NEW INSTANT RELATIVE DETECTIN FOR MINIMUM ERROR PROBABILITY OVER NOISY BINARY-CODED CHANNELS

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RÉSUME

La nouvelle découverte relative instantanée est un nouvel approche qui minimise simultanément l'erreur zéro bite et les probabilités de l'unité bite erreur. L'échantillonnage d'un bruit dans des périodes de communications silencieuses, permet de prénommer un ensemble de seuils correspondant aux conditions actuelles de bruit instantané. Les décisions sont basées sur la comparaison relative de situations uniquement dans l'espace du bruit et du signal-plus-bruit simultanément. Un groupe puissant de criteriums pour décisions logiques permet des décisions échecs au cas de situations nonsûres qui sont accomplies par un code de poid constant pour corriger des érreurs désorientées. Les résultats analytiques obtenus prouvent la superiorité de cet approche.

SUMMARY

The new instant relative detection is a novel appoach which simultaneously minimizes the zero bit-error and the one bit-error probabilities. Sampling noise in silent communication periods allows to preset a group of thresholds corresponding to actual instant noisy conditions. Decisions are based on relative comparison of situations in the space of noise only and signal-plus-noise simultaneously. a powerful set of logical decision criteria allows decision failures in nonsure situations, which are implemented by a constant weight code for correcting undirectional errors. Achieved analytic results prove the superiority of this approach.

1- BASIC IDEA

In nearly all known digital communication systems using error detecting and /or correcting codes, decisions concerning the outcomes of the detection process take place independent of the decisions of code decoding algorithm. In addition, the binary symmetric channel (BSC) model is far from being true. A realistic model is to consider a noisy channel as a generalized binary asymmetric channel (BAC) with time varying parameters. This model can match the environmental noisy condition of any arbitrary channel, e.g. mobile radio links, where the channel is subject to noise, fading or a combination of both.
The new instant relative detection technique can fit the above-mentioned model in nearly real-time behaviour. More over, the detection results are fed to the decoder which is capable on the basis of those results, to minimize the decoding errors while the decoding algorithm is executed. The constant-ratio high-rate codes are implemented due to their high detection capabilities.This will yield an instant relative detector - constant weight code matched pair. To achieve the above mentioned goals simultaneously the silent periods between messages are exploited for sampling the received noise at the same bit rate. If this possiblility is not existing, a single time slot (test slot) may be alloted for this purpose each frame. The sum of squares of these samples voltage will have a chi-square distribution. For large enough number of samples and on the basis of the central limit therorem, the sampling distribution of the r.m.s value of noise voltage will be Gaussian. Consequently, the measured root mean square voltage $\hat{V}$ will be an estimator of $\sqrt{\psi_0}$ (the noise standard deviation). The output of the envelope detector in the presence of noise only will have a Rayleigh pdf given by:

$$P(R) = \frac{R}{\psi_0} \exp\left(-\frac{R^2}{2\psi_0}\right) \qquad (1)$$

When a sine wave of amplitude A is presnet with noise in the same IF amplifier, the output of the envelope detector will have the pdf given by Rice as

$$P_s(R) = \frac{R}{\psi_0} \exp\left(-\frac{R^2 + A^2}{2\psi_0}\right) I_0\left(\frac{RA}{\psi_0}\right) \qquad (2)$$

where $I_0\left(\frac{RA}{\psi_0}\right)$ is the modified Bessel function of zero order and argument $\frac{RA}{\psi_0}$. The probability of detection P_D is the probability that the signal-plus-noise envelope will exceed a preset threshold V and is given by:

$$P_D = \int_V^{\infty} P_s(R)\,dR \qquad (3)$$

In setting such a threshold we are subject to either the situation that the noise envelope may exceed this threshold yielding an error in zero (assuming postive logic) with a probability of false alarm P_f or to the fall of signal-plus-noise envelope below this threshold yielding an error in one with a probability of miss P_m. P_f and P_m are given by

$$P_f = \int_V^{\infty} \frac{R}{\psi_0} \exp\left(-\frac{R^2}{2\psi_0}\right) dR = \exp\left(-\frac{V^2}{2\psi_0}\right) \qquad (4)$$

$$P_m = 1 - P_D \qquad (5)$$

By sampling the received noise we are able to set a group of m decision thresholds corresponding to different false alarm probabilities and update these thresholds each frame. This enables us to process signals in the space of noise-only and the space of signal-plus-noise simultaneously. Decisions regarding signal presence (one) or absence (zero) are carried out on relative logic basis which consequently will cancel the effect of the error in estimating $\sqrt{\psi_0}$.

