



A Comparison of Matched Field Processing Schemes  
Operating in the Time and Frequency Domains

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## RESUME

Les modèles de propagation acoustique sous-marine se sont améliorés au point où les méthodes de champs appareillés peuvent maintenant s'en servir pour produire une détection et localisation de sources très améliorées comparées à celles possibles en se basant sur l'assomption d'ondes planes. Les méthodes de champs appareillés partent habituellement de la matrice de covariance du signal et bruit estimés à la fréquence d'intérêt. La matrice de covariance représente la moyenne des termes spectraux et interspectraux après une transformation de Fourier de la série temporelle. Des sources stationnaires peuvent être résolues ou détectées par traitement de signal appliqué à la matrice de covariance. Cependant, quand une source est en mouvement dans un environnement multimodal ou à trajets multiples, cette sommation produit une matrice de covariance dans laquelle le signal couvre un espace vectoriel plus grand et en conséquence n'est plus bien séparée du bruit. En conséquent, la résolution de sources acoustiques en mouvement sera réduite comparée à celle obtenue pour

des sources stationnaires.

Cette étude examine la possibilité de réduire la perte de gain d'antenne et l'augmentation de l'ambiguïté qui résulte lorsque l'on performe la moyenne temporelle des matrices de covariance d'une source acoustique en mouvement dans les méthodes de champs appareillés. La solution proposée est une méthode de champs appareillés opérant dans le domaine temporel, et qui retient le concept de fréquence simple avec son gain de signal à bruit pour un signal à bande étroite, et ses commodités pour le modelage. L'algorithme de traitement du signal ressemble une transformation de Fourier modifiée par le modèle de propagation. Le rendement de méthodes de champs appareillées opérant dans les domaines des temps et des fréquences sont comparés pour des sources en mouvement ou stationnaires.

## SUMMARY

Underwater acoustic propagation models have improved to the point where they can now be employed in matched field processing (MFP) to produce much improved source detection and localization over that obtainable on the basis of plane wave assumptions. MFP usually proceeds from the covariance matrix of signal plus noise estimated at the frequency of interest. The covariance matrix represents an averaging of spectral and cross-spectral terms after Fourier transformation of the time sequence. Stationary sources may be resolved or detected by signal processing applied to the covariance matrix. However, when an acoustic source is moving in a multimodal or multipath environment, this averaging produces a covariance matrix in which the signal spans a larger vector space and is no longer as

well separated from the noise. As a result, the resolution of moving acoustic sources by orthogonal techniques will be reduced compared to that obtained for stationary sources.

This study investigates the possibility of reducing the loss in resolution and array gain of MFP that results from averaging in time to form the covariance matrix. The proposed solution is a MFP scheme operating in the time domain that retains the single frequency concept with its attendant signal-to-noise gain and modelling convenience. The signal processing algorithm resembles a Fourier transform modified by the propagation model. Performance of MFP schemes operating in the time and frequency domains are compared for moving and stationary sources.

## INTRODUCTION

When an array of sensors is employed in underwater acoustics a single plane wave signal is usually sought by the signal processing method used. When more than one not necessarily plane wave path is present some better means of using the signal information in the time sequence is required. Special

schemes which make use of more of the signal paths have been described in the literature. Recently, more generally applicable Matched Field Processing (MFP) schemes, that are suitable for stationary sources, have been developed.<sup>1</sup> In MFP the received signals are matched with the signal predicted with a propagation model. When the acoustic source is in motion the existing schemes suffer a loss in gain and an increase



in the ambiguity of the source position. Tolstoy<sup>2</sup> has taken source motion into account to some extent in an attempt to reduce the increased ambiguity. In this paper, source motion is completely modelled and the benefits of taking it into account are presented.

## THEORY

### Introduction

MFP may be employed to detect the presence of signals in noise and to determine the location of a signal source. This paper is concerned with how the signals are compared. By choosing a time sequence for the field representation and directly comparing the time sequences, both narrowband gain and array gain may be obtained simultaneously. The following sections describe the propagation model, the MFP as it is usually done, and the new MFP scheme which operates in the time domain, and compare the performance of the processing schemes for both stationary and moving sources.

