Multicriterion Decision Model and Algorthm of Image Reconstruction from Projections

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Summary

In this paper we propose a multicriterion decision approach to iamge reconstruction from projections. Our model and algorithm are applied to shepp-Logan head phantom reconstruction and computer pcitures are given on VAX-11/730 microcomputer.

1. Introduction

The problem of reconstructing a multidimensional object from a set of its projections arises in, among other fields, computerized tomography (CT) , radio astronomy, electron microscopy, and synthetic aperture radar(SAR). In CT, the object frequently is a two-dimensional slice ofthe human body. A large number of different techniques have been proposed for image reconstruction, utilizing a number of different models and assumption, the most commonly used approaches are transform methods, series expansion reconstruction methosd, et. al[1]. Wang Yuan Mei and Lu Wei Xue had proposed multicriterion optimization model and iteration algorithms for image reconstruction from projections [2]. In this paper, we discuss

Table 1. Multicriterion Decision Process

Ideal Point	$-g^*_1=4.916$				$g^*_2=0.00476$		$g^*_3=0.001$
Terms	w^k_1	w^k_2	w^k_3	$-g_1(x^k)$	$g_2(x^k)$	$g_3(x^k)$	$d_i=\sum_{j=1}^{3} w^k_j(g_j(x^k)-g_j)$
k=0	0.2	0.3	0.5	4.620	0.0153	0.00621	0.06757
k=1	0.27	0.23	0.5	4.750	0.00832	0.00561	0.04792
k=2	0.36	0.24	0.40	4.820	0.00542	0.00272	0.03580
k=3	0.40	0.15	0.45	4.880	0.00490	0.00124	0.01453

a) 128×128 test image

b) 128×128 image reconstruction by Pyramid expansion

Fig.1 original Image and Recostruction

set k=k+1

3). If the decision maker is able to select the best-compromise solution from x^c_i, or X^c_i contain g^*, stop; the best-compromise solution is found otherwise go to step 4).

4). Change x^c_i by setting weights $w^k=(w^k_1, w^k_2, w^k_3)$, and go to step 1).

3. Simulation Results

We developed an unexplored model and algorithm for multicriterion decision problem for image reconstruction from projections. We first discussed the theory behind the algorithm, and then described a five-step implementation. We next gave a reconstruction, done the shepp and logan head phantom[3]. The shepp and Logan head phantom incorporates brain's intensity constraint, and hence gives a realistic, demanding test of an image reconstruction algorithm. Fig.1 gives an exact 128x128 head phantom and reconstrcuted image, which is obtained by pyramid expansion from 16×16 digital reconstruction. our multicriterion decision process based on 16x16 digitation and 256 projection data (16 views and 16 ray), is shown in table 1. Our model and algorithm showed that multicriterion decision method can significantly improve the performace of entropy oprimization for image reconstruction from projections.

references

1. Gabor T.Herman, Image Reconstruction from projections: the Fundamentals of Computerized Tomography, Academic Press, NY, 1980

2. Wang Yuanmei and LuWeiXue, "Multicriterion Image Reconstruction and Impementation", Computer Vision, Grphics and Image Processing, 1989, to appear.

3. A.Rosenfeld and A,C.kak, " Digital Picture Processing", Academic press, NY, 1982

multicriterion decision approach to image reconstruction from projections.

We often encounter decision making problems with multicriterion in image reconstruction from projections. In recent years, Authers have been devoting attention to the multiplicity of objectives in computerized tomography. Namely, it often becomes necessary to attain some mutually conflicing goals, such as, to maximizethe entropy of image fiels, to minimize the sum of nonuniformity function and peakeness function of image, and square error function of original image and reconstrction. Problems of this type are formulate as multicriterion decision making. An important class of multicriterion decision making is multicriterion optimization or vector optimization. Here we suggested a multicriterion decision making model and iterated algorithm for image reconstruction from projections, and the efficiency of the method will be verified through some experiments in the latter part of this paper.

