2D FIR EIGENFILTERS: A LEAST SQUARES APPROACH

Soo-Chang Pei and Jong-Jy Shyu

Department of Electrical Engineering, National Taiwan University, Taipei, Taiwan, Rep. of China

RESUME

La methode de 1D filtre propre est étendue à faire le plan de FIR filtre de dimension deux. Par minimizer la mesure d'erreur quadratique dans le band de 2D fréquence, le vecteur propre d'un matrix convenable est calculé pour l'obtenir les coefficients de filtre. Cette methode est non seulement simple et aussi optimal dans le sense du moindre carré. Quelques exemples numeriques de 2D filtre de forme quelconque sont donné pour montrer l'efficacité de cette methode.

SUMMARY

The 1D eigenfilter approach is extended for designing two dimensional FIR filters. By minimizing a quadratic measure of the error in the 2D frequency band, an eigenvector of an appropriate matrix is computed to get the filter coefficients. This method is not only simple and also optimal in the least square sense. Several numerical design examples of 2D arbitrary shape filters are illustrated to show the effectiveness of this approach.

I. INTRODUCTION:

Recently Vaidyanathan and Nguyen introduced a new method to design 1D linear phase FIR digital filters by eigenfilter [1], which is formulated by minimizing a quadratic measure of the error in the passband and stopband. The method is based on the computation of an eigenvector of a real, symmetric and positive-definite matrix, and the corresponding eigenvector of the smallest eigenvalue is the desired filter coefficients we want. The advantage of this eigenfilter approach over the McClellan-Parks algorithm [2] is that it is general enough to incorporate both time and frequency domain constraints; also the design time is comparable to the McClellan-Parks algorithm.

In this paper, the 1D eigenfilter approach is extended for designing two dimensional FIR filters. The method is proposed by minimizing a quadratic measure of the passband and stopband error in the 2D frequency domain. The total error function can be formulated as

$$E=(1-\alpha)\iint_p[D(w_1,w_2)-H^*(w_1,w_2)]^2dw_1\,dw_2+\alpha\iint_s[H^*(w_1,w_2)]^2dw_1\,dw_2$$

$$=(1-\alpha)E_p+\alpha E_s \qquad (1)$$

where $D(w_1,w_2)$: Desired frequency response.

$H^*(w_1,w_2)$: Actual amplitude response of the 2D filter $H(Z_1,Z_2)$.

- E_p: Passband error.
- E_s: Stopband error.
- p: Passband region.
- s: Stopband region.
- α: Parameter controls the relative accuracies of approximation in the pass and stopbands, and is in the range $0\leq\alpha\leq1$.

Eq.(1) can be reformulated properly such that it has the following form:

$$E = A^tQA \qquad (2)$$

where t is the vector transpose operation, Q is a real, symmetric and positive-definite matrix; and A is a real vector related to the 2D filter impulse response $h(n_1,n_2)$ in some manner. By Rayleigh Principle [3], the eigenvector A associated with the smallest eigenvalue of matrix Q minimizes the total error E.

This approach is optimal in the least square sense, and also can design any arbitrary shape 2D FIR filters. In Section II, we present a detailed discussion of this design technique, and some typical 2D FIR filter design examples are given in Section III. To show the flexibility and the effectiveness of this approach, we present some special 2D arbitrary filters in Section IV for demonstration. Finally Section V gives a summary.

II. DESIGN TECHNIQUE:

A 2D FIR filter with the impulse response $h(n_1,n_2)$, $n_1=0,1,\ldots,N_1-1$ and $n_2=0,1,\ldots,N_2-1$ has a frequency response

$$H(w_1,w_2)=\sum_{n_1=0}^{N_1-1}\sum_{n_2=0}^{N_2-1}h(n_1,n_2)e^{-jn_1w_1}e^{-jn_2w_2} \qquad (3)$$