### Propagation model

For simplicity and efficiency a normal mode model was chosen. The depth dependence of the normal modes may be represented as,

$$y_{m1}(z_s) = (P_m)^{-1/2} \{ E_1 \exp(j\gamma_1(z-h_1)) + F_1 \exp(-j\gamma_1(z-h_{1+1})) \} \quad (1)$$

$$\text{where } \gamma_1 = \{ (\omega/c_{f1})^2 - \eta_1^2 \}^{1/2} \quad (2)$$

The received signal at the  $i^{\text{th}}$  hydrophone will be,

$$Y_i(t) = \sum_{s=1}^u \sigma_s \sum_{m=1}^q A_m \exp(j(\eta_m R_{is} - \pi/4 + \Phi_m - \omega t)) + \sigma_n N_i \quad (3)$$

where,

$$A_m = \rho_1 \omega^2 (8\pi R_{is} \eta)^{-1/2} y_{m1}(z_s) y_{mk}(z_r) \exp(-\alpha R_{is}), \quad (4)$$

$\eta_1$  = mode wavenumber  $1^{\text{th}}$  mode,

$R_{is}$  = source range to  $i^{\text{th}}$  receiver,

$\Phi_{i1}$  = mode phase fluctuation,

$\sigma_s^2$  = power of  $s^{\text{th}}$  source,

$\sigma_n^2$  = power of noise,

$N_i$  = noise at  $i^{\text{th}}$  receiver,

$\alpha^m$  = attenuation of  $m^{\text{th}}$  mode,

$c_{f1}$  = compression speed in  $i^{\text{th}}$  layer,

$\omega$  = angular frequency,

$u$  = number of sources,

$\rho$  = water density,

$z_s$  = depth of source,

$z_r$  = depth of receiver,

$h_1$  = depth to top of  $1^{\text{th}}$  layer,

$q$  = number of modes.

### MFP in the Frequency Domain

When the signal source is stationary, it is computationally efficient and optimal to perform MFP in the frequency domain (FDMFP) because the signal received at array elements consists of a pure tone of

constant amplitude in time. Several authors have described this process.<sup>1,3,4</sup> FDMFP as employed in this study was first described by Klemm<sup>1</sup> and was further developed by Ozard<sup>3,5</sup> and is the basis of the brief description that follows. A generalized beamformer (GB) was chosen for the FDMFP as it corresponds most closely with the Time Domain Matched Field Processing (TDMFP) described later. For the GB the power estimator  $P$  as a function of range, depth, and bearing may be written as,

$$P(r, z, \theta) = \text{TRACE}(R(r, z, \theta)Q) \quad (5)$$

where  $R$  is the expected covariance matrix of the signal and  $Q$  is the measured covariance matrix.  $Q$  and  $R$  are formed by Fourier transforming the measured and the modelled time sequences. To detect and localize a source, Equation 5 is evaluated for all possible ranges, depths and bearings.

Array gain (A.G.) is calculated on the basis of the ratio,

$$\text{A.G.} = \{ \text{Max}[P_{s+n}(r, z, \theta)] / \text{Max}[P_n(r, z, \theta)] \}_{\text{out}} / \{ P_{s+n} / P_n \}_{\text{in}} \quad (6)$$

where max indicates that the maximum value of Equation 5 is evaluated for signal and noise or for noise as appropriate. In the limit of large signal-to-noise ratios A.G. approaches the signal-to-noise gain.

### MFP in the Time Domain

Time domain matched field processing (TDMFP) is performed on the basis of the expected or test time sequence  $T(t_i) = Y(t_i)$  (signal alone) and the measured time sequence  $M(t_i)$  (signal and noise) by evaluating the power  $C(r, z_s, \theta)$  for all possible source positions.

$$C(r, z, \theta) = \left\{ \sum_{i=1}^N \sum_{j=1}^M [M(t_i)T(t_j)] / \sqrt{\sum_{i=1}^N \sum_{j=1}^M T(t_i)^2} \right\}^{1/2}, \quad (7)$$

where  $M$  is the number of points in the measured time series and  $N$  is the number of hydrophones.

