CONTRIBUTION TO PREDICTOR OPTIMIZATION IN DPCM

Zoran S. BOJKOVIĆ

Faculty of transport and traffic engineering,
University of Belgrade, Vojvode Stepe 305, YUGOSLAVIA

RÉSUMÉ

Dans cette communication on considère la contribution pour l'optimisation du predicteur en systèmes de modulation par impulsions codées defferentielles MICD. L'optimisation du predicteur peut produire un changement concernant la densité de probabilité du signal a l'entrée du quantificateur. Le maximum gain par le report signal à bruit à cause de l'optimisation du predicteur en MICD est la function de la densité de probabilité du signal à l'entrée du quantificateur avant et après l'optimisation.

ABSTRACT

This paper seeks to provide contribution to predictor optimization in differential pulse code modulation DPCM systems. The predictor optimization may produce a change in the signal probability density function which is the quantizer input. The maximum improvement in signal-to-noise ratio due to predictor optimization in DPCM depends on the prediction error probability density function before and after optimization

1. INTRODUCTION

Differential pulse code modulation DPCM and its variants are important techniques for signal coding. The efficiency of DPCM coders depends upon the prediction algorithm as well as the structure of the quantizer. An optimum quantizer is defined to have a minimum entropy for a given average distortion measured between the sequences of real-valued input values and the corresponding discrete-valued output values [1].

Given a source and a DPCM system, we will say that its quantizer is matched to the prediction error if the quantizer is the Max quantizer (i.e. the minimum mean-squared-error MMSE quantizer) for the stationary probability density of the prediction error process [2].

On the other hand, the analytical result obtained by R.A. Mc Donald, shows that the linear prediction can provide a maximum of 10 dB increase in signal-to-q uantization noise ratio SNR and that 6 dB of this gain can be obtained with a first order predictor [3]. This result is based on the assumption that a decrease in mean-square prediction error is directly reflected as an increase in SNR and that SNR is an appropriate performance indicator.

In the first part of this paper, we deal with theory concerning DPCM systems. Maximum possible improvement due to prediction in DPCM will be analysed in the second part. At the end, some numerical rezults which can be satisfied in practice will be presented.

2. THEORY

The DPCM transmitter and receiver are shown in Figures 1 and 2, respectively. The quantizer input in Figure 1 is given by

$$e(k) = s(k) - \hat{s}(k/k-1) \quad .... \quad (1)$$

where $\hat{s}(k/k-1)$ is the predicted value at time k, while

$$\hat{s}(k/k-1) = \sum_{i=1}^{N} a_i \, \hat{s}(k-i) \quad .... \quad (2)$$

The $a_i (i=1, 2, \ldots, N)$ are predictor coefficients, and

$$\hat{s}(k) = \hat{s}(k/k-1) + e_q(k) \qquad (3)$$

Here, $e_q(k)$ is a quantized version of $e(k)$ and can be expressed in the form

$$e_q(k) = e(k) + n_q(k) \quad .... \quad (4)$$

where $n_q(k)$ denotes quantization noise which may be highly correlated with $e(k)$ and $s(k)$. From equations (3) and (4) and using equation (1), we obtain

$$\hat{s}(k) = s(k) + n_q(k) \quad .... \quad (5)$$

The sequence $(\hat{s}(k))$ is the feedback signal in the transmitter and in the absence of channel errors, $(\hat{s}(k))$ is the receiver output and the feedback signal in the receiver.

The most widely used indicator of DPCM system performance is the signal-to-noise ratio

$$SNR = \frac{\overline{s^2(k)}}{\overline{[s(k)-\hat{s}(k)]^2}} = \frac{\overline{s^2(k)}}{\overline{n_q^2(k)}} \quad .... \quad (6)$$

where "——" denotes a time averaging operation. In order to investigate predictor performance, SNR can be

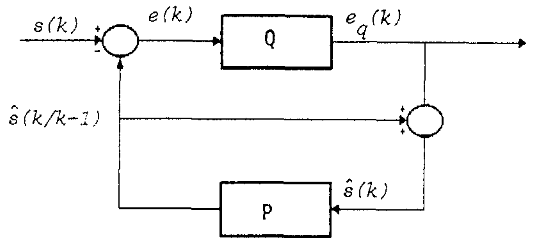

Figure 1. DPCM system transmitter

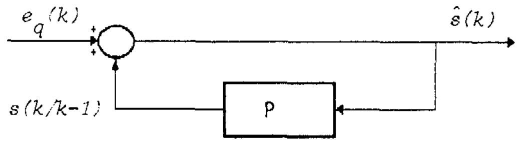

Figure 2. DPCM system receiver

express in the form

$$SNR = \frac{\overline{s^2(k)}}{\overline{e^2(k)}} \cdot \frac{\overline{e^2(k)}}{\overline{n_q^2(k)}} = \frac{SPER}{Q\ (N)} \quad \ldots\ldots\ (7)$$

where $SPER$ is the signal-to-prediction error ratio, i.e.