Let $\hat{V}_k$ be the estimate of $\sqrt{\psi_0}$ in the kth frame test slot, then the instant decision threshold $V_j(k)$ corresponding to an arbitrary probability of false alarm P_{fj} is given by:

$$V_j(k) = \left(2 \ln \frac{1}{P_{fj}}\right)^{1/2} \hat{V}_k \qquad (6)$$

with $j = 1, 2, \ldots, m$

If the test slot is the first time slot in each frame, processing of received signals will take place in near-real time.

2- LOGIC CONCEPTS

2-1 The Instant Relative Detection

On applying the new instant relative detection approach it is expected to achieve one of the following three outcomes of the detectin process depending on the situation in both noise-only and signal-plus-noise spaces simultanmeously.

A. Reception failure (noise only)

This decision yields a zero. It is declared in one of the following simultaneous situations in the noise-only and signal-plus-noise spaces respectively:

$$\hat{V}_k < V'_1 \qquad \text{while } R < V'_1 \qquad (7)$$

$$\hat{V}_k > V'_j \qquad \text{while } R < V'_j \qquad (8)$$

where R is the signal-plus-noise voltage envelope.

B. Reception success (one decision)

It is declared (with the exception of the bottom threshold V'_1) in the following simultaneous situations:

$$\hat{V}_k > V'_j \qquad \text{while } R < V'_j \qquad (9)$$

for all $j,\ j \neq 1$

c. Decision failure (erasure)

$$\hat{V}_k < V'_1 \qquad \text{while } V'_2 > R > V'_1 \qquad (10)$$

$$V'_j > \hat{V}_k > V'_{j-1} \quad \text{while } V'_j > R > V'_{j-1},\ n > J > 1 \qquad (11)$$

$$\hat{V}_k > V'_n \qquad \text{while } R > V'_n \qquad (12)$$

2-2 The Decoding Algorithm

The implemented code is kOn consisting of k ones out of n bits constant-weight code. For a code vector of length n, the decoder will receive k_c reception successes (ones), k_n reception failures (zeros) and k_d decision failures respectively such that:

$$k_c + k_n + k_d = n \qquad (13)$$

The decoding algorithm will be governed by the following decoding logical criteria:

A- Criteria for correct decoding

$$k_c = k\ ,\ k_n = n - k \quad \text{while } k_d = 0 \qquad (14)$$

$$k_c < k\ ,\ k_n = n - k \quad \text{while } k_d = k - k_c \qquad (15)$$

then k_d are considered to be ones.

$$k_c = k\ ,\ k_n < n - k \quad \text{while } k_d = n - k - k_n \qquad (16)$$

and k_d are considered to be zeroes.

B- Criteria for error detection

$$k_c \neq k \qquad \text{while } k_d = 0 \qquad (17)$$

$$k_c < k \qquad \text{while } k_d \neq k - k_c \ \text{(high errors in ones)} \qquad (18)$$

$$k_c > k \qquad \text{while } k_d \neq k_c - k \ \text{(high errors in zeroes)} \qquad (19)$$

The basic decoding logical criteria closely meet practical channels with extremely low decoding error probability.

3- EVALUATION OF SYSTEM PROBABILITIES

3-1 Instant Relative Detection Probabilities

On the basis of logical criteria (7-12), the space consisting of all events associated with the instant relative detection process consist of two sets which are not mutually exclusive. These are the detection-miss and the reception failure-false alarm sets respectively.

