GENERALISED TRANSFORMATIONS IN NONLINEAR IMAGE RESTORATION

M.E. Zervakis and A.N. Venetsanopoulos

University of Toronto
Toronto, Ontario
M5S 1A4, Canada

RESUME

Dans cet article, un algorithme général pour la restoration d' images produites par des systémes non-linéaires est introduit. L' algorithme proposé conduit à ce qui est appelé la "technique de transformation generalisée". Il peut etre utilisé non seulement pour la restoration des systèmes de formation d' images multiplicatifs, mais aussi pour la restoration d' une classe générale de systemes de formation d' images. Afin de démontres les propriétés et les avantages de l' approche de transformation généralisé, le filtre de restoration basé sur l' application directe du critere MMSE en modèle de bruit multiplicatif est obtenu. Le critère MMSE, appliqué a l' image reçue ou bien transformée, résulte en une dégradation de la structure detaillée. L' incorporation d' adaptivité locale dans l' algorithme de restoration introduit est aussi considérée. La formulation d' un critére combiné incorporant les critéres MMSE et LSE est proposée. Enfin, les techniques introduites sout comparées à l' approche MMSE direct.

SUMMARY

In this paper, a general algorithm is introduced for the restoration of images formed through nonlinear systems. The approach introduced, derives the so called "generalised transformation technique". The algorithm proposed can be employed for the restoration of not only the multiplicative, but also a general class of image formation systems. In order to reveal some of the properties and advantages of the generalised transformation approach, the restoration filter based on the direct application of MMSE criterion in the multiplicative noise model, is also derived. The MMSE criterion applied on either the received or the transformed image, results in a degradation of the detailed structure. The incorporation of local adaptivity in the restoration algorithm introduced, is also addressed. The formulation of a combined criterion incorporating the MMSE and the LSE criteria is proposed. Comparisons of the techniques introduced with the direct MMSE approach are presented.

1. INTRODUCTION

Many nonlinear image formation models can be represented by the general equation

$$g(x,y) = q\,(\int_{-\infty}^{\infty} h(x-x_1,y-y_1) f(x_1,y_1)\, dx_1 dy_1 \bigcirc n(x,y)\,) \quad \text{(1a)}$$

where g, f, and n are the received, the original image, and the noise process, respectively. Furthermore $h(.,.)$ is the system's point spread function (psf) and $q(.)$ is the system's point nonlinearity. The operation denoted by $\bigcirc$ is usually either addition, or multiplication. In the discrete case, under the stacked (vector) notation [1], this equation can be written as:

$$g = q(\,Hf \bigcirc n\,)\ . \quad \text{(1b)}$$

In many image formation systems, this model can be transformed to a model, in which there exists a linear relationship among some nonlinear version $p(g)$, $s(Hf)$, and $t(n)$, of the terms g, Hf, and n, respectively. This relationship is represented by:

$$g_p = p(g) = s(Hf) + t(n) = s(Hf) + n_t\ . \quad \text{(2)}$$

The additive noise model, for example, can be classified under this class, with $s(.)$, $p(.)$, and $t(.)$ the unit transformations. As further examples, consider the multiplicative, the exponential, and the nonlinearly transformed multiplicative noise models:

$$g = (Hf) \odot n\ , \quad \text{(3a)}$$

$$g = (Hf)^k \odot n\ , \quad \text{(3b)}$$

$$g = q[\,(Hf) \odot n\,]\ , \quad \text{(3c)}$$

where $\odot$ denotes here point-by-point multiplication, and $q(.)$ is an invertible transformation. By applying the logarithmic transformation, these models reduce to one involving signal and noise interacting in an additive fashion:

$$\ln[\,g\,] = \ln[\,Hf\,] + \ln[\,n\,]\ , \quad \text{(4a)}$$

$$\ln[\,g\,] = k\ln[\,Hf\,] + \ln[\,n\,]\ , \quad \text{(4b)}$$

$$\ln[\,q^{-1}(g)\,] = \ln[\,Hf\,] + \ln[\,n\,]\ , \quad \text{(4c)}$$

Early attempts to the solution of the multiplicative problem were based on inverse filtering followed by a linear operator, such as a Wiener filter. Considering the application of a fixed-coefficient linear filter the "homomorphic" technique was proposed [2]. According to this approach, the image model (3a) is transformed to (4a). Then, a linear filter is employed for the removal of the noise. The estimate of $\ln(Hf)$ is transformed through the inverse nonlinearity $\ln^{-1}(.)$, to yield an estimate of Hf. Finally, the estimate of the original image is obtained through inverse filtering. Several drawbacks of these methods render them of limited use in image restoration. The linear filter affects not only the spectrum of the noise, but also that of the original image. Furthermore, in the presence of some portion of the noise power, the inverse solution produces unacceptable results.