$$SPER = \frac{\overline{s^2(k)}}{\overline{e^2(k)}} = \frac{\overline{s^2(k)}}{\overline{(s(k) - \hat{s}(k/k-1))^2}} \quad \ldots\ldots\ (8)$$

$Q(N)$ is the normalized quantizing noise power at the output of N-level quantizer

$$Q(N) = \frac{\overline{n_q^2(k)}}{\overline{e^2(k)}} \quad \ldots\ldots\ldots\ldots\ldots\ (9)$$

When it is assumed that the quantization is fine so that the quantization noise in $\hat{s}(k)$ can be ignored, then $\hat{s}(k) = s(k)$ and equation (2) becomes

$$s(k/k-1) = \sum_{i=1}^{N} a_i s(k-i) \quad \ldots\ldots\ldots\ (10)$$

where the "^" has been removed to indicate that $(s(k))$ rather than $(\hat{s}(k))$ appears on the right side of equation (10).

Another form for equation (7) is used in DPCM system analysis. Namely, using equation (10) and defining

$$\varepsilon(k) = s(k) - s(k/k-1) \quad \ldots\ldots\ldots\ (11)$$

equation (7) gives

$$SNR = \frac{\overline{s^2(k)}}{\overline{\varepsilon^2(k)}} \cdot \frac{\overline{\varepsilon^2(k)}}{\overline{n_q^2(k)}} = \frac{SNRI}{Q(N)} \quad \ldots\ldots\ (12)$$

where

$$SNRI = \frac{\overline{s^2(k)}}{\overline{\varepsilon^2(k)}} \quad \ldots\ldots\ldots\ldots\ (13)$$

represents the SNR improvement and $Q(N)$ is previously defined, only this time the quantizer input is $\varepsilon(k)$.

3. MAXIMUM POSSIBLE IMPROVEMENT DUE TO PREDICTION

The main result of the present paper originates from the following theorem.

Theorem: The signal-to-noise ratio improvement in DPCM system SNRI $\geqslant$ SPER, where SPER represents the signal-to-prediction error ratio. The only assumption required to establish

$$SNRI \geqslant SPER \quad \ldots\ldots\ldots\ldots\ldots\ (14)$$

is that the predictors are optimal in the minimum mean squared error MMSE sense.

Proof: We have to demonstrate that the SNRI from equation (13) is greater than or equal to the SPER in equation (8). Since the numerators in equations (8) and (13) are identical, it sufficies to show that the denominator of equation (13) is less than or equal to the denominator of equation (8). Replacing the time averaging operation with the ensemble expectation, we have to show tacking into account equation (11) that

$$E\ ((s(k) - s(k/k-1))^2) \leqslant E\ ((s(k) - \hat{s}(k/k-1))^2) \quad (15)$$

where it is assumed

$$s(k/k-1) = E\ (s(k)\ /\ S(k-1)) \quad \ldots\ldots\ldots\ (16)$$

as well as

$$\hat{s}(k/k-1) = E\ (s(k)\ /\ \hat{S}(k-1)) \quad \ldots\ldots\ldots\ (17)$$

with $S(k-1) = (s(j),\ j= 1,\ 2,\ \ldots,\ k-1)$ and $\hat{S}(k-1) = (\hat{s}(j),\ j=1,\ 2,\ \ldots,\ k-1)$ assuming that the predictors are chosen to be optimal in the MMSE sense.

Left side of the equation (15) gives

$$E\ ((s(k) - E\ (s(k)\ /\ S(k-1))^2) = E\ ((s(k) - E\ (s(k)\ /\ S(k-1),\ \mathcal{N}(k-1)))^2) \quad \ldots\ldots\ldots\ (18)$$

where $\mathcal{N}(k-1) = (n_q(j),\ j=1,\ 2,\ \ldots,\ k-1)$

Equation (18) holds since regardless of the properties of the quantization noise, $\mathcal{N}(k-1)$ cannot provide more information about $s(k)$ than $S(k-1)$.