Assuming N_1 and N_2 are odd, and the impulse response symmetry due to linear phase condition is

$$h(n_1,n_2)=h(N_1-n_1-1,n_2) \qquad n_1=0,1,\ldots,\frac{N_1-1}{2}$$
$$h(n_1,n_2)=h(n_1,N_2-n_2-1) \qquad n_2=0,1,\ldots,\frac{N_2-1}{2} \qquad (4)$$

then Eq.(3) can be rewritten in the form [4]

$$H(w_1,w_2)=e^{-j\frac{N_1-1}{2}w_1}e^{-j\frac{N_2-1}{2}w_2}\sum_{n_1=0}^{\frac{N_1-1}{2}}\sum_{n_2=0}^{\frac{N_2-1}{2}}a(n_1,n_2)\cos n_1w_1\cos n_2w_2$$

$$=e^{-j\frac{N_1-1}{2}w_1}e^{-j\frac{N_2-1}{2}w_2}H^*(w_1,w_2) \qquad (5)$$

where $H^*(w_1,w_2)$ is the amplitude response of $H(w_1,w_2)$, and $a(n_1,n_2)$ is related to $h(n_1,n_2)$, the filter impulse response, by

$$\begin{aligned} &a(0,0)=h(\tfrac{N_1-1}{2},\tfrac{N_2-1}{2}) \\ &a(0,n_2)=2h(\tfrac{N_1-1}{2},\tfrac{N_2-1}{2}-n_2) \quad n_2=1,\ldots,\tfrac{N_2-1}{2} \\ &a(n_1,0)=2h(\tfrac{N_1-1}{2}-n_1,\tfrac{N_2-1}{2}) \quad n_1=1,\ldots,\tfrac{N_1-1}{2} \\ &a(n_1,n_2)=4h(\tfrac{N_1-1}{2}-n_1,\tfrac{N_2-1}{2}-n_2) \quad n_1=1,\ldots,\tfrac{N_1-1}{2} \text{ and } n_2=1,\ldots,\tfrac{N_2-1}{2} \end{aligned} \tag{6}$$

For simplicity, let $N_1=N_2=N$ and define

$$A=[a(0,0),a(1,0),\ldots,a(\tfrac{N-1}{2},0) \vdots a(0,1),\ldots,a(\tfrac{N-1}{2},1) \vdots \ldots \vdots a(0,\tfrac{N-1}{2}),a(1,\tfrac{N-1}{2}),\ldots,a(\tfrac{N-1}{2},\tfrac{N-1}{2})]^t \tag{7a}$$

and

$$C(w_1,w_2)=[1,\cos w_1,\ldots,\cos\tfrac{N-1}{2}w_1 \vdots \cos w_2,\cos w_1\cos w_2,\ldots,\cos\tfrac{N-1}{2}w_1\cos w_2 \vdots \ldots \vdots \cos\tfrac{N-1}{2}w_2, \cos w_1\cos\tfrac{N-1}{2}w_2,\ldots,\cos\tfrac{N-1}{2}w_1\cos\tfrac{N-1}{2}w_2]^t \tag{7b}$$

Hence we can write the amplitude response $H^*(w_1,w_2)$ as

$$H^*(w_1,w_2)=A^t\cdot C(w_1,w_2) \tag{8}$$

Then from Eq.(1) and (8), the stopband error can be defined as

$$E_s=A^t[\iint_s C(w_1,w_2)\cdot C^t(w_1,w_2)dw_1dw_2]A \tag{9a}$$

and the passband error is

$$E_p=A^t\{\iint_p (C(w_1',w_2')-C(w_1,w_2)][C(w_1',w_2') - C(w_1,w_2)]^t dw_1dw_2\} A \tag{9b}$$

where (w_1',w_2') is the reference frequency point in the passband we choose to approach the passband to this desired reference frequency response, i.e. $D(w_1,w_2)=A^tC(w_1',w_2')$ in Eq.(1).

