COMPARISON OF SIGNAL SEPARATION METHODS BASED ON SVD AND GSVD

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RÉSUME

Dans cette conférence deux principes de traitement de signaux, qui essaient de séparer des signaux en trouvant la meilleure combinaison lineaire d'une série de signaux enrégistrés, ont été présentés et comparés. Les méthodes sont basées sur deux conceptions mathématiques et sur la Décomposition (Générale) en Valeurs Singuliers. Elles ont été appliquées aux problèmes de l' extraction de l' électrocardiogramme foetal et de l'amélioration des signaux de diction.

SUMMARY

In this paper two signal processing principles, that try to separate signals from a set of recorded signals by finding the optimal linear combination of these data signals, are presented and compared. The methods are based on two mathematical concepts and on the computation of the (Generalized) Singular Value Decomposition. They are applied to the problems of FECG extraction from potential signals on the maternal skin and to speech enhancement.

1 Introduction

Many detection, control and estimation problems start from a set of captured, recorded or measured signals that reflect a kind of "global perception" of a situation. This "perception" is often a superposition of many different influences, characterized by signals. Some of these signals are desired, as they contain useful information, while others are undesired, since they disturb the perception of desired signals in one way or another.

Where the global perception holds the information of many phenomena occuring simultaneously, it is often the aim to observe one specific phenomenon individually. In that case signal separation techniques have to be developed that eliminate the undesired influences and obtain the best possible individual observation of the desired phenomenon. The techniques that will be presented in this paper, try to find the optimal linear combination of the recorded signals in order to reach that goal, because simple filtering techniques are not always suitable.

Example 1 : <u>FECG extraction</u>

During pregnancy it is impossible to record the fetal electrocardiogram (FECG) by an invasive method since the fetal body is inaccessible. In this case observations (potential signals) are taken from electrodes put on the maternal abdomen. These observations are however a superposition of the desired phenomenon (the electrical activity of the fetal heart) and some undesired phenomena (the electrical activity of the maternal heart, maternal muscle activity, net-interference (50 Hz), respiration, ...).

Example 2 : <u>Speech enhancement</u>

In applications such as mobile telephone or cockpit conversations, speech (the desired signal) is corrupted by stationary noise (the undesired signal), e.g. the motor running at constant rotation speed. In this case it is the aim to suppress as much as possible the noise and to enhance the signal level. For this example a set of signals can be obtained from multiple microphones in some array structure. Also in this case the optimal linear combination of the recorded signals is looked for.

2 Mathematical concepts

In this section we will define some mathematical concepts and at the same time we will show how the (Generalized) Singular Value Decomposition ((G)SVD) of a matrix provides crucial information about some extremal values for these concepts. Each $p \times q$ matrix A can be considered as a sequence of vectors (its columns $\mathbf{a}_i$) in a p dimensional space $\Re^p$. The energy $E_{\mathbf{e}}[A]$ of matrix A in the direction of unit vector $\mathbf{e}$ in $\Re^p$ is then defined as : ([4], [7])

$$E_{\mathbf{e}}[A] = \sum_{i=1}^{q} (\mathbf{e}^t\, \mathbf{a}_i)^2 = \|\mathbf{e}^t A\|^2 \qquad (1)$$

with $\|\cdot\|$ the Euclidean norm. Furthermore, the SVD of A is given by [5]

$$A_{p,q} = U_{p,p}\, \Sigma_{p,q}\, V^t_{q,q} \qquad (\text{for } p < q) \qquad (2)$$

in which U and V are orthogonal matrices and Σ is a real pseudo-diagonal matrix, that contains the so called nonnegative singular values σ_i of A. It has been proven that this SVD provides p orthogonal directions (the columns $\mathbf{u}_i$ of U) in $\Re^p$ for which the oriented energy of A is extremal. Indeed

$$E_{\mathbf{u}_i}[A] = \|\mathbf{u}_i^t A\|^2 = \sigma_i^2 \qquad (3)$$

$$\forall\ \mathbf{e} = \sum_{i=1}^{p} \gamma_i\, \mathbf{u}_i \ :\ E_{\mathbf{e}}[A] = \sum_{i=1}^{p} \gamma_i^2\, \sigma_i^2 \qquad (4)$$

In the case of two matrices $A_{p,q}$ and $B_{p,k}$ a more relative concept can be defined : the energy ratio of matrix A to B, called the oriented signal-to-signal ratio $R_{\mathbf{e}}[A,B]$ in the direction of unit vector $\mathbf{e} \in \Re^p$, is defined as : ([3], [4])

$$R_{\mathbf{e}}[A,B] = \frac{E_{\mathbf{e}}[A]}{E_{\mathbf{e}}[B]} = \frac{\|\mathbf{e}^t A\|^2}{\|\mathbf{e}^t B\|^2} \qquad (5)$$

