# Optimum coherent radar detection in K–distributed clutter

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## RESUME

Dans cet article nous abordons la détection d'un signal certain dans un bruit avec une enveloppe répartie selon la loi Kappa et l'évaluation de ses performances. La modélisation du bruit comme un processus aléatoire sphériquement invariant conduit au calcul du rapport de vraisemblance et, par conséquence, à la synthèse du détecteur optimal. Nous montrons que le détecteur optimal doit calculer la norm du vecteur des obsérvations et la distance entre le même vecteur et celui du signal; après il doit élaborer ces deux quantités avec un dispositif non linéaire et non inertiel et enfin comparer la differénce avec un seuil. Les performances du détecteur optimal ont eté obtenus par simulation: nous les avons aussi comparées avec les performances du détecteur linéaire. Nous avons aussi étudié l'influence de la corrélation du bruit et de la fréquence doppler du signal sur les performances. La comparaison entre les deux détecteurs présentés montre que le système optimal est superieur surtout lorsque le signal est faible.

## SUMMARY

This paper deals with the synthesis of the optimum receiver for known signal in the presence of K-distributed disturbance, and with the assessment of its performance. Modelling the background noise as a Spherically Invariant Random Process yields closed form expressions for the joint pdf's of any order, thus enabling a Neyman–Pearson design of optimum detector. We show that the optimum detection amounts to processing, via a zero-memory non-linearity, the distances of the received vector both from the origin and from a stored replica of the useful signal and to comparing the difference to a threshold. The optimum detector's performance assessment of the optimum detector is evaluated via computer simulations: for sake of comparison, the performance the conventional receiver under the same disturbance is also considered. An analysis of the optimum receiver operating characteristics shows that a marked improvement is achievable over the conventional receiver, at least in the region of low and moderately high detection probabilities. In particular, the larger the deviation from Gaussian distribution, the better the detectability of weak signals. The effect of the clutter correlation properties has also been investigated, as well as the influence of non-zero doppler shift of the target echo.

## 1 Introduction

The theory of optimum detection of targets embedded in clutter is well established if the baseband equivalent of the clutter can be modelled as a complex Gaussian process, namely with Rayleigh amplitude and uniform phase [1]. However, in some situations of practical interest, such as high-resolution and/or low grazing operating radars, the Gaussian assumption becomes inconsistent with real data, as the experimental clutter amplitude distribution exhibits far higher tails than predicted by Rayleigh distributions.

To model this non-Gaussian clutter involves at least fitting experimental data to theoretical amplitude probability density functions (pdf) and matching the measured spectral properties to suitable covariance matrices or equivalent second-order characteristics. This level of specification, however, although sufficient for the analysis of some incoherent processors, is still inadequate for the analysis of coherent cancellation techniques and, even more so, for the design of optimum or *ad hoc* processors. These tasks require a "coherent" process specification, namely providing joint probability distributions for the in phase and the quadrature clutter components.

Recently some coherent models have been proposed, compatible with the most common non-Rayleigh envelope distributions, namely the lognormal [2,3] the Weibull [4,5], and the K-distribution [6].

In the present paper we focus on the K-model, as such a distribution satisfies the constraint requested for the envelope of a complex Spherically Invariant Random Process (SIRP) and leads to a clutter model that is consistent with the well-assessed composite surface scattering theory [7].

Previous work on K-distributed clutter was confined to the analysis of conventional radar processors, namely the optimum ones in Gaussian clutter [8,9]. Here we consider the design and the analysis of the optimum detector for known signal in the presence of K-distributed clutter.

The paper is organized as follows. Section 2 reviews the problem of modelling a process with pre-assigned envelope pdf and arbitrarily given correlation functions as a complex SIRP. Section 3 describes the synthesis of the optimum receiver for a known signal in K-distributed clutter. The evaluation of optimum receiver performance and its comparison to the conventional receiver, that is the optimum one as long as noise is Gaussian, are presented in Section 4. Concluding remarks and some hints for future research are in Section 5.

## 2 Clutter modelling

To design the optimum detector of a target in clutter requires modelling the disturbance as a complex random process. Two different approaches can be followed.