This enables us to write the total error E as a quadratic in A, this will lead to the eigenformulation $E=A^tQA$, and the matrix Q is

$$Q=(1-\alpha)\iint_p [C(w_1',w_2')-C(w_1,w_2)][C(w_1',w_2')-C(w_1,w_2)]^t dw_1dw_2+\alpha\iint_s C(w_1,w_2)C^t(w_1,w_2)dw_1dw_2 \tag{10a}$$

For N odd, the size of matrix Q is $(\frac{N+1}{2})^2 x (\frac{N+1}{2})^2$, and the elements of Q are given by

$$\begin{aligned} Q(n',m')=&\tfrac{1-\alpha}{\pi}\iint_p [C(w_1',w_2')-\cos n_1w_1\cdot\cos n_2w_2] [C(w_1',w_2')-\cos m_1w_1 \cos m_2w_2]dw_1dw_2+ \\ &\tfrac{\alpha}{\pi}\iint_s (\cos n_1w_1 \cos n_2w_2 \cos m_1w_1 \cos m_2w_2)dw_1dw_2, \\ &n'=n_1x\tfrac{N+1}{2}+n_2,\ n'=0,1,\ldots,(\tfrac{N+1}{2})^2-1, \\ &m'=m_1x\tfrac{N+1}{2}+m_2,\ m'=0,1,\ldots,(\tfrac{N+1}{2})^2-1 \end{aligned} \tag{10b}$$

Once the matrix Q is found according to the design requirement, we can compute the eigenvector A which corresponds to the smallest eigenvalue, it is easy to get the filter impulse response $h(n_1,n_2)$ from the eigenvector A by Eq.(6).

The similar procedures can be applied in 2D cases for N even, the amplitude frequency response now is

$$H^*(w_1,w_2)=\sum_{n_1=1}^{N/2}\sum_{n_2=1}^{N/2} a(n_1,n_2)\cos(n_1-\tfrac{1}{2})w_1\cos(n_2-\tfrac{1}{2})w_2 \tag{11}$$

where $a(n_1,n_2)=4h(\frac{N}{2}-n_1,\frac{N}{2}-n_2)$, $n_1=1,2,\ldots,\frac{N}{2}$ and

$$n_2=1,2,\ldots,\tfrac{N}{2} \tag{12}$$

III. 2D FILTER DESIGN EXAMPLES:

Example 1: Design of 2D circular and elliptical bandpass filters.

We take $(w_1',w_2')=(\frac{w_{p1}+w_{p1}'}{2},0)$ as the reference frequency point. The specification for this bandpass design is shown in Fig.1(a), and Fig.1(b) gives as a 23x23 2D circular bandpass filter with $w_{p1}=w_{p2}=0.36\pi$, $w_{p1}'=w_{p2}'=0.64\pi$, $w_{s1}=w_{s2}=0.16\pi$, $w_{s1}'=w_{s2}'=0.84\pi$, Fig.1(c) shows a 26x26 2D elliptical bandpass filter with $w_{p1}=0.3\pi$, $w_{p1}'=0.54\pi$, $w_{s1}=0.16\pi$, $w_{s1}'=0.7\pi$, $w_{p2}=0.5\pi$, $w_{p2}'=0.74\pi$, $w_{s2}=0.36\pi$ and $w_{s2}'=0.9\pi$.

Example 2: Design of 2D fan type filter.

Fig.2(a) presents the specification of a 2D fan filter in which w_s is the stopband cutoff frequency, and the frequency response of a 23x23 2D fan filter is shown in Fig.2(b) with $w_s=0.16\pi$.

IV. CONCULSIONS:

In this paper, the 1D eigenfilter approach is extended for designing two dimensional FIR filters; This approach is simple and powerful, we get very good performance in the resultant 2D filters. Several numerical design examples are illustrated to show the effectiveness of this approach.

REFERENCES

[1] P.P. Vaidyanathan and T.Q. Nguyen, "Eigenfilters: a new approach to least-squares FIR filter design and applications including Nyguist filters," IEEE Trans. Circuits and Systems, Vol.CAS-34, pp.11-23, Jan. 1987.