Various improved restoration techniques, based on local statistics have been proposed in the literature. Lee [3] employed the local statistics estimated in a small window centered at each image point, in order to incorporate adaptivity to a local linear least squares error (LLSE) estimation scheme. The multiplicative case is approximated with an additive model through first order Taylor series expansion. Kuan et.al. [4] considered the additive, signal-dependent noise model, where the noise may not be the original noise process corrupting the image, but an appropriate (signal dependent) modification of this process. [4] derives the nonstationary mean, nonstationary variance model, by making the assumption of uncorrelated image (diagonal autocovariance matrix). Bernstein [5] extended the results of [4] to more involved kinds of noise, proposing an adjustable window, which depends on the spatial signal activity.

The previous algorithms are called "local" algorithms. They can only utilise information confined in a small area around the pixel to be enchanced. Global approaches that act on the whole image at once, through a filter that takes under consideration local characteristics (edges), combine the advantages of both local and global techniques. Towards this direction, [6] proposed an adaptive scheme for the enchancement of noisy radar images corrupted by multiplicative noise. [7] proposed a two-dimensional (2-D) homogeneous and isotropic random field for the description of a class of images possessing pronounced edge structures.

In this paper, a global algorithm for the restoration of images formed through nonlinear systems is introduced. Specifically, image formation models that can be brought to the form (2) through a transformation, are considered. The approach introduced in section 2, results in the so called "generalised transformation technique", which derives its name from the mapping from g to $p(g)$. If the nonlinear function $s(.)$ is logarithmic, then the nonlinear mean square error restoration filter can be analytically derived [8]. The statistics of the noise process $t(n(.))$ can be accurately computed (with respect to these of $n(.)$) in the case of a white noise process. Otherwise, the same statistics can be estimated from the transformed data $p(g)$. The algorithm proposed can be employed for the restoration of not only the multiplicative, but also a general class of image formation systems. In order to reveal some of the properties and the advantages of the generalised transformation approach, the restoration filter based on the direct application of MMSE criterion in the multiplicative model, is derived in section 3. It is indicated that this approach can be applied only in the pure multiplicative noise case. The global techniques that are based on the application of the MMSE criterion on either the received (g), or the transformed image ($p(g)$) result in degradation of the detailed structure. The incorporation of local adaptivity in the global restoration algorithm introduced in section 2 is discussed in section 4. The formulation of a combined criterion incorporating the MMSE and the LSE criteria in portions controlled by an indicator of the spatial signal activity, is proposed. The restoration algorithm based on this approach offers the flexibility of applying either MMSE filtering, or inverse filtering, or no filtering at all, depending on the presence of detailed structure in the area under consideration. Comparisons of the techniques introduced with the direct MMSE approach are presented in section 5.

2. GENERALISED TRANSFORMATIONS

Let the image formation model be described by (2). The additive-noise nonlinear image restoration problem is posed as the estimation of the original signal f that results in the transformed image g_p. The noise process n_t is considered as signal independent, whose statistics can be analytically derived. Considering the multiplicative model (3a), for example, a particularly interesting probability density function (pdf) that represents the noise process n, is the log-normal:

$$F_n[n] = \frac{1}{\sqrt{2\pi}\, n\sigma} \exp\left\{ -\frac{(\ln(n)-m)^2}{2\sigma^2} \right\} . \tag{5a}$$

This noise model results in a transformed noise process $n_t = t(n)$, whose pdf is the Gaussian. Some interesting properties of the log-normal process can be found in [9].

$$F[t(n)] = e^{t(n)} F_n[e^{t(n)}] = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left\{ -\frac{(t(n)-m)^2}{2\sigma^2} \right\} . \tag{5b}$$