2. Multicriterion Decision Making Model and Algorithm

Assuming no prior preference structure, we can formulate a multictiterion decsion problem (MDP) for iamge reconstruction from projectionsin general as

$$DR\ \{g_1(x),\ g_2(x),\ g_3(x)\} \qquad (1)$$

$$\text{subject to } x \in X \qquad (2)$$

where $x = \{x \in R^n \mid x \geqslant 0\}$, DR stands for the appropriate decision rules, g_1 is negative entropy function of image field, $g_1(x) = x^T \ln x$, $g_2(x)$ is sum of nonuniformity and peakenness functions, i. e., $g_2(x) = 1/2\alpha x^T S x + 1/2 x^T x$, here S is the smoothed matrix[1]. $g_3(x)$ is the square error funtion of original image and reconstruction, i. e., $g_3(x) = 1/2\beta(y-Ax)^T(y-Ax)$, y is a m-dimensional projection data vector, A is mxn projection matrix.

The multicriterion decision problem(MDP) we are interested in can be stated in the most general term as follows: Based on the decision criteria $g(x)=(g_1(x), g_2(x), g_3(x))$, choose the best alfernatives x from X. Our general formulation of MDP(1)-(2) may transform into the so-called a surrogate MDP of the form

$$\min\ \{ d_p(g(x),\ g^*) = \sum_{j=1}^{3} w_j (g_j(x)-g^*)^p \} \qquad (3)$$

subject to $x \in X$

where $1 \leqslant p < \infty$, $g*$ is the goal vector, w_j is the weight or priority given to the jth criterion, and $d_p(\cdot)$ represents the distance between $g(x)$ and g^*.

Conceptually, once $g*$ and $w = (w_1, w_2, w_3)$are given, and (3) is formnlated for any chosen value of $1 \leqslant p < \infty$, any proper optimization technique can be applied to the surrpgate MDP (3) to determine the best-compromise solution of the original MDP. Let us now return to the surrogate formulation of a multicriterion decision problem(3) . Here we shall maintain the preference structure which stipulates that the best-compromise solution is one which minimizes the combined $d_p(g(x), g*)$ from the given levels $g*$ as in (3).

Given a weight vector w, x_p^c is a compromise solution of an MDP with respect to p if and only if it solves

$$\min\ \{ d_p(g(x), g*) = \sum_{j=1}^{3} w_j (g_j(x)-g^*)^p \} \qquad (4)$$

for $1 \leqslant p < \infty$. The compromise set X given the weight W, is defined as the set of all compromise solutions x_p^c, $1 \leqslant p < \infty$. More precisely,

$$X_w^c = \{x \in X \mid x \text{ sdves(4)given } w \text{ for some} 1 \leqslant p < \infty\} \qquad (5)$$

Theorem 1 . Let x^* solve (3) for any $1 \leqslant p < \infty$ when either (i)x^* is a unigue solution of (3) , or (ii) $w_j > 0$ for j=1, 2, 3 holds. then x^* is a noninferior solution of MDP.

By Theorem 1, if $w > 0$, x_p^c is always a noninferior solution for any $1 \leqslant p < \infty$. Hence the compromise set X_w^c is a subset of the set of noninferior solution X^*.

hence the method of image reconstruction from projection based on MDP model has the following basic steps

0). Ask the decision maker to give $w^0=(w_1^0, w_2^0, w_3^0)$, α, β, $S^{(n \times n)}$, y, $A^{(m \times n)}$ and the ideal point $g*=(g*_1, g*_2, g*_3)$ by considering test image.

$g_1 = \sum_{j=1}^{n} x_{0j} \ln x_{0j}$, $g_2 = 1/2\alpha x_0^T s x_0 + 1/2 x_0^{T,0}$

$g_3 = 0$.

set k=0

1). Scale the weight $w_j^k = w_j^k / g_j$, j=1, 2, 3.

2). Set p=1, construct the compromise set x_p^c by finding the set of optimal solution of

$$\min_{x \in X} \left(\sum_{j=1}^{3} w_j^k (g_j(x) - g_j^*) \right)$$

where $X = \{x \in R^n \mid x \geqslant 0, \sum_{j=1}^{n} x_j = 1\}$