[2] J.H. McClellan, T.W. Parks and L.R. Rabiner, "A computer program for designing optimum FIR linear phase digital filters" IEEE Trans. Audio Electroacoust, Vol.AU-21, pp.506-526, Dec. 1973.

[3] B. Nobel and J.W. Daniel, Applied Linear Algebra, Englewood Cliffs, NJ: Prentice-Hall, 1977.

[4] L.R. Rabiner and B. Gold, Theory and Application of Digital Signal Processing, Englewoods Cliffs, NJ: Prentice-Hall, pp.472-478, 1975.

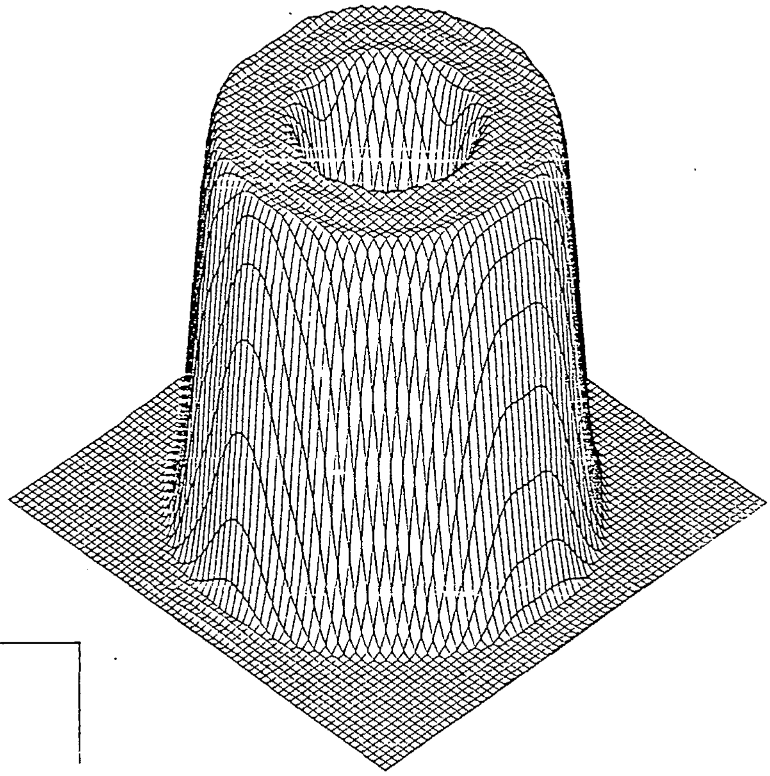

Fig.1(b) A 23x23 2D circular band-pass filter with
$w_{p1}=w_{p2}=0.36\pi$, $w'_{p1}=w'_{p2}=0.64\pi$,
$w_{s1}=w_{s2}=0.16\pi$, $w'_{s1}=w'_{s2}=0.84\pi$.