The estimator that has been proposed for the nonlinear restoration problem has the form [8]:

$$\hat{f} = W\, r(g_p) , \tag{6}$$

where W and $r(.)$ are a linear and a nonlinear transformation, respectively. The MMSE criterion results in the estimator:

$$W\, E\left\{ r(g_p)\, r(g_p^t) \right\} = E\left\{ f\, r(g_p^t) \right\} . \tag{7}$$

This is the general form of the MMSE estimator that is composed from a nonlinear followed by a linear transformation. Under the assumptions that the nonlinear transformation $r(.)$ is the inverse of the model's nonlinearity $s(.)$, and that

$$s(x) = \alpha \ln(x) , \tag{8}$$

the nonlinear function can be approximated by:

$$r(g_p) = r(\, s(Hf) + n_t\,) = Hf + \frac{1}{\alpha}(Hf) \odot n_t , \tag{9}$$

where $\odot$ denotes point-to-point multiplication. Notice that for the pure multiplicative model, $r(.)=s^{-1}(.)=p^{-1}(.)$, so that

$$g = r(\, s(Hf) + n_t\,) \cong Hf + (Hf) \odot n_t .$$

The noise corrupting an image formation system is usually stationary. Hence, the noise process n_t is assumed to yield a homogeneous autocorrelation matrix, even though its mean is not restricted to be zero. Assuming that the mean of the noise process is $\bar{n}_t$, after some algebra one gets:

$$E\{\, r(g_p)\, r(g_p^t)\,\} = (1+\frac{2\bar{n}_t}{\alpha})\, HR_fH^t + \frac{1}{\alpha^2} HR_fH^t \odot R_t \tag{10a}$$

where R_f and R_t are the auto-correlation matrices of the original image f and the noise n_t, respectively. Furthermore,

$$E\{\, f\, r(g_p^t)\,\} = (1+\frac{\bar{n}_t}{\alpha})\, R_f H^t . \tag{10b}$$

Introducing (10) into the MMSE equation (7):

$$W = \kappa R_fH^t\, [\, \kappa_1\, HR_fH^t + \kappa_2\, HR_fH^t \odot R_t\,]^{-1} , \tag{11}$$

where κ, κ_1 and κ_2 denote the corresponding coefficients in (10a) and (10b). In the case of zero mean noise n_t, the scalars κ and κ_1 are equal to unity, so that:

$$W = R_fH^t\, [\, HR_fH^t + \frac{1}{\alpha^2} HR_fH^t \odot R_t\,]^{-1} . \tag{12}$$

Assuming that the Psf is space invariant and that stationarity assumption holds, all matrices involved in (11) or (12) are Toeplitz and, consequently, they can be approximated by circulant matrices. Hence, Fourier transform (FT) techniques are applicable in the implementation of the linear transformation W. For the multiplicative model, the coefficients of W in the FT domain are given by:

$$W(\omega_1,\omega_2) = \frac{P_f(\omega_1,\omega_2)\, H^*(\omega_1,\omega_2)}{|H(\omega_1,\omega_2)|^2\, P_f(\omega_1,\omega_2) + \frac{1}{\alpha^2} P_{tf}(\omega_1,\omega_2)}$$

where $P_f(\omega_1,\omega_2)$ is the power spectrum of f, $H(\omega_1,\omega_2)$ is the FT of the Psf. Furthermore, $P_{tf}(\omega_1,\omega_2)$ denotes the N^2 eigenvalues of the corresponding matrix ($HR_fH^t \odot R_t$) in (12).

Summarising, the proposed "generalised transformation technique" is characterised by the following steps.

Generalised Transformation Technique

i. Estimate the auto-correlation matrix R_t of the transformed noise process. Compute also, the DFT $H(\omega_1,\omega_2)$.
ii. Evaluate the power spectrum $P_f(\omega_1,\omega_2)$ of f, through (10a).
iii. Evaluate the nonlinear MMSE filter by implementing (11).
iv. Apply the filter on the data $r(g_p)$.

For the multiplicative noise case notice that all, except the second step, are performed in the observed (g) data domain; no transformation is required.

