



**MOTION ESTIMATION USING POINTWISE  
MULTIGRID MEASUREMENTS**

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**RÉSUMÉ**

On introduit une méthode différentielle du premier et deuxième ordre pour la mesure ponctuelle du déplacement des images qui permet une définition complète du vecteur sauf qu'aux points singuliers. On améliore l'estimation du champ déplacement avec des techniques de filtrage linéaire, non-linéaire, de sous-échantillonnage et de recursion. Les résultats sont présentés par des simulations avec des images naturelles: l'image différence à mouvement composé montre une corrélation spatiale très réduite, ce qui valide la qualité de la méthode proposée.

**SUMMARY**

A first and second order differential pointwise motion vector measurement is proposed and investigated so that complete motion evaluation is obtained except at singular pixels. Motion field estimation is improved by linear, non-linear filtering and multigrid recursion. Research results on natural image sequences are presented: displaced frame differences luminances show very little residual structuring thus validating the capabilities of the proposed algorithm.

**1. Introduction**

Motion estimation methods are based either on block matching or recursive differential measurements /CAF/.

Extensions of the differential method using a second derivative algorithm have been proposed by /NAG/ and more recently by /KRA/: they allow pointwise motion measurement at the expenses of an increase of noise effects.

The aim of the current research is to merge the advantages of the second derivative method with the ones of the gradient method. Large displacements are not well estimated by differential methods so the proposed method exploits also multigrid recursion /CRI/.

Moreover, as estimation methods are going to be increasingly introduced

in hierarchical image processing systems, it is felt that they should be able to provide the upper processing layer with reliable, even if uncomplete, motion field estimations. Here is another goal of the proposed method.

In section two the pointwise two-component motion measurement algorithm will be introduced; in section three the proposed multigrid estimation processing will be presented while in section four some simulation results will be presented.

**2. Pointwise motion measurement algorithm**

The optical flow proposed by /HOR/ is an elegant way of expressing the



bound of the image luminance  $I(P,t)$  on its velocity components  $(v_x, v_y) = \vec{V}(P,t)$ , where  $P$  is the position of the image pixel at time  $t$ :

$$(1) \quad \vec{V}(P,t) \cdot \vec{\nabla} I(P,t) = -\partial I(P,t) / \partial t$$

where  $\vec{\nabla} I(P,t)$  is the luminance surface gradient and  $\partial I(P,t) / \partial t$  is its partial time derivative. /CAF/ assumes that  $\vec{V}(P,t)$  changes slowly so that he proposes a linear regression for estimating the two components  $(v_x, v_y)$  and intraframe recursion in the case of large displacements.

Krause /KRA/ has proposed the use of a vector, the luminance gradient  $\vec{\nabla} I(P,t)$ , to determine the two components of  $\vec{V}(P,t)$ :

$$(2) \quad H(I) \vec{V}(P,t) = -\partial (\vec{\nabla} I(P,t)) / \partial t$$

where

$$H(I) = \begin{vmatrix} I_{xx} & I_{xy} \\ I_{xy} & I_{yy} \end{vmatrix} .$$

This method is very sensitive to image noise so it requires filtering and block averaging, nevertheless the simulation results are not very satisfactory.

If we consider the overdetermined linear system of (1) and (2) we see that it provides the solution of the pointwise motion measurement except at the singular pixels where the gradients of the source luminance  $I(P,t)$  and that of its partial derivatives  $I_x(P,t)$ ,  $I_y(P,t)$  are parallel.

The system is graphically represented by the three velocity lines  $I$ ,  $I_x$ ,  $I_y$  of Fig. 1. These lines, in absence of noise, should intersect in a single point; because of the higher reliability of line  $I$  we assume point  $A$  as the one representing the measurement of the pointwise velocity.

### 3. The displacement field estimation algorithm

The pointwise measurement is limited to pixels having the

absolute value, of at least one first and one second derivative, larger than a given threshold  $T$ , moreover the differential methods are valid only for small displacement, The estimation of the displacement field therefore requires a suitable interpolation algorithm.

The noise effects are reduced by linear filtering of the source image and linear averaging of the pointwise measurements while the sparseness of the displacement measurement can be reduced by non linear averaging of the measurements. The basic steps of the estimation algorithm are as follows:

- a) Changed/no-changed image segmentation is applied to the original frame difference and the two image luminances are then low-pass filtered.
- b) The subsampled displacement measurement algorithm is then applied at each pixel of the original grid assuming that this is plausible due to the non ideality of the filter. A sparse displacement field is obtained.
- c) These measurements are averaged on a 5x5 block, if at least half of the block pels are classified as changed and extremes values are discarded. Moreover the averaged displacement is validated only if the corresponding displaced frame difference is smaller than the corresponding frame difference, otherwise no local displacement is assumed.
- d) The previously obtained measurements are used for recursion on the original non filtered image for measuring recursion displacements.
- e) Step c) is applied to the resulting displacement vector.

The algorithm has been applied on a two level pyramid, still larger displacements can be recursively estimated on a three level pyramid using stronger filtering and subsampling, while higher pyramid levels are not justified due to low correlation.