The first approach, developped in [2,4], is a generalization of the Wiener generation scheme of real processes with pre-assigned first-order pdf and covariance function. As outlined in Fig. 1, it amounts to processing the envelope of a complex correlated Gaussian sequence through a zero memory non linearity (ZMNL), leaving unchanged the phase process. The ZMNL converts the Rayleigh input pdf into the desired non-Rayleigh distribution. The correlation properties of the output process can be controlled by a suitable linear filtering of the input white Gaussian sequence provided that an analytical relationship between the covariance functions of the desired sequence $x(k)$ and the Gaussian sequence $y_G(k)$ can be established. This approach turns out to be hopelessly complex if the output envelope is to have the K-distribution:

$$f_R(r) = \frac{4}{\Gamma(\nu)}\left(\frac{\nu}{\sigma^2}\right)^{\nu+1} r^{\nu} K_{\nu-1}\left(\frac{2\nu r}{\sigma^2}\right) r \geq 0 \qquad (1)$$

where $\Gamma(\cdot)$ is the Eulerian function, $K_\nu(\cdot)$ is the modified second kind Bessel function of order $\nu$ and $\sigma^2$ is the common power of the quadrature components. Indeed a closed-form expression for the ZMNL does not exist for this case, and hence the said relationship in terms of covariance functions cannot be found. In the second approach, proposed in [10], a complex Gaussian process $y_G(k)$ is modulated by an highly correlated exogenous process $s(k)$ as shown in Fig.2. This achieves the desired envelope pdf, while not appreciably affecting the correlation properties of $y_G(k)$. This allows for an independent adjustment of the correlation properties, by linearly filtering the complex white Gaussian signal $w_G(k)$, and of the desired envelope distribution, by selecting a suitable pdf of the the modulating process. If the exogenous process is approximately constant in the observation time the resulting process $x(k)$ is a SIRP [6](Fig.3). This approach models either stationary or non-stationary processes, according to whether the linear filter is time invariant or not.

One appealing feature of this approach is its physical interpretation in the light of the composite surface scattering theory [7]. With reference to sea clutter, this theory assumes

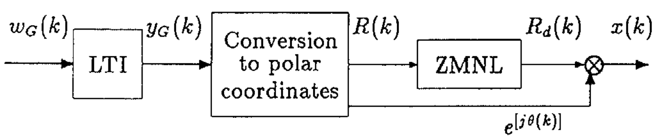



Figure 1: Model for a complex non-Gaussian correlated sequence.

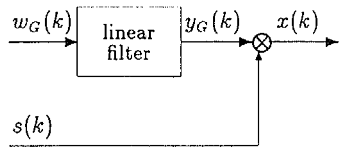
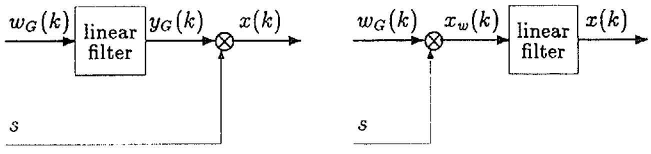



Figure 2: Exogenous model for a complex non-Gaussian correlated sequence.




a) pre-modulation filtering b) post-modulation filtering

Figure 3: SIRP model for a complex non-Gaussian correlated sequence.

that a slightly rough surface, corresponding to the Gaussian component $y_G(k)$ sequence of Fig.3, is superimposed upon a larger swell structure, corresponding to the process $s(k)$. Such physical interpretation is confirmed by experimental results in the case of K-distributed clutter [8,11].

Not all envelope pdf's can be assumed as the amplitude distribution of a complex SIRP since, as suggested by the representation of Fig.3, the envelope of a SIRP is the product of a Raileigh variable times an independent non-negative random variate. The envelope pdf (1) fulfills such a constraint and hence can be assumed as the envelope distribution of a SIRP [6]. Modelling the K-distributed clutter as a SIRP, allows a complete specification of the process. The joint pdf of the $N$ complex noise samples $\mathbf{n}$ is [6]:

$$f_{\mathbf{n}}(\mathbf{x}) = \frac{2^{1-\nu}}{|\mathbf{M}|\pi\Gamma(\nu)}(\sqrt{2\nu}\ \|\mathbf{x}\|_{\mathbf{M}})^{\nu-N} K_{\nu-N}(\sqrt{2\nu}\ \|\mathbf{x}\|_{\mathbf{M}}) \qquad (2)$$

where $\mathbf{M}$ is the covariance matrix of the zero-mean noise samples, and

$$\|\mathbf{x}\|_{\mathbf{M}} = (\mathbf{x}^T\mathbf{M}^{-1}\mathbf{x})^{\frac{1}{2}} \qquad (3)$$

is the norm of the vector $\mathbf{x}$ induced by the inverse covariance matrix. In the presence of uncorrelated, unit variance observations, this norm reduces to the usual Euclidean norm.