3. MMSE RESTORATION

Let us consider the multiplicative noise model (3a) and the application of the MMSE criterion. Introducing the diagonal matrix Λ_n with elements the elements of the noise process, the problem can be modified to:

$$g = \Lambda_n\, H\, f$$

The (linear) MMSE criterion requires the solution of :

$$\min \ ||f-\hat{f}||^2 = ||f-Wg||^2 \ .$$

This function is minimised at:

$$W = R_{fg} R_g^{-1} \ .$$

The correlation matrix between f and g is given by:

$$R_{fg} = E \{ f f^t H^t \Lambda_n \} = R_f H^t \overline{\Lambda}_n \ , \qquad (13a)$$

where, the noise is considered independent of the image. Furthermore,

$$R_g = (H R_f H^t) \odot R_n \ . \qquad (13b)$$

Consequently,

$$W = R_f H^t \overline{\Lambda}_n [(H R_f H^t) \odot R_n]^{-1} \ .$$

Since the noise process n is of non-zero mean $\overline{n}$,

$$W = R_f H^t \overline{\Lambda}_n [(H R_f H^t) \odot (\overline{n}\, \overline{n}^t + C_n)]^{-1} \ .$$

In the case of uncorrelated noise process with mean equal to $\overline{n}$,

$$W = \overline{n} R_f H^t [\overline{n}^2 H R_f H^t + (H R_f H^t) \odot C_n]^{-1} \ . \qquad (14)$$

The MMSE criterion, in the case of multiplicative noise, produces an analytic solution. However, as the image formation model becomes more complicated, the MMSE approach requires more information and complicated manipulations. The estimation of the original image in model (3b), for example requires knowledge of higher order statistics, whereas that in (3c) requires the utilisation of a nonlinear estimator. The generalised transformation technique bypasses these problems by considering the estimation problem in the transformed data domain.

4. ADAPTIVE NONLINEAR RESTORATION

The MMSE criterion (in either the data or the transformed data domain) minimises in the mean square error sense the distance between the original and the restored image. However, the human eye can tolerate high noise levels at areas of high spatial detail (edges). In such areas, it prefers the preservation of the spatial structure to noise suppression. Hence, the notion of the trade-off between restoration and feature preservation becomes evident. Kuan [4] argues that the Wiener filter produces good results if it is applied in a local fashion, so that it locates and preserves edges. A local restoration scheme, however, is not utilising the full amount of correlational information contained in the data. Alternatively, we argue that the global MMSE approach produces good estimates as long as local edge information is utilised. Under this formulation the MMSE filter utilises global information, even though it does alter its form at each image pixel, in order to avoid the full application of the Wiener filter at areas close to edges. Consider the transformed version (9) of the data. The operator $r(.)$ restricts the nonlinear noise effects in the signal-dependent noise term. Since the human eye is not distracted by noise at the regions of edges, the noisy version $r(g_p)$ of Hf can be employed as to provide information concerning the edges. Furthermore, the inverse filter does not destroy the characteristics of the original image; it does, however, amplify the noise, producing (in high noise contamination) noisy estimates. Thus, in the case of light noise corruption, the restoration algorithm may also favour the local structure in the inverse filter. Within these guidelines, the minimisation of the following trade-off criterion is proposed:

$$E \left\{ (1-\alpha) ||f-\hat{f}||^2 + \alpha(1-\beta) ||C_1 [r(g_p) - H\hat{f}] ||^2 + \alpha\beta ||C_2 [r(g_p) - \hat{f}] ||^2 \right\}$$

The characteristics to be preserved are introduced through the inverse solution, and/or the transformed image and they affect the result through the scalar α. The variable β may be either binary $\{0,1\}$, or continuous in $[0,1]$, and incorporates a preference between the inverse and the transformed image.