A. The detection-miss set

The set include the events of reception success, signal miss and decision failure in signal presence with probabilities $P_c(m)$, $P_m(m)$, and $P_{dc}(m)$ respectively, such that :

$$P_c(m) + P_m(m) + P_{dc}(m) = 1 \qquad (20)$$

It could be proved that in general for m thresholds it is valid that:

$$P_c(m) = \sum_{j=2}^{m-1} \left(P_{D_j} - P_{D_{j+1}}\right)\left(1 - P_{f_{j-1}}\right) + P_{D_m}\left(1 - P_{f_m}\right) \quad (21)$$

where P_{D_j} and P_{f_j} are the probabilities of signal detection and flase alarm at the j^{th} threshold respectively.

$$P_m(m) = \sum_{j=2}^{m-1} P_{m_j}\left(P_{f_j} - P_{f_{j+1}}\right) + P_{m_1}\left(1 - P_{f_2}\right) + P_{f_m} P_{m_m} \quad (22)$$

P_{m_j} is the probability of miss at the j^{th} threshold.

$$P_{dc}(m) = \sum_{j=2}^{m-1} \left(P_{D_j} - P_{D_{j+1}}\right)\left(P_{f_j} - P_{f_{j+1}}\right) + \left(P_{D_1} - P_{D_2}\right)\left(1 - P_{f_2}\right) + P_{D_m} P_{f_m} \quad (23)$$

B- The reception failure-false alarm set

This set include the events of signal absence, false alarm and implicit false alarm in decision failure with probabilities $P_n(m)$, $P_f(m)$ and $P_{D_n}(m)$ respectively such that:

$$P_n(m) + P_f(m) + P_{dn}(m) = 1$$

$$P_n(m) = \sum_{j=2}^{m-1} \left(1 - P_{f_j}\right)\left(P_{f_j} - P_{f_{j+1}}\right) + \left(1 - P_{f_1}\right)\left(1 - P_{f_2}\right) + P_{f_m}\left(1 - P_{f_m}\right) \quad (24)$$

$$P_f(m) = \sum_{j=2}^{m-1} \left(P_{f_j} - P_{f_{j+1}}\right)\left(1 - P_{f_j}\right) + P_{f_m}\left(1 - P_{f_m}\right) \quad (25)$$

$$P_{d_n}(m) = \sum_{j=2}^{m-1} \left(P_{f_j} - P_{f_{j+1}}\right)^2 + \left(P_{f_1} - P_{f_2}\right)\left(1 - P_{f_2}\right) + P_{f_m}^2 \quad (26)$$

From the set of equations (21- 26), the following main conclusions are achieved

- There is a slight improvement by increasing of m. Thus two or maximally three thresholds may give near optimum results.
- The probability of false alarm is kept constant and equal to that corresponding to the second threshold. The probability of miss depends on the selection of the lowermost threshold while the probability of detection corresponds to that of the second threshold. The probability of decision failure $P_{dc}(m) \approx \left(P_{D_1} - P_{D_2}\right)$.

This means that if those decision failures are properly implemented by the code we can achieve an arbitrary high probability of detection and arbitrary low probability of false alarm simultaneously.

3-2 Relative Detector - Constant Weight Code Matched Pair Probabilities

On decoding a constant-weight code vector having k ones and n-k zeroes received from a noisy channel and detected by the instant relative detection new technique the expected inputs to the decoder will be as follows:

- k_c correctly detected ones, k_m missed and k_{dc} yielded decision failures such that: $k_c + k_m + k_{dc} = k$
- k_n correctly decided zeroes, k_f erroneously changed to ones and k_{dn} yielding decision failures such that: $k_n + k_f + k_{dn} = n - k$

On the basis of the decoding logical criteria (14-19) and independent errors, the following probability equations are achieved:

A- Probability of correct decoding $P_k(m,n)$

$$P_k(m,n) = \left[P_c(m) + P_{dc}(m)\right]^k P_n^{n-k}(m) + P_c^k(m)\left[P_n(m) + P_{dn}(m)\right]^{n-k} - P_c^k(m) P_n^{n-k}(m) \quad (27)$$