## 3 Optimum detection of a known signal in K–distributed noise

The problem of detecting a known signal embedded in additive disturbance can be stated in terms of the following hypotheses test:

$$\begin{cases} H_0: & \mathbf{r} = \mathbf{n} \\ H_1: & \mathbf{r} = \mathbf{s} + \mathbf{n} \end{cases} \qquad (4)$$

where vectors $\mathbf{r}$, $\mathbf{s}$, $\mathbf{n}$ are composed of samples from the received signal, the target signal and the disturbance respecively.

The Neyman–Pearson receiver, namely the one which maximizes the detection probability ($P_d$) for a given false–alarm–probability ($P_{fa}$), amounts to comparing the log–likelihood ratio,

$$\Lambda = \log\left[\frac{f_{\mathbf{n}}(\mathbf{r}-\mathbf{s})}{f_{\mathbf{n}}(\mathbf{r})}\right] \qquad (5)$$

to a threshold, to be set according to the required false–alarm rate.

Substituting the joint pdf (2) into (5) the Neyman-Pearson test reduces to:

$$g_\nu(\|\mathbf{r}-\mathbf{s}\|_{\mathbf{M}}, N) - g_\nu(\|\mathbf{r}\|_{\mathbf{M}}, N) \underset{H_0}{\overset{H_1}{\gtrless}} T \qquad (6)$$

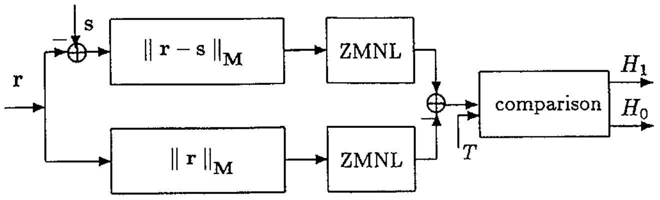



Figure 4: Optimum coherent detector in K-distributed clutter.

where:

$$g_\nu(x, N) = \log\{x^{\nu-N} K_{\nu-N}(\sqrt{2\nu}x)\} \qquad (7)$$

As outlined in the block diagram of Fig.4, the optimum processor computes first the distances of the received vector $\mathbf{r}$ from the origin (no useful signal present) and from a stored replica of the target echo $\mathbf{s}$, respectively. Then such distances are warped through the Zero-Memory-Non-Linearity (7) and, finally, the difference between the warped distances is compared to the detection threshold $T$.

This receiver can be considered as a generalization of the optimum receiver for correlated Gaussian clutter to which it is equivalent as the $\nu$ parameter increases to infinity. In fact, letting $\nu \to +\infty$, the input noise distribution tends to the Gaussian one [6] and the ZMNL $g_\nu(x, N)$, suitably normalized, approaches a square law:

$$g_\infty(x, N) = -\frac{x^2}{2\ln 10} - N \log 2\pi \qquad (8)$$

An alternative realization of the optimum receiver relies on whitening the input noise. The resulting receiver structure is basically the same as in Fig.4, using the Euclidean norm instead of the norm induced by $\mathbf{M}$ and replacing $\mathbf{s}$ and $\mathbf{r}$ with their filtered versions:

$$\mathbf{s}' = \mathbf{A}\mathbf{s} \qquad \mathbf{r}' = \mathbf{A}\mathbf{r} \qquad (9)$$

where $\mathbf{A}$ is the matrix of the whitening transformation. From an operational point of view the computational complexity of the two receivers is essentially the same. However the whitening approach is preferable in carrying on the performance analysis via computer simulation, since a single set of white K-distributed vectors can be generated to serve as patterns for both cases of uncorrelated and correlated noise.