The coefficient α controls the restoration quality that is compromised at each pixel, to retain some desirable structure. Therefore, α is a function of the position and introduces the spatial feature to be preserved from the inverse solution $H^- r(g_p)$, or the transformed data $r(g_p)$. Image features to be preserved are, usually, the areas of gray-level transitions (edges). Hence, the scalar $0 \leq \alpha \leq 1$ has, usually, the form of an edge detector; $\alpha=1$ at sharp edges, and $\alpha=0$ at flat image areas. Furthermore, since the original image is correlated with g and $r(g_p)$, it is expected that similar correlation applies between the estimate $\hat{f}$, and the transformed data $r(g_p)$. The restored image $\hat{f}$ is expected to be close to the original one, f. Therefore, the pixels of the residual $(f-\hat{f})$ are expected to be little correlated. However, since the Psf matrix H increases the inter-pixel correlation, it is also expected that a non-negligible correlation scheme applies between $H\hat{f}$ and $r(g_p)$. These facts are expressed with the presence of matrices C_1 and C_2, which are used as whitening operators. Since $(f-\hat{f})$ is considered uncorrelated, such a whitening operator does not appear in the first norm of the criterion. Under this interpretation, the matrices C_1 and C_2 introduce spectral information in the adaptive algorithm.

In the presence of strong noise, the inverse solution is of unacceptable quality so that only information derived from $r(g_p)$ itself is employed ($\beta=1$). Hence the following combined criterion is considered for the demonstration of the adaptive algorithm:

$$\min \ E \left\{ (1-\alpha) ||f-\hat{f}||^2 + \alpha ||C [r(g_p) - \hat{f}] ||^2 \right\} . \qquad (15)$$

Considering an estimator of the form (6), and optimising the criterion over W:

$$W = (1-\alpha) [(1-\alpha) I + \alpha C^t C]^{-1} R_f H^t [H R_f H^t + H R_f H^t \odot R_t]^{-1} + \alpha [(1-\alpha) I + \alpha C^t C]^{-1} C^t C \ . \qquad (16)$$

The form (16) introduces a general class of nonlinear restoration filters. Depending on the choice of α and C, the role of the first or the second quadratic form in the objective function, is enhanced. Hence, several interesting filtering structures are obtained for different combinations of the controlling variables. Under the present restoration structure (16), one particular filter has to be designed at each position i,j. The process, however, is highly simplified if no spectral knowledge is considered. For the sake of simplicity, the case $C^t C = I$ is further considered. In this case, only one filter has to be designed. Its application on various image pixels is controlled by the spatially varying α. Hence, for the i-th pixel (in vector notation),

$$\hat{f}_i = \{ W^i r(g_p) \}_i = (1-\alpha_i) \{ R_f H^t [H R_f H^t + H R_f H^t \odot R_t]^{-1} \}_i + \alpha_i \{ r(g_p) \}_i \ . \qquad (17)$$

The implementation of the algorithm is performed in two steps. First, the MMSE filter is designed for the whole image. This filter can be implemented through FT techniques. Then, it is applied on image pixels, in portions controlled by the local value of α_i. Two simple forms of edge indicators, one based on the local and the other on the robust statistics, can be found in [9]

5. EXAMPLES

In this section, two specific examples of the application of the generalised transformation technique are presented. The first one is an 1-D example that demonstrates the advantages of the adaptive technique proposed over the MMSE approach on the multiplicative model. The original signal assumes values in the range of 3 to 200, and its extent is 256 points. The signal is convolved with a triangular psf whose extent is nine (9) points. A white log-normal noise process affects the resulting signal in a multiplicative fashion. The signal to (multiplicative) noise ratio (SNR) is 38dB. The distorted is presented in Figure 1a (solid line), along with the original signal (dashed line). The signal restored through the MMSE criterion applied on the multiplicative model is presented in Figure 1b. The smoothing effect on the detailed structure (edges) is evident. The adaptive algorithm results in the signal presented in Figure 1c. Close to the edges, this algorithm favours the structure of the inverse solution. Hence, the edges are reliably reconstructed, to the expense of some noise preservation at areas of high signal activity.

The second example concerns the application of the adaptive algorithm on a 2-D image. The 256 x 256 test image in Figure 2a.is convolved with a 2-D psf that simulates defocussed lens with circular aperture, whose diametric extent is five (5) points. A multiplicative log-normal noise process corrupts this model, yielding a SNR equal to 34 dB. The distorted image is presented in Figure 2b. Figure 2c depicts the result of the linear MMSE restoration filter, whose mean square error from the original image is 17.46. The adaptive algorithm proposed results in Figure 2d. This algorithm favours the structure of the received image (Figure 2b) at areas close to edges. The resulting mean square error is 18.91. Even though the adaptive algorithm yields a little higher mean square error than the linear MMSE approach, it preserves the edges better, as can be seen by comparing Figure 2d to 2c. The difference is more pronounced at the edges of the tubes, where the signal activity is high.