Tacking into account that

$\hat{S}(k-1) = S(k-1) + \mathcal{N}(k-1) = (s(j) + n_q(j),$
$j=1,\ 2,\ \ldots,\ k-1)$

and noting that $\hat{S}(k-1)$ is a transformation of the sets $S(k-1)$ and $\mathcal{N}(k-1)$ and hence cannot provide more information about $s(k)$ then $S(k-1)$ and $\mathcal{N}(k-1)$ individually, it follows that

$$E\left((s(k) - E(s(k) / S(k-1))^2\right) =$$
$$= E\left((s(k) - E(s(k) / S(k-1), \wp(k-1)))^2\right) \leqslant$$
$$\leqslant E\left((s(k) - E(s(k) / S(k-1) + \wp(k-1)))^2\right) =$$
$$= E\left((s(k) - E(s(k) / \hat{S}(k-1)))^2\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (19)$$

which establishes equation *(15)*. Thus, from equations *(8)*, *(13)* and *(15)*, we obtain the result that

$$SNRI \geqslant SPER$$

Hence, SNRI represents the maximum possible improvement due to prediction in DPCM.
This completes the proof.

4. NUMERICAL RESULTS

The performance of a suboptimum predictor and an optimum predictor are to be compared based on equations *(12)* and *(13)*. Assume that the suboptimum predictor achieves an SNRI denoted by $SNRI_1$ and that the corresponding error sequence given by equation *(11)* has a Gaussian distribution. If the quantizer is four-level, MMSE quantizer from Max [4] then $Q(N)$ in equation *(12)* is $Q(4) = 0{,}1175$. Then the SNR of this DPCM system from *(12)* is

$$SNR_1 = \frac{SNRI_1}{0{,}1175} = 8{,}51\ SNRI_1 \quad \ldots\ldots\ldots (20)$$

S uppose now that the predictor is chosen to maximize the SNRI and that the maximum SNRI achieved by this optimum predictor is SNRI' where by assumption

$$SNRI' \geqslant SNRI_1$$

Under the same assumptions that led to equation *(20)*, the estimated value of the SNR for this DPCM system with the optimized predictor is

$$SNR_2 = 8{,}51\ SNRI' \quad \ldots\ldots\ldots\ldots\ldots (21)$$

Since $SNRI' \geqslant SNRI_1$, it will be $SNR_2 \geqslant SNR_1$.

Assume now that the sequence $(\varepsilon(k))$ is gamma distributed. For a gamma distribution, $Q(N)$ with $N=4$, is $Q(4) = 0{,}2326$ [5]. Therefore the SNR of the system with the optimized predictor and gamma-distributed quantizer is according to equation *(12)*

$$SNR' = \frac{SNRI'}{0{,}2326} = 4{,}30\ SNRI' \quad \ldots\ldots\ldots (22)$$

Comparing equations *(21)* and *(22)*, it is evident that the SNR of the system after optimization is about 3 dB less than what would be predicted if the change in *pdf* of $\varepsilon(k)$ were ignored, i.e

$$\frac{SNR'}{SNR_2} = \frac{4{,}30}{8{,}51} = 0{,}505$$

or

$$10 \log_{10} \frac{SNR'}{SNR_2} = -2{,}97\ dB$$

If the SNR of the original nonoptimized system given by equation *(20)* is compared to the SNR of the optimized system in, equation *(22)*, in order to have $SNR' \geqslant SNR_1$, it is necessary that $SNRI' \geqslant 1{,}98\ SNRI_1$. That is, in order for the optimized system to have a grater SNR than the nonoptimized system, SNRI' must exceed SNRI by about 3 dB. Of course, this is only true under the assumption of the example concerning four-level quantizer, as well as the fact that the MMSE Gaussian quantizer is replaced with a MMSE quantizer optimized for a gamma-distributed input. It seems likely that these assumptions can be satisfied in practice.

5. CONCLUSION

The DPCM predictor optimization may cause a change in the prediction error probability density function that is ignored by standard analyses. Optimizing the DPCM predictor in the MMSE sense can produce a change in the SNR that is much greater than or much less than the increase obtained in the SNR improvement or the SPER. Finally, the minimum improvement in the SNR due to predictor optimization in DPCM depends not only of the source correlation properties but also on the prediction error probability density function before and after optimization.

REFERENCES

[1] P. Noll, R. Zelinski: "Bounds on Quantizer Performance in the Low Bit-Rate Region", IEEE Trans. on Commun., Vol. Com - 26, No. 2, February 1978, pp 300-304.

[2] M. Naraghi-Pour, D.L. Neulroff: "Mismatched DPCM Encoding of Autoregressive Processes", IEEE Trans. on Inf. Theory, Vol. 36, No. 2, March 1990, pp 296-304.

[3] R.A. Mc Donald: "Signal-to-noise and idle channel performance of differential pulse code modulation systems - Particular applications to voice signals", Bell Syst. Tech. J., Vol. 45, September 1966, pp 1123-1151.

[4] J. Max: "Quantizing for minimum distortion", Trans. Inform. Theory, Vol. IT-6, pp 7-12, March 1960.

[5] M.D. Paez and T.H. Glisson: "Minimum mean-squared error quantization in speech PCM and DPCM systems" IEEE Trans. Commun., Vol. COM-20, pp 225-230, April 1972.