In recent work, a link has been found between this concept and the Generalized SVD (GSVD) of matrices A and B, given by

$$\begin{aligned} A &= X^{-t}\, D_A\, U_A^t \\ B &= X^{-t}\, D_B\, U_B^t \end{aligned} \qquad (6)$$

with U_A and U_B orthogonal matrices and D_A and D_B pseudo-diagonal matrices. The elements of the set $\sigma(A,B) = \left(\frac{\alpha_1}{\beta_1}, \ldots, \frac{\alpha_{r_B}}{\beta_{r_B}}\right)$, with $r_B = \text{rank}(B)$ and α_i (β_i) the diagonal elements of D_A (D_B), are referred to as the generalized singular values of A and B. The GSVD of A and B now provides r_B non-orthogonal directions (the columns $\mathbf{x}_i$ of X) in $\Re^p$ for which the oriented signal-to-signal ratio is extremal, since ([3], [4])

$$R_{\mathbf{e}}[A,B] = \left(\frac{\alpha_i}{\beta_i}\right)^2 \quad \text{for } \mathbf{e} = \frac{\mathbf{x}_i}{\|\mathbf{x}_i\|} \qquad (7)$$

The two defined concepts and their relation with SVD and GSVD will play a key rôle in solving signal separation problems.

3 The signal model

Since in multichannel signal processing applications the set of recorded signals is often found as a linear combination of some original signals, called "source" signals, and corrupted by additive noise, the model of a recorded signal $m_i(t)$ can be expressed as

$$m_i(t) = \sum_{j=1}^{r} t_{ij}\, s_j(t) + n_i(t) \quad \text{for } i = 1, \ldots, p \qquad (8)$$

with p the number of measured signals and r the total number of source signals involved. The transfer coefficients t_{ij} in the model essentially depend on the signal transfer from source to measurement point.

Suppose a number of different more-dimensional phenomena $X, Y, Z, \ldots$ are observed, where more-dimensional means that the phenomenon can be described by more than one source signal. This means that the total number r of source signals in the model consists of x source signals describing phenomenon X, y source signals describing Y, etc., with $x, y, \ldots$ the dimension of each phenomenon.

4 Two signal separation principles

4.1 SVD of the data matrix M

Suppose p signals are recorded as the superposition of three phenomena X (undesired), Y (desired) and Z (of minor importance). The first thing to realize is to record the p signals such that the desired signals are certainly stronger present in the recordings than the signals of minor importance. Second, p_x signals (with $p_x \geq x$) that pick up the undesired phenomenon individually and as strongly as possible, have to be added to the set of p recorded signals. After sampling and digitizing the extended signal set, a data matrix M with $p + p_x$ rows and q columns can be constructed.

The SVD of that data matrix provides $p + p_x$ orthogonal directions of extremal oriented energy : ([2], [7], [8]) the first x columns $\mathbf{u}_i$ of U correspond to energies of the undesired phenomenon, while the following y columns of U are the directions of extremal energies for the desired phenomenon. Projection of the given vectorsequence (the columns of the data matrix) onto those y directions gives linear combinations of the $p + p_x$ data signals that contain no contribution from the undesired phenomenon anymore.

Example 1 : <u>**Fetal ECG extraction**</u>

We will illustrate this signal separation principle for the separation of the cutaneously recorded maternal and fetal electrocardiogram ([3], [6], [8]). The main problem is however the very strong and undesired maternal ECG that is omnipresent in these recordings. Figure 1 shows a typical signal, recorded at the maternal abdomen. The fetal ECG is clearly visible, but seriously disturbed by the much stronger MECG.

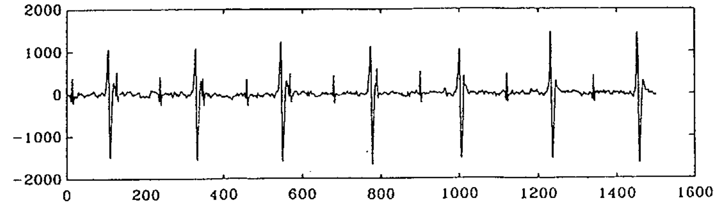

Figure 1: Potential signal recorded on the maternal abdomen and containing both MECG and FECG

Since it has been verified that, sufficiently far from the adult heart, its source dimension is three, at least three signals that pick up the maternal ECG only (close to the mother's heart) have to be added. The SVD of the resulting data matrix provides three directions of extremal oriented energy corresponding with the maternal heart, while the fourth column of U contains the coefficients in the linear combination of the four recorded signals, that results in an MECG-free fetal signal (see Figure 2).