When TDMFP is performed, the innermost sum in equation (7) represents the narrowband gain of the Fourier transform in FDMFP. Narrowband gain is not included in the FDMFP array gain. In order to make a comparison with FDMFP the narrowband gain was subtracted from the calculated TDMFP array gain. It was also necessary to perform TDMFP for several time segments corresponding to the averaging in the formation of the covariance matrix in FDMFP. The power obtained for successive segments when TDMFP was applied was then summed at each position at which a peak or peaks were obtained and Equation (6) was applied to calculate the array gain.



### ARRAY GAIN VS SPEED

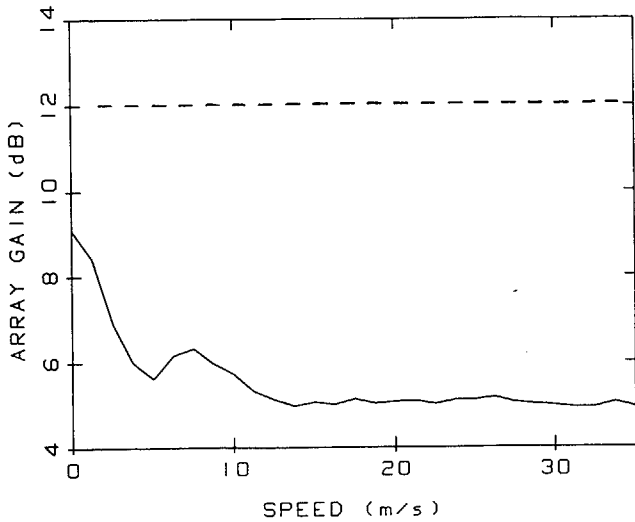


Figure 1. Calculated array gain for a moving acoustic source, continuous line FDMFP and broken line TDMFP.

### SIMULATION RESULTS

#### Effect of Source motion on array gain

The motivation for an alternate to the FDMFP scheme is apparent when array gain is calculated as a function of source speed for a low redundancy horizontal array. The 3.6-km long array consisted of fifteen elements and was placed at a depth of 430 m in water 500 m deep. The sediment velocity was 3500 m/s. Figure 1 shows clearly that modest speeds produce substantial losses in array gain when FDMFP is used. These losses in gain result because the signal is reconstructed on the basis of a stationary source and the time variations of the phase and amplitude of the modes are not taken into account. Figure 1 also shows array gain as a function of source speed for TDMFP under conditions identical to those used for FDMFP. Consistent recovery of all