6. CONCLUSION

In conclusion, a general algorithm is introduced for the restoration of images formed through nonlinear systems. The approach introduced is called "generalised transformation technique". It can be employed for the restoration of a general class of image formation systems, including the multiplicative noise model. In order to preserve the detailed structure, the incorporation of local adaptivity in the restoration algorithm is also addressed. The formulation of a combined criterion incorporating the MMSE and the LSE criteria is proposed. Comparisons of the techniques introduced with the direct MMSE approach are presented.

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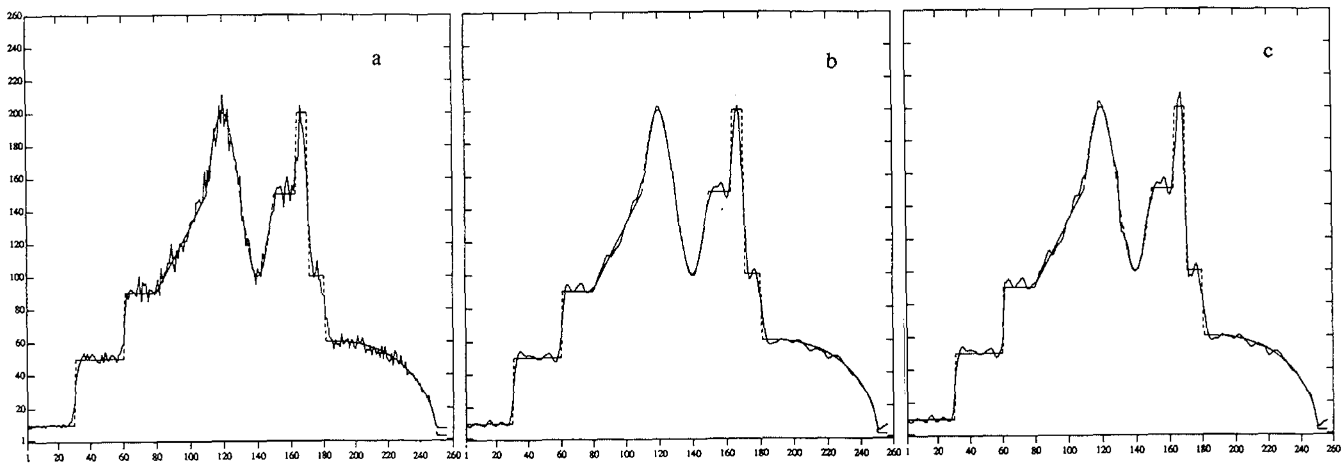
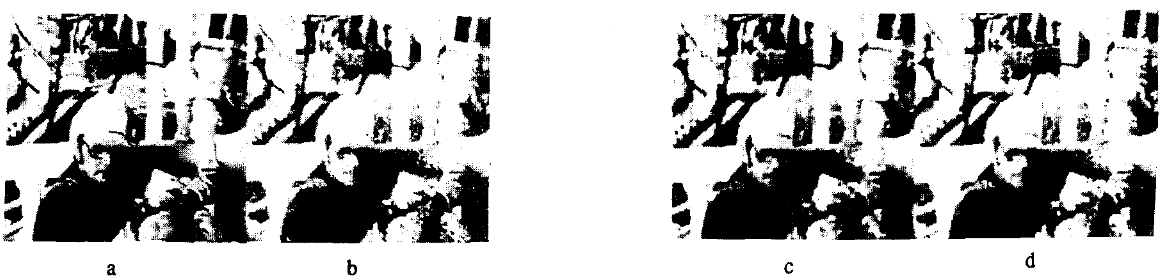


Figure 1
Restoration of 1-D multiplicative noise model.

a) Received signal; b) Signal restored through direct application of the MMSE criterion on the multiplicative model, and c) restored through the adaptive "generalised transformation technique".



a b c d

Figure 2
Scene from a factory;

a) Original and b) Received image; c) Image restored through direct application of the MMSE criterion on the multiplicative model, and d) restored through the adaptive "generalised transformation technique".