Analytical Evaluation of the Performance of the Square Law Receiver for Gaussian Targets in Gaussian Clutter

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RESUME

Cet article analyse les performances des détecteurs quadratiques en rélation aux courbes caracteristiques de travail, lorsque le signal et le bruit sont des processus aléatoires Gaussiens.

Les résultats théoriques, valids pour un récepteur optimale, sont étendus et appliqués ou cas de filtres MTI pratiquement utilisés.

Une méthode pour l'évaluation et le projet des récepteurs est fournie.

SUMMARY

The paper analyses the performance of Square Law Detectors in terms of Receiver Operating Characteristics, when both the signal and the noise are Gaussian stochastic processes.

The theoretical results valid for the optimum receiver are extended and applied to the case of some commonly used MTI filter.

An evaluation and design methodology is outlined.

1 Introduction

The problem of evaluating the performance of the square law receiver (SLD) in coloured Gaussian noise has been extensively studied in the radar literature in the case of deterministic signals with unknown parameters. The paper analyses and provides exact analytical means to evaluate the operating characteristcs of the SLD when both the signal and the noise are Gaussian stochastic processes originated from frequency spread targets; this can be the case of propeller aircraft or helicopter detection in clutter or of the discrimination between two clutter-like objects by meteorological radars.

The SLD is assumed to process M pulses according to a batch processing scheme whilst the sampling time can also vary from pulse to pulse. The basic theory of detection of Gaussian in Gaussian signals has been developed in classical works such as in Middleton [1] or in Van Trees [2]; they have demonstrated that in this case the SLD is the optimum receiver. The analytical study of the optimum SLD performance has been investigated by Kanter [3] for a deterministic signal (Swerling models) and by Dillars and Rickard [4].

The extension to the non-Gaussian case has been recently presented in [6] under the condition of finite fourth order moments.

The paper applies and extends the theoretical results valid for the optimum SLD to the general case of an SLD having any assigned linear filter, including the most widely used MTI schemes. Any given M-dimensional hermitian matrix can be used to model the clutter and the signals; the SLD operating characteristics are derived in closed form by performing an eigenvalue analysis. On the basis of this mathematical model, it is possible to define an evaluation and design methodology for the SLD class under different and composite models of the clutter and of the target signals.

2 Detection of Gaussian signals in coloured noise

The frequency-spread target is modelled as a complex M-dimensional stochastic vector $\mathbf{s}\epsilon C^M$ representative of a zero-mean circularly complex Gaussian process $s(t)$, whose covariance matrix is:

$$\mathbf{S} = E\{\mathbf{s}\mathbf{s}^+\} \quad (1)$$

where E denotes expectation and $^+$ transpose conjugate. An additive clutter plus thermal noise process is similarly represented by a zero-mean M-dimensional circularly complex vector $\mathbf{c}\epsilon C^M$ with covariance matrix:

$$\mathbf{C} = E\{\mathbf{c}\mathbf{c}^+\} \quad (2)$$

Being the clutter and the signal independent, the observed signal vector $\mathbf{r}\epsilon C^M$: $\mathbf{r} = \mathbf{s} + \mathbf{c}$ has the following covariance matrix:

$$\mathbf{R} = E\{\mathbf{r}\mathbf{r}^+\} = \mathbf{C} + \mathbf{S} \quad (3)$$

Two hypotheses are considered by the SLD:

$$\begin{array}{lll} H_0: & \mathbf{r} = \mathbf{c} & \text{i.e. signal absent} \\ H_1: & \mathbf{r} = \mathbf{c} + \mathbf{s} & \text{i.e. signal present} \end{array} \quad (4)$$