B- Probability of decoding error $P_e(m,n)$

The penalty payed for the application of criterion (15) yields the following probability of decoding error:

$$P_{e1}(m,n) = \sum_{k_n=0}^{n-k-1} \sum_{k_f=0}^{n-k-k_n} \frac{(n-k)!}{k_n!\,k_f!\,k_{dn}!} P_n^{k_n}(m) P_f^{k_f}(m) P_{dn}^{k_{dn}}(m) \cdot \sum_{k_c=0}^{2k-n+k_n} \frac{k!}{k_m!\,k_c!\,k_{dc}!} P_m^{k_m}(m) P_c^{k_c}(m) P_{dc}^{k_{dc}}(m) \quad (28)$$

and on the application of criterion (16) the decoding error probability is yielded:

$$P_{e0}(m,n) = \sum_{k_n=0}^{n-k-1} \sum_{k_f=1}^{n-k-k_n} \frac{(n-k)!}{k_n!\,k_f!\,k_{dn}!} P_n^{k_n}(m) P_f^{k_f}(m) P_{dn}^{k_{dn}}(m) \cdot \sum_{k_m=0}^{k_f} \frac{k!}{k_m!\,k_c!\,k_{dc}!} P_m^{k_m}(m) P_f^{k_f}(m) P_{dc}^{k_{dc}}(m) \quad (29)$$

In evaluating the overall probability of decoding error, the intersection of the subsets associated with the probabilities $P_{e0}(m,n)$ and $P_{e1}(m,n)$ is the subset in which $k_{dc} = k_{dn} = 0$

The probability of that event is given by:

$$P_{e0,1}(m,n) = \sum_{k_n=0}^{n-k-1} \frac{(n-k)!}{k_n!\,k_f!} P_n^{k_n}(m) P_f^{k_f}(m) \cdot \frac{k!}{k_m!\,k_c!} P_m^{k_m}(m) P_c^{k_c}(m) \quad (30)$$

The total probability of decoding error will be

$$P_e(m,n) = P_{e0}(m,n) + P_{e1}(m,n) - P_{e0,1}(m,n) \quad (31)$$

C- Probability of error detection

$$P_d(m,n) = 1 - \left[P_c(m,n) + P_e(m,n)\right] \quad (32)$$

4- ANALYSIS OF ACHIEVED RESULTS

A comparison between the behaviour of the noncoherent envelope detector used in O.O.K systems implementing constant weight codes and the new instant relative detector - constant weight matched arrangement is illustrated in Fig.1 and Fig. 2 respectively.

At S/N ratio of 13.7 db the optimum thereshold yield

$P_m = P_f = 10^{-3}$. For the same S/N ratio, two thresholds $P_{f_1} = 10^{-2}$ and $P_{f_2} = 10^{-4}$ and the corresponding probabilities of detection P_{D_1} and P_{D_2} were extracted to establish the instant relative detector with $m = 2$. The probabilities $P_c(2)$, $P_m(2)$, $P_{dc}(2)$, $P_n(2)$ and $P_{dn}(2)$ were computed on the basis of formulas 21, 22, 23, 24 and 26 respectively. The achieved results were substituted in (27) to obtain the probability of correct decoding $P_k(n)$. Fig. 1 illustrates the dependence of $P_k(n)$ on n (code length) for the highest rate constant ratio codes 4O7, 4O8, 5O9, 5O10, 6O11 and 6O12 successively. $P_k(n)$ is evaluated for conventional detection and for the new matched pair. It is quite obvious the very high improvement achieved in the probability of correct decoding on the application of the new approach. Fig. 2 deals with the same case as in Fig. 1 but the probability of decoding error $P_e(n)$ is here evaluated. It is noticed that a slight increase in the probability of decoding error over the conventional case. The increase arises due to the application of the decoding criteria (14 - 19) yielding the probability equations (28 - 31). On comparing the high gain in the probability of correct decoding with the exclusively small increase in the decoding error probability, it is clear that the effeciency of the new approach is very high.

5- REALIZATION OF THE NEW DETECTOR-CODE MATCHED PAIR

Simplified scheme is shown in Fig. 3. The received signals are detected by an m-thresholds instant relative detector. A sensor of silent communication periods enables the estimation of noise voltage r.m.s value. Decision results are stored in a buffer. Decisions concerning the actually received signals are applied simultaneously with buffer results to the relative logic (R-logic) decision device. Since decisions involve three kinds of variables, the stream of output decision functions z_1 and z_2 is applied to a pair of shift registers acting as series-to -parallel converters. The outputs of those pairs are applied to the decision PROM on the basis of criteria (14 - 19). An additional bit is alloted to the case of detected error E_d. After the n-bit code word is received the decision results are written into an n + 1-bit parallel-to-series shift register and transmitted to destination.