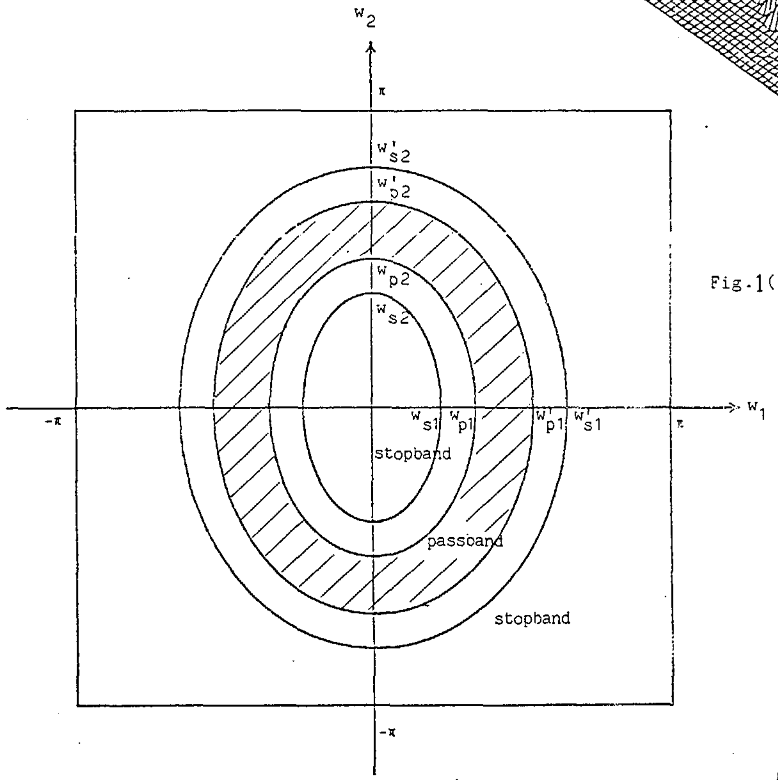

Fig.1(a) Specification of 2D circular and elliptical band-pass filters in Example 1.

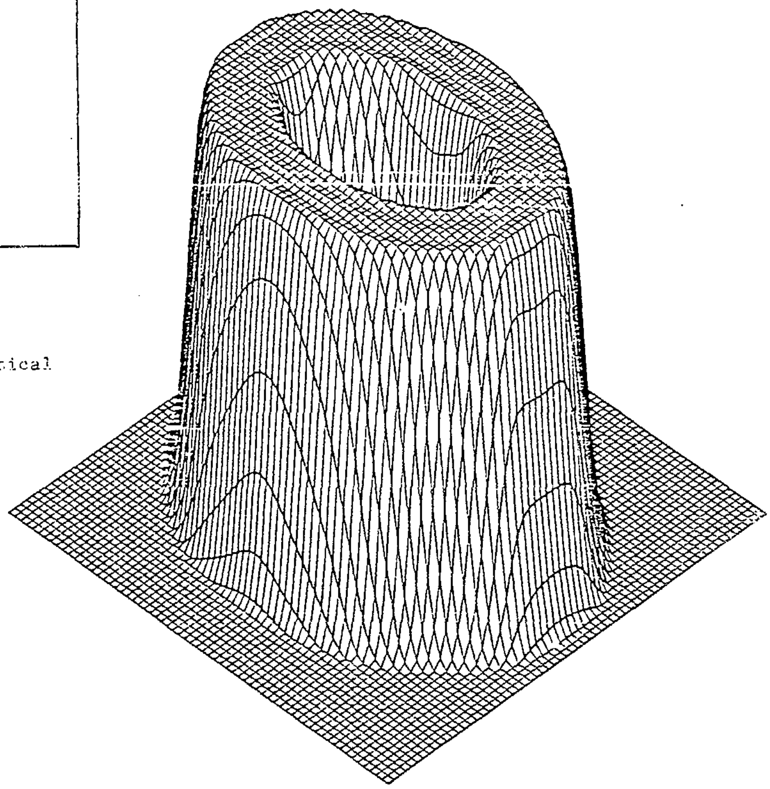

Fig.1(c) A 26x26 2D elliptical band-pass filter with
$w_{p1}=0.3\pi$, $w'_{p1}=0.54\pi$, $w_{s1}=0.16\pi$, $w'_{s1}=0.7\pi$
$w_{p2}=0.5\pi$, $w'_{p2}=0.74\pi$, $w_{s2}=0.36\pi$, $w'_{s2}=0.9\pi$.

