\omega_1=0.4\pi$			$\omega_2=0.6\pi$		
n=2		n=1	n=2		n=1
model (3)	model (2)		model (3)	model (2)	
.000036	.000034	.000035	.000025	.000026	.000025

Table 1

$\omega_1=0.4\pi$			$\omega_2=0.6\pi$		
n=2		n=1	n=2		n=1
model (3)	model (2)		model (3)	model (2)	
.011945	.004128	.003749	.008612	.002845	.002549

Table 2

Table 3 presents typical results illustrating the efficiency of the algorithm in detecting the number of sinusoids. Two equal power sinusoids of frequencies $\omega_1=0.4\pi$ and $\omega_2=0.6\pi$ are present in the signal. The decrease in the residual power is considerable from the first to the second row and from the second one to the third one. In the subsequent passes through steps 2-4 of the algorithm the same frequencies are repetedly discovered, but the corresponding power decrease is not substantial.

i	p_i	$\hat{\omega}_i$
1	1018	.400052
2	750	.599999
3	297	.399799
4	277	.599976
5	256	.401448
6	251	.599023

Table 3

Table 4 presents typical results illustrating the efficiency of the algorithm in detecting the number of sinusoids when their amplitudes differ significantly, namely by the factor of 10. The partial SNR for the weaker sinusoid is -3 dB. It appears that less cascades are needed to detect both sinusoids in the case when α_{if} is somwwhat lower. This is due to the transient notch filter response, the duration of which increases with increasing α_i. The example illustrates the fact that α_i should be carefully chosen with respect to the available data length.

$\alpha_{if}=0.99$			$\alpha_{if}=0.977$		
i	p_i	$\hat{\omega}_i$	i	p_i	$\hat{\omega}_i$
1	720	.400001	1	720	.400001
2	197	.399996	2	163	.400192
3	163	.400237	3	145	.404114
4	155	.396935	4	141	.599602
5	149	.406536	5	125	.390981
6	144	.410411	6	121	.410626
7	143	.599903	7	120	.376663
	126			120	

Table 4

Properties of the recursive algorithm described in section 2.2 can be seen from an example in which the signal is composed of three sinusoids ($\omega_1=0.38\pi$, $\omega_2=0.4\pi$, $\omega_3=0.6\pi$) and white additive noise (partial SNR's are -5 dB), and the cascaded model used in identification consists of five notch blocks. The frequency estimates obtained in the blocks (label 1 corresponds to the first block, etc.) are represented in Figure 1. Obviously, curves 1-3 give accurate estimates of ω_1, ω_2 and ω_3. Estimates of the power of residuals obtained by (6) are represented in Figure 2 (curve 0 gives the signal power, curve 1 the residual power at the first block, etc.). The curves 3, 4 and 5 coincide indicating in an obvious way that three sinusoids are present in the signal.

Figure 3 gives an illustration of the capabilities of the proposed recursive method in estimating time varying frequencies. The true frequencies are varying linearly in time, starting from 0.3π, 0.35π and 0.4π and ending at 0.5π, 0.55π and 0.6π, respectively, with partial SNR's of 0 dB.

4. CONCLUSION

In this paper the algorithms for simultaneous estimation of both frequencies of noisy sinusoids and their number are proposed starting from the generalized least-squares method, and assuming the signal model structure in the cascaded notch form. Experimental results show that both methods are very efficient even in the case of a large number of sinusoids with unequal powers. Further efforts have to be more oriented towards theoretical justification of the approach.

5. REFERENCES

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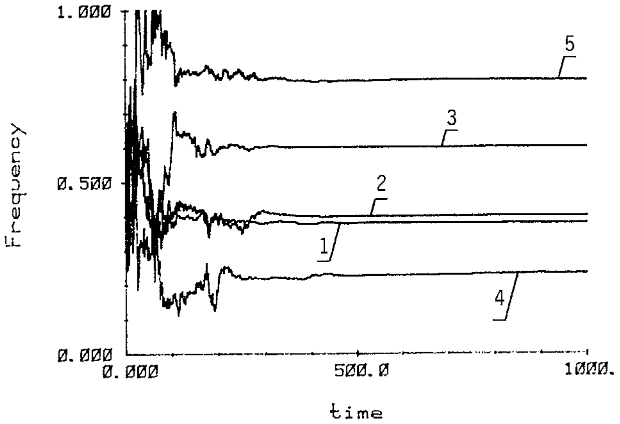

Fig. 1. Frequency estimates (w_1=0.38π, w_2=0.4 π and w_3=0.6π, SNR's=-5dB)

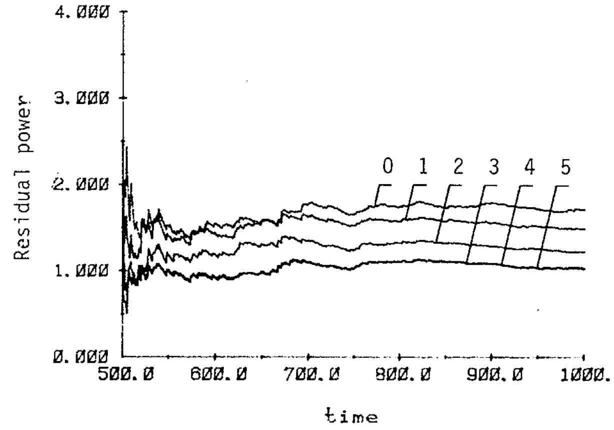

Fig. 2. Residual power estimates

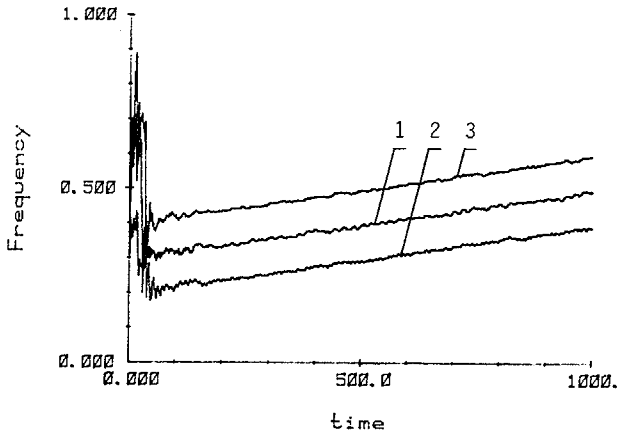

Fig. 3. Chirp frequency tracking