The SLD computes the filtered vector $\mathbf{v} = \mathbf{H}\mathbf{r}$ and integrates the samples by a dot product operation:

$$z = E\{\mathbf{v}^+\mathbf{v}\} \quad (5)$$

From [2], the characteristic function $F_z(\xi)$ associated with the complex quadratic form z is

$$F_z(\xi) = \prod_{i=1}^{M} (1 + 2\lambda_i\xi)^{-1} \quad (6)$$

whereλ_i are the eigenvalues of the hermitian matrix

$$\mathbf{K} = \mathbf{H}\mathbf{R}\mathbf{H}^+ \quad (7)$$

The pdf $f(z)$ is obtained through an inverse transform of F; the corresponding probability distribution can be obtained as:

$$Pr\{z > \gamma\} = \int_{\gamma}^{\infty} f(t)dt \quad (8)$$

whereγ is the threshold of the detector. After some algebraic manipulations and when $\mathbf{K}$ has simple eigenvalues, it is possible to demonstrate that the SLD operating characteristics take the form:

$$\begin{cases} Pr[z > \gamma] = \sum_{i=1}^{M} a_i exp\left(-\frac{\gamma}{2\lambda_i}\right) \\ a_i = \prod_{i=1;i\neq j}^{M} \frac{\lambda_j}{\lambda_j - \lambda_i} \end{cases} \quad (9)$$

where λ_i are the eigenvalues of $\mathbf{K}$. When the eigenvalues are corresponding to the H_0 hypothesis eq.(9) gives the false alarm probability P_{fa},while if the eiegenvalues are corresponding to the H_1 hypothesis eq.(9) gives the detection probability P_d.

It can be shown that equation (9) can be extended to the case when the eigenvalues of $\mathbf{K}$ have multiplicity greater than one. In fact the general expression for the SLD takes the form:

$$\begin{cases} Pr[z > \gamma] = [det(\mathbf{K})]^{-1}\cdot \\ \cdot\sum_{l=1}^{M}\sum_{k=1}^{\mu_l} \frac{r_{lk}}{(\mu_l - k)!(b_l)^{\mu_l - k + 1}}\Gamma(\mu_l - k + 1,\ b_l\gamma) \\ b_l = \lambda^{-1} \end{cases} \quad (10)$$

where Γ denotes the gamma function, μ_l is the multiplicity of the eigenvalue of order l, r_{lk} is the residual of order lk. In [1] it has been demonstrated that the filter matrix $\mathbf{H}_{opt}$ which optimises the SLD performance for assigned clutter and signal models is such that:

$$\mathbf{H}_{opt}^+\mathbf{H}_{opt} = \mathbf{C}^{-1} - (\mathbf{C} + \mathbf{S})^{-1} \quad (11)$$

3 Parametric description of signals and clutter

The target and the signal are assumed to be modelled as wide sense stationary Gaussian processes with zero-mean and autocorrelation function $r(t)$. In principle any $r(t)$ can be adopted; a widely used model both for the signal and the clutter is the Gaussian shaped acf model:

$$r(t) = r(0)exp\{-2(\pi\sigma t)^2\}exp\{-j2\pi f t\} \quad (12)$$

where $r(0)$ is the process power, σ is the spreading parameter and f is the Doppler frequency.

The M-dimensional covariance matrix $\mathbf{C}$ can be obtained by sampling the corresponding continuous process. If uncorrelated processes are assumed, different covariance matrices can be added to obtain a composite clutter matrix $\mathbf{C}$; additional receiver noises (e.g. A/D converters, round-off errors) can be included in this way. The K-th time-around clutter effect can be taken into account by putting to zero the first k rows and columns of the covariance matrix.

4 MTI filters

In this paragraph it is shown how some well known MTI filters can be evaluated according to the above described detection model. To each casual discrete-time stationary linear filter, having an impulse response $[h(k); k > 0]$ it can be associated an M-dimensional square Toeplitz matrix $\mathbf{H}$ which transforms the first M samples of the input sequence into an output vector:

$$H(i,l) = \begin{cases} 0 & 1 < i < l < M \\ h(i-l) & 1 \leq l \leq i \leq M \end{cases} \tag{13}$$

Equation (13) can be used to model both FIR and IIR classical MTI filters; for example the Double Delay Line Canceler (DDLC) can be modeled as:

$$\begin{cases} h(0) = 1;\ h(1) = -2;\ h(2) = 1 \\ h(k) = 0\ otherwise \end{cases} \tag{14}$$