#### 4. Results

Fig. 2 shows the original image, while Fig. 3 shows the simulation results. Top pictures respectively show the frame difference (right) and the motion compensated frame difference (left); we notice that very little structuring has been left over.

The bottom right picture shows the pels where no pointwise displacement measurement has been possible (white areas); the bottom left picture shows pels where displacement estimation results in nonzero displacement vectors (white areas); pels that have been given zero displacement correspond to black areas.

The algorithm has been able to estimate the displacement of about 75% of the changed pels using a two dimensional filter suited for 1:2 subsampling, a threshold  $T = 5$  has been used on space derivatives.

#### 5. Conclusions

The paper has proposed a modified version of the differential motion estimation algorithm which exploits also the luminance second order derivatives and picture pyramid recursion. The experimental results show that the motion compensated prediction error is no more structured, so that the

displacement field can be assumed to be coherent with the physical motion of the objects and useful for different types of applications.

#### References

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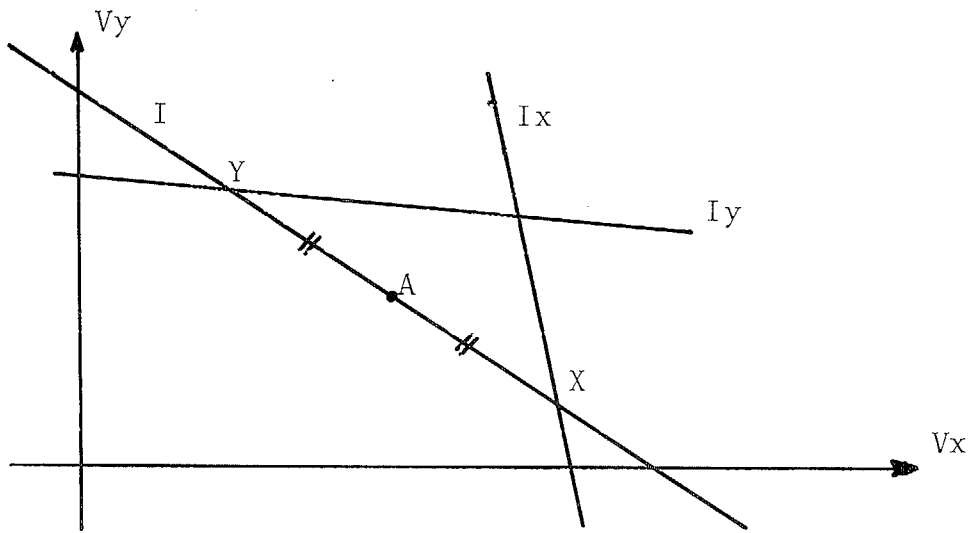


Fig.1. Velocity lines  
 Luminance velocity line I  
 Horizontal edges velocity line Ix  
 Vertical edges velocity line Iy



Fig.2 Source image

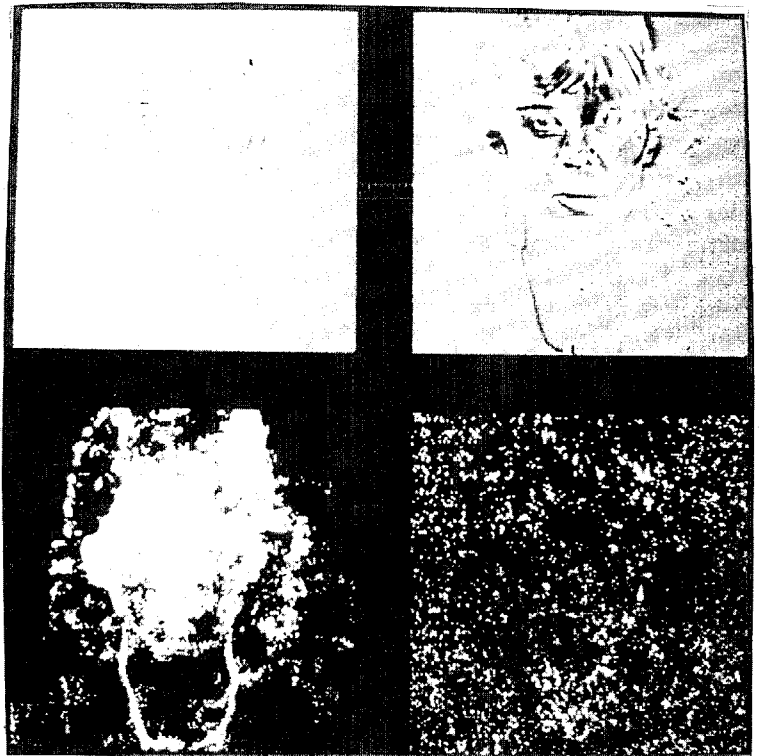


Fig.3 Simulation results  
 upper-left: motion compensated frame differences  
 upper-right: frame differences  
 bottom-left: motion estimated pels  
 bottom-right: pels where motion measurement is not possible for  $T=5$