# 4 Performance assessment

The present section is devoted to a comparative analysis of the optimum and the conventional receiver, when both operating in K-distributed noise. Since the statistical characterization of the test variable of the optimum processor turns out to be hopelessly complex, the assessment of the receiver performance requires simulation techniques. Standard Montecarlo counting has been used in estimating detection probabilities, whereas an extrapolative method based on generalized extreme value theory has been adopted for estimating false-alarm probabilities so as to avoid generating and processing an enormous number of observations [12]. Moreover a suitable interpolation has been performed on data provided by simulation, in order to improve the readability of the results.

On the other hand, integral expressions for $P_d$ and $P_{fa}$ of the conventional detector can be obtained. The conventional detector performs the following binary hypothesis test:

$$\mathbf{s}^{\mathbf{T}}\mathbf{M}^{-1}\mathbf{r} \underset{H_0}{\overset{H_1}{\gtrless}} T \qquad (10)$$

Since the received vector $\mathbf{r}$ is the product of a Gaussian vector times a random modulating variate (see Fig.3a), the LHS of the previous equation turns out to be itself the product of the same modulating variate times a Gaussian variable and hence is a a SIRV of size one (closure property of SIRP's under linear transformations [6]). Therefore, $P_{fa}$ and $P_d$ are easily found by averaging the corresponding probabilities in the Gaussian case over the modulating variate distribution $s$ referred to in [6].

The Receiver Operating Characteristics (ROC's), namely plots of $P_d$ versus the average signal-to-noise (SNR) ratio, are shown in Figures 5, 6 and 7 for several values of the shape parameter $\nu$ of the noise distribution, for $P_{fa} = 10^{-6}$ and for $N = 8$ integrated pulses.

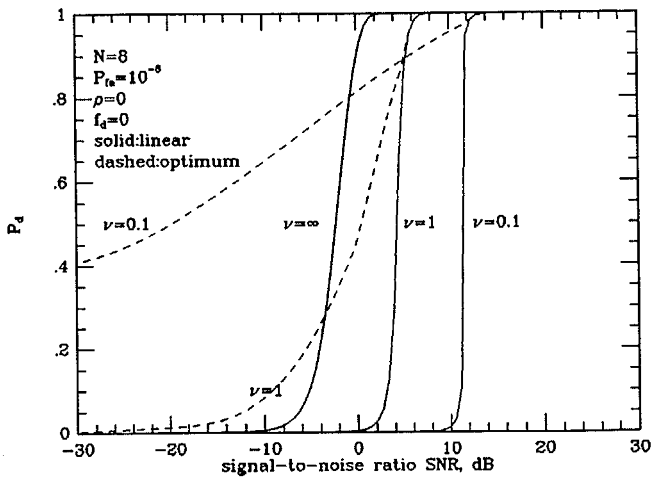



Figure 5: Optimum and conventional receiver performances in uncorrelated K-clutter