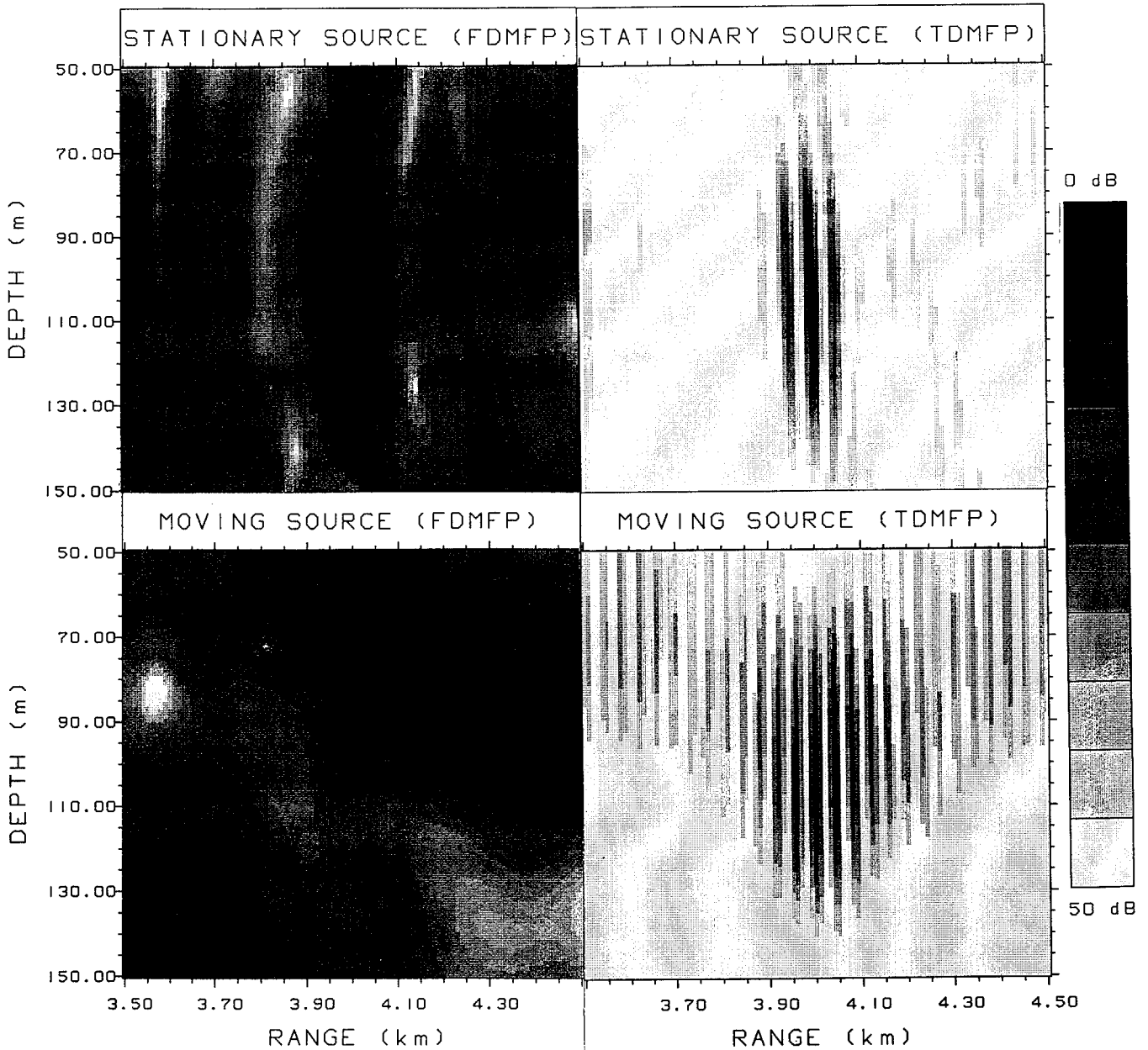


Figure 2. Depth-range ambiguity surface for a 20 dB signal-to-noise ratio with the source at a depth of 100 m, a range of 4.0 km and a bearing of 45°.



signal energy and reduction of noise levels for a spatially white noise sample lead to a motion-independent array gain.

Acoustics, NATO ASI Series C, 151, 269-279, May 1985.

#### Effect of Source Motion on ambiguity level

Figure 2 shows the ambiguity surface for a 20 Hz source and a 20 dB signal-to-noise ratio. The source position is correctly located with FDMFP for the stationary source at 4000-m range and 100-m depth (at a bearing of  $45^\circ$ ) but incorrectly located when the source moves 2400 m (10 m/s for 240 s) during the averaging time used to form the covariance matrix. Furthermore many strong ambiguities are present outside the region shown in the figure when the source is in motion. Figure 2 also illustrates that the source position is correctly identified regardless of whether the source is in motion when TDMFP is used. The source position shown in the figure corresponds to the starting position for the linear track segment. The resolution of the TDMFP also exceeds that of FDMFP especially in the presence of source motion.

#### CONCLUSIONS

A FDMFP scheme suitable for a stationary source has been demonstrated to suffer a loss in array gain in the presence of source motion of ten metres per second over several minutes. Source motion also produced increased ambiguity of source position. A TDMFP scheme was devised that provided array gain similar to that of the FDMFP scheme for a stationary source. For TDMFP, array gain was unchanged in the presence of source motion. Source motion increased the ambiguity of source position with TDMFP but not as much as for FDMFP.

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