Non stationary filters can be used as well; they are useful for instance to compensate for the multiple time-around clutter effects. When the weighted MTI filtering in the DFT domain is adopted (WDFT), a bank of detectors is obtained; the k-th filter of the bank can be expressed as:

$$\begin{cases} \mathbf{H}_{(k)} = \mathrm{diag}(h_1, h_2, \ldots, h_l) \\ h_l = w(l)exp[-j(2\pi\frac{kl}{M})] \\ 1 \leq l \leq M \end{cases} \tag{15}$$

where $w(.)$ denotes the weighting window coefficient. A possible adaptive MTI scheme makes use of a clutter whitening filter whose coefficents are estimated in real-time on the basis of the data available from the surrounding range-azimuth cells. In our model an Ideal Clutter Whitening Filter (ICWF) of order p $(k < M)$ is defined as the autoregressive filter related to the clutter covariance matrix when only the first p rows and columns are considered; in essence the ICWF filter is obtained as

$$h(k) = \begin{cases} \mathbf{Q}_p(k+1,1) & 0 \leq k \leq p \\ 0 & p \leq k \leq M \end{cases} \tag{16}$$

where $\mathbf{Q}_p(.,.)$ is the p-dimensional inverse of the reduced clutter covariance matrix $\mathbf{K}$. For Gaussian acf the expression of the AR samples are available in closed form [5]. More complex receiver structures con be built by combining different kinds of filters in cascade or in parallel. As an example, the classical Moving Target Detector described in [7] can be obtained by combining the DDLC and the WDFT blocks described above. Another interesting example is obtained when the DDLC is substituted by the ICWF.

5 Numerical results

The diagrams of Fig.1,3 and 4 are related to the following numerical case:

- Signal to Clutter Ratio = 10 dB,
- Clutter spreading = 625 Hz,
- Signal spreading = 1250 Hz,
- Signal Doppler frequency = 1875 Hz,
- PRF = 5 Khz,
- Number of pulses = 8;

while the diagram of Fig.2 are the same with the exception of:

- Clutter spreading = 20 Hz,
- Signal spreading = 100 Hz,
- Doppler frequency = 1500 Hz

References

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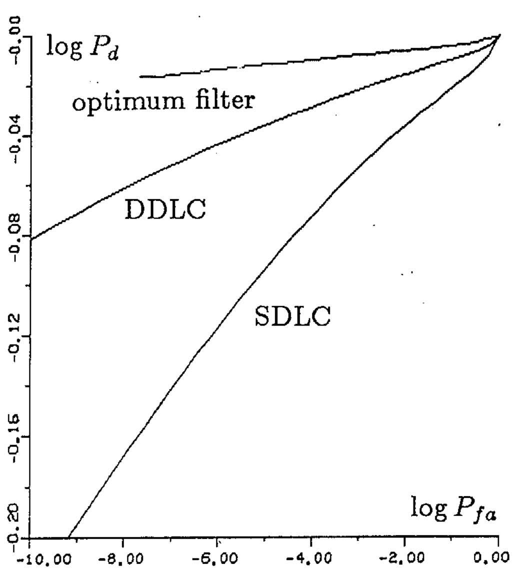

Figure 1: *Receiver Operating Curves*

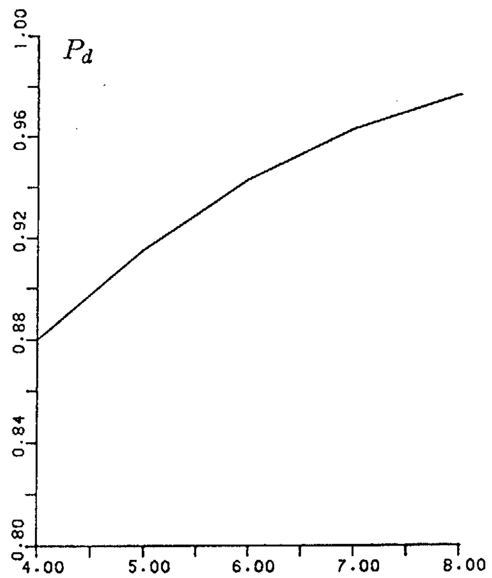

Figure 2: *Probability of detection versus the number of integrated pulses for a DDLC receiver* ($P_{fa} = 10^{-6}$)