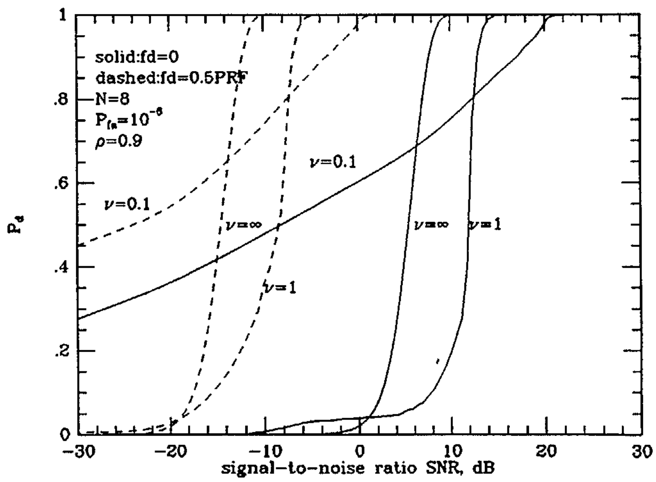



Figure 6: Optimum detector performance in correlated K-clutter

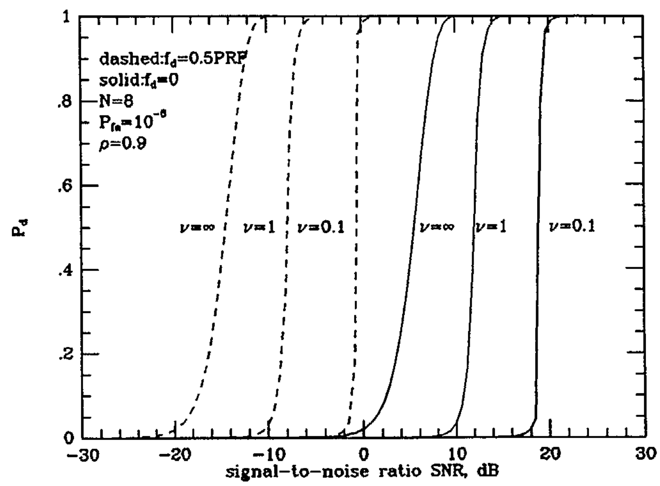



Figure 7: Conventional detector performance in correlated K-clutter

In Fig.5 plots of $P_d$ versus SNR are shown for both the optimum and the conventional receiver in the presence of uncorrelated noise: the curve labelled as $\nu = \infty$ corresponds to both receivers, since, in such a limit case, the K-distributed clutter becomes Gaussian.

Fig.5 shows that clutter spikyness (low values of $\nu$) sharpens the "threshold effect" exhibited by the conventional detector operating in Gaussian environment. In other words, the detection of weak signals is almost completely inhibited, while the actual strength of the targets is immaterial, when the receiver operates above a certain value of the SNR (threshold SNR). The threshold SNR raises as $\nu$ decreases, thus resulting in a remarkable detection loss: for example, in passing from $\nu = \infty$ to $\nu = 0.1$, the detection loss is about 13 dB (see Fig.5).

The optimum receiver largely outperforms the conventional one, the amount of such an improvement depending on clutter spikyness and SNR. More precisely, for low values of $\nu$, the optimum receiver outperforms the conventional one over the whole range of $P_d$'s, while, as $\nu$ increases, an appreciable improvement is gained only for weak signals. The value of $\nu$ is also relevant to the optimum performance in that the threshold effect becomes less marked as $\nu$ decreases, and in particular it is always less marked than that corresponding to $\nu = \infty$.

If the assumption of uncorrelated clutter no longer holds, the performance of both the optimum and the conventional receiver changes, depending on the clutter bandwidth as well as on the target doppler shift $f_d$. To account for such factors we provide in Figs. 6 and 7 two sets of curves describing the performances of the optimum and of the conventional receivers in the presence of exponentially correlated noise. The extreme cases of $f_d = 0$ and $f_d = PRF/2$ (half the Pulse Repetition Frequency) are considered, corresponding to the minimum and the maximum spectral separation between the target and the clutter. From Figs.6 and 7, it is seen that, for high one-lag correlation coefficient of the noise values, the overall shape of the ROC's is practically the same as in the uncorrelated noise both for optimum and for conventional detector and therefore the above comments apply also to the case of correlated noise. Yet, the ROC's shift toward lower or higher signal-to-noise ratios according to the value of the target doppler frequency. Indeed, due to the high correlation, the power of the clutter tends to concentrate in the low-frequencies region, thus masking zero-doppler signals. Conversely, if the signal possesses a nonzero doppler frequency, then such a masking effect reduces, as a consequence of increased separation between the spectra of the clutter and of the signal.

## 5 Conclusions

In this paper, we face the synthesis of the optimum receiver for a known signal embedded in K-distributed noise and the assessment of its performance. The structure of the optimum detector can be reduced to conventional one, but for the presence of a zero-memory non linear processor.

The computed ROC's show a marked improvement over the conventional detector, both in the case of white noise and in the case of correlated noise, especially for highly spiky clutter and low signal-to-noise ratios, the actual gain decreasing noticeably as the probability of detection approaches unity and/or the shape parameter grows up.

A possible drawback of the proposed scheme is that its optimality relies on two assumptions:

- at the design stage, an estimate of $\nu$ is available, in order to determine the optimum ZMNL;
- at the operational stage, the current SNR value is given, in order to set the detection threshold.

To remove these drawbacks calls for further investigations, aiming to the synthesis of robust sub-optimum receivers in the general case of incomplete statistical description of clutter and/or target.

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