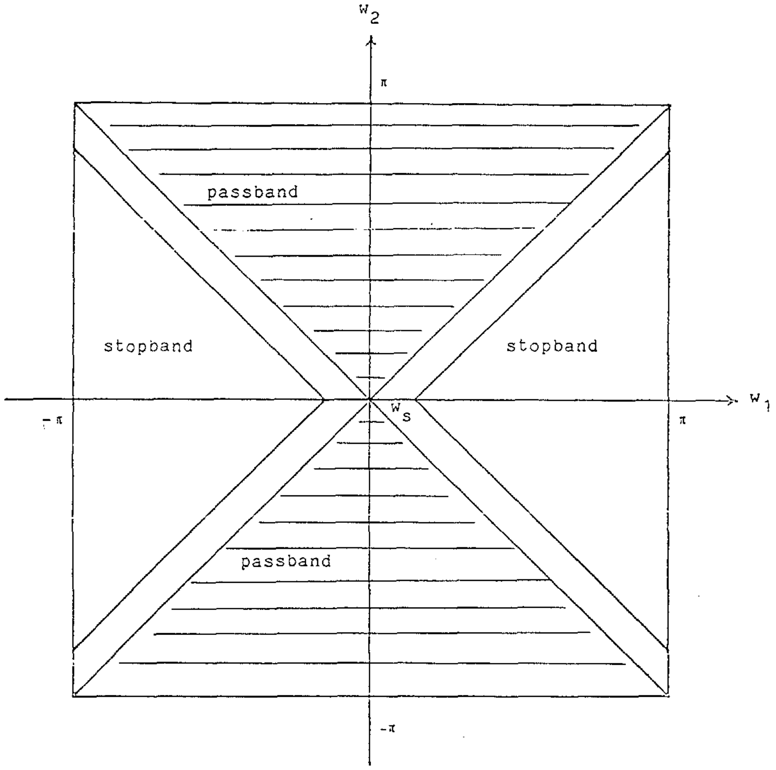

Fig.2(a) Specification of 2D fan type filter in Example 2.

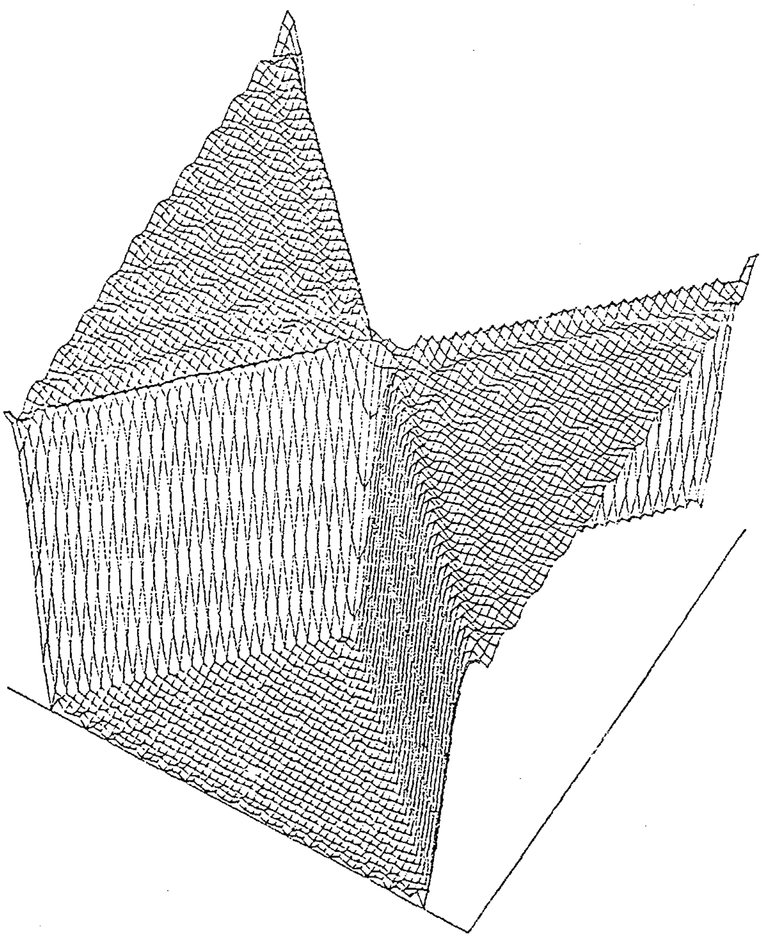

Fig.2(b) A 23x23 2D fan type filter with w_s=0.16π.