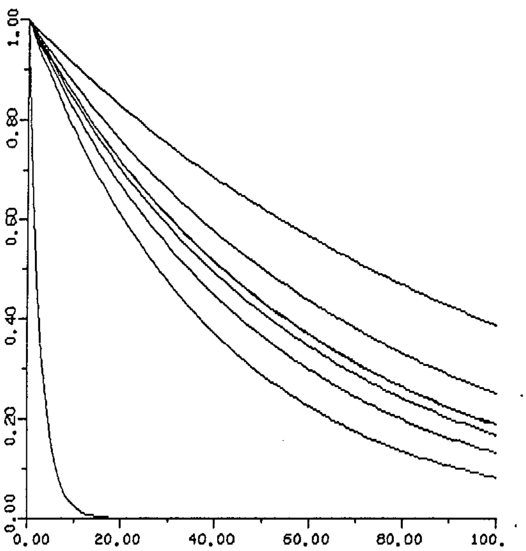

Figure 3a: P_d *versus threshold* γ *for a DDLC followed by a Hamming weighted DFT for* $\log P_{fa} = -0.2171\gamma$. *The topmost curve refers to the best output.*

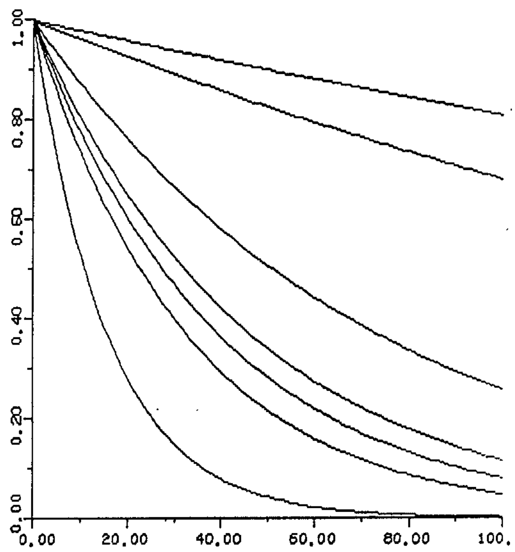

Figure 3b: P_d *versus threshold* γ *for a IWCF followed by a Hamming weighted DFT for* $\log P_{fa} = -0.2171\gamma$. *The topmost curve refers to the best output.*

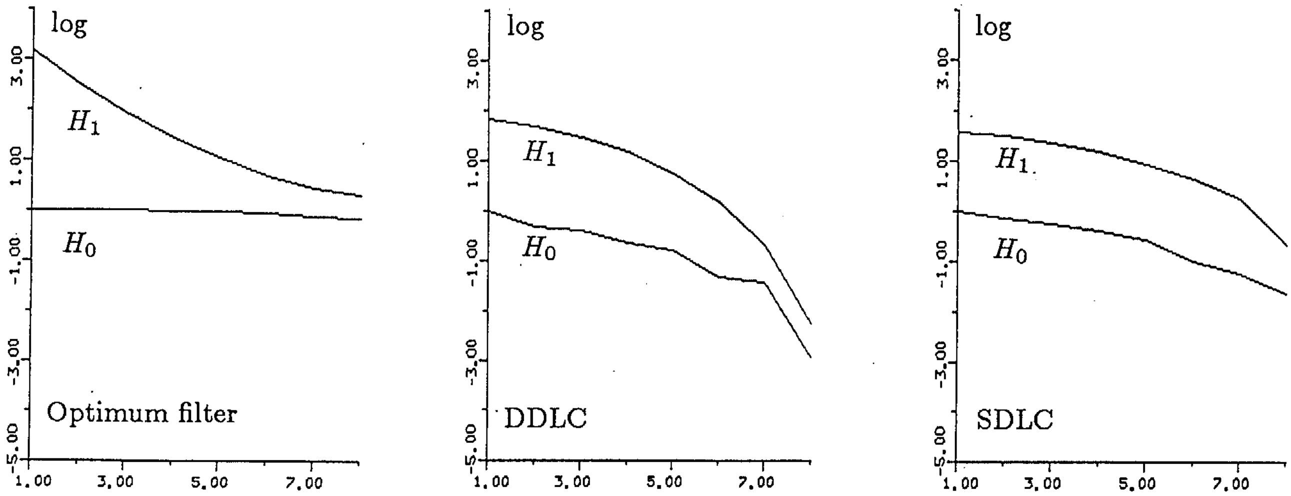

Figure 4: *Eigenvalues spectra for the two hypoteses* H_0 *and* H_1.