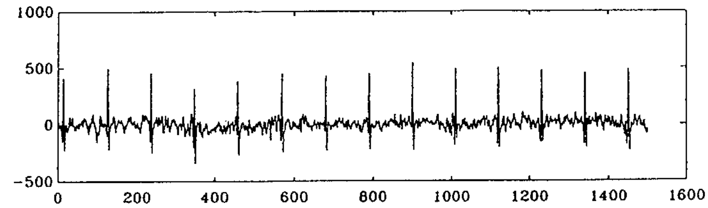

Figure 2: The resultant MECG-free fetal signal after applying the first signal separation principle.

4.2 GSVD of selected data intervals

In some signal processing applications it occurs that, over a specific time interval, the undesired phenomenon contributes more in the recorded signals than the desired ones (e.g. for the pulse-like ECG signals the maternal QRS-peaks often appear in between two fetal QRS-peaks (see Figure 1) and in the speech application, noise dominates if speech is absent (see Figure 3)). In that case we can construct from the datamatrix M a matrix $A_{p,k}$ as a sequence of such intervals. This matrix A then contains mainly contributions from the undesired phenomenon only. The GSVD of the matrix pair (M, A) now finds some non-orthogonal directions $\mathbf{x}_i$ of extremal oriented signal (in M)-to-signal (in A) ratio. Projection of the recorded signals in M onto some of these directions $\mathbf{x}_i$ results in an elimination of the undesired signals.

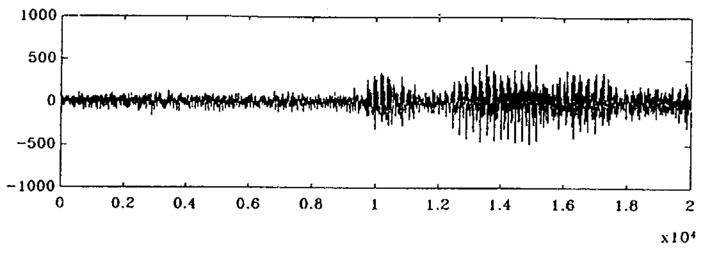

Figure 3: Speech signal with pure noise in the initial part.

Example 1 : Fetal ECG extraction

Suppose p signals are recorded at the maternal abdomen, containing both MECG and FECG. After arranging these signals in a $p \times q$ data matrix M, a matrix A is constructed as a sequence of several maternal QRS-intervals, not coinciding with fetal complexes. The coefficients in the linear combination of the recorded signals are found by the GSVD of the pair (M, A).

Example 2 : Speech enhancement

At the moment we are doing tests on speech signals, recorded using multiple microphone systems, and corrupted by stationary noise. From the recorded speech signals in data matrix M, some intervals are selected, where speech is absent and only the noise is present, and arranged in a new matrix B.. The GSVD of (M, B) results in an enhancement of the speech-to-noise ratio of about a factor 2.

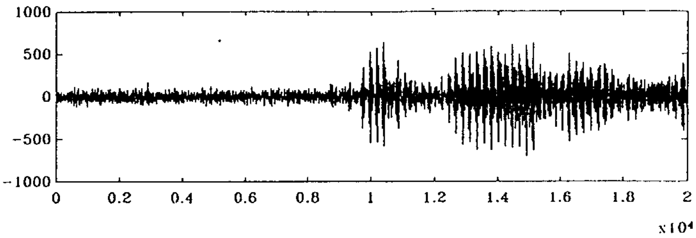

Figure 4: Resultant signal after applying the signal separation technique to a set of recorded and delayed speech signals

5 Comparison of the principles

Both signal separation problems basically look for an optimal linear combination of the recorded signals. For the first one this linear combination is optimal with respect to extremal oriented energy, while the second method looks for a combination with an extremal ratio of desired to undesired signals. The first principle requires more measurement points than the second one because of the necessary addition of a number of extra signals. On the other hand, the second method asks a larger computational work than the first one, and requires furthermore the visual selection of some intervals. In order to permit a real-time separation of the recorded signals, an adaptive on-line algorithm has been designed to compute the SVD ([1], [2]). After arranging the computational work in a special way, this algorithm can also be used for the GSVD-based method [3].

6 Conclusion

In this contribution the use of the (Generalized) Singular Value Decomposition for signal processing and specifically for signal separation purposes is advocated. The SVD and GSVD are not only excellent tools in formulating and describing new geometrical concepts (oriented energy, oriented signal-to-signal ratio), they are also extremely useful in efficiently computing reliable and elegant solutions for many signal processing problems. Two signal separation principles are presented and compared in this paper, and applied to the problems of FECG extraction and speech enhancement.

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