6- CONCLUSIONS

The two novel basic concepts of instant relative detection and the matched detector-code pairs briefly introduced in this paper proved to add an essential improvement in binary communication systems behavior. This improvement is achieved without the necessity to any addend redundancy. The new instant relative detection approach allows the optimum detection of signals in near-real time response with the actual dominating noisy conditions. The decision failures are exploited to add the erasure correction capability to the implemented code.

7- REFERENCES

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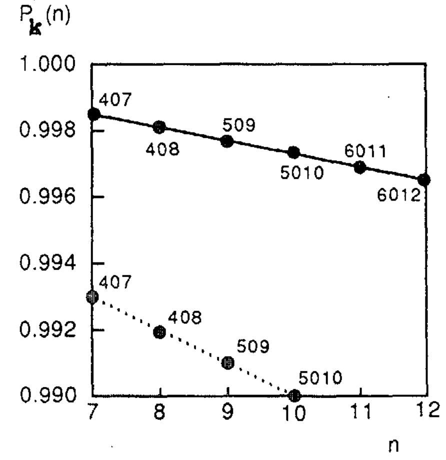

Fig. 1 Comparison of correct decoding probabilities for the highest rate constant-weight codes

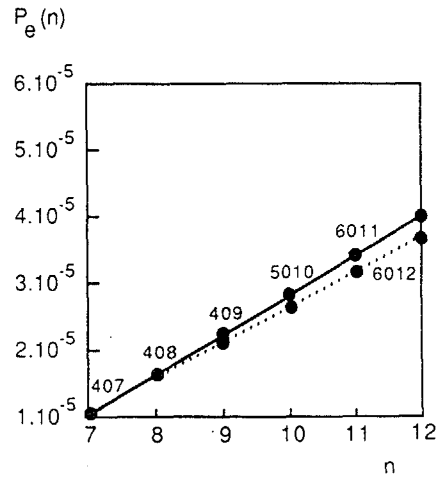

Fig. 2 Comparison of decoding error probabilityies for the same conditions in Fig.1

—— R-logic detector-constant weight code matched pair

····· Normal constant weight error detecting code

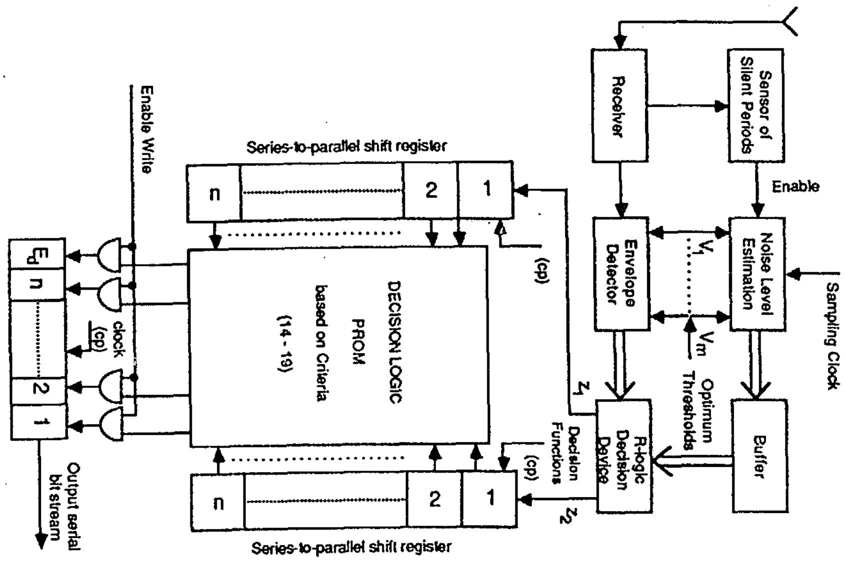

Fig. 3 Simplified hardware realization of the proposed R-logic detector constant-weight code matched pair.