

LOSSES AND RECOVERY OF EFFECTIVE
PSEUDOPower IN PRODUCT MODULATION
AND MULTIRATE FILTERING

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RÉSUMÉ

The phenomenon of losses of the effective pseudopower in product modulation and in multirate filtering has been investigated. The negative influence of this phenomenon on the functional properties of the considered systems and the problem of recovery of the effective pseudopower in wave digital filters (WDF's) has been discussed. Arrangements with additional, periodically time-varying feedback between the modulated signal (representing the incident wave) and the reflected signal, initially applied for the pseudopower recovery in multirate filtering, have been adopted also for a product modulation. The arrangements under consideration have been described by means of the so-called conversion functions. Their analytical expressions have been derived and, on this basis, a method for the construction of the frequency characteristics has been proposed. The design of the considered arrangements has been investigated. Suitable strategies for optimization of filter coefficients have been proposed.

ABSTRACT

1. Introduction

In many applications of digital signal processing, digital filtering is connected with product modulation or with sampling rate alteration (interpolation or decimation) [1, 2, 3]. It has been shown [4, 5, 6] that in such processes losses of effective input signal pseudoenergy occur even for classical pseudolossless filters [7]. Such losses influence in a negative way the functional properties of the system. However, using an additional, periodically time-varying feedback between the modulated signal (representing the incident wave of a pseudolossless wave digital filter) and the reflected signal, it is possible to recover the pseudoenergy which would otherwise be lost in a process of sampling rate increase or decrease [4, 5]. In order to achieve acceptable passband attenuation of such arrangements called wave digital filters with recovery of effective pseudopower, optimization of their coefficients is necessary [5]. Among the most important advantages thus obtained are the following: improved filtering properties, broader stability margin under looped conditions, and greater dynamic range.

This same technique can also be implemented for a product modulation [8, 9] but the effects achieved in this case may be not so spectacular as those for sampling rate alteration. This is not only because of an unperfect pseudopower recovery, even after a careful optimization, but also due to the fact that, in the case of product modulation, the selectivity requirements for the filters (which are time-varying systems) may occur to be much stronger than those for the classical time-invariant filters. These and related aspects of the pseudopower recovery in a product modulation

and in multirate filtering will be discussed in the present paper.

2. Product modulation and multirate filtering

We will consider an arrangement of Fig. 1 which may be a part of a greater signal processing system. This arrangement contains a premodulation filter $H_1(e^{j\omega T})$, a product modulator with the carrier signal $q(nT)$, $n = 0, \pm 1, \pm 2, \dots$, and a postmodulation filter $H_2(e^{j\omega T})$, where T is the sampling period. The input signal $x(t_{1n})$, $t_{1n} = t_{10} + nT$, is transformed by the transmittance $H_1(e^{j\omega T})$ into the effective signal $x_u(nT)$. Then $x_u(nT)$ is multiplied by $q(nT)$ in order to obtain the modulator output signal $x_q(nT)$. Finally, the filter $H_2(e^{j\omega T})$ reduces the spectrum to the desired bands and in this way produces the output signal $y(t_{2n})$, $t_{2n} = t_{20} + nT$. About the carrier signal $q(nT)$ we will assume that it is periodic with period $T_o = MT$ where M is an arbitrary positive integer. Notice that if the signal $q(nT)$ is composed of sequences $\dots, 1, 0, 0, \dots, 0, \dots$ then the modulation in Fig. 1 may be interpreted as sampling rate alteration. Thus, the arrangement in Fig. 1 represents both processes: product modulation and sampling rate alteration but for interpolation we omit the premodulation filter and for decimation — the postmodulation filter.

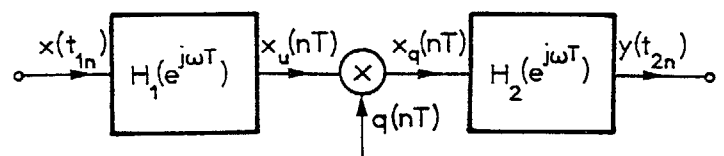


Fig. 1. An arrangement under consideration



In practice, the most important modulation scheme is the so-called multiplier-free sinusoidal modulation for which the carrier signal $q(nT)$ is composed of sinusoidal sequences containing elements ± 1 and 0 only [2, 9].

Theorem 1: All possible essentially different multiplier-free sinusoidal sequences representing one period of the sinusoid

$$q(nT) = \sqrt{2}|Q| \cos(2\pi M^{-1}n + \vartheta) \quad (1)$$

$$-\pi < \vartheta \leq \pi, \quad n = 0, \pm 1, \pm 2, \dots$$

are those listed in Table 1.

Table 1. Multiplier-free sinusoidal sequences

Sequence	M	$P_q = M_1/M$	$\sqrt{2} Q $
(1)	1	1	$\sqrt{2}$
(1, -1)	2	1	$\sqrt{2}$
(1, -1, 0)	3	2/3	$2/\sqrt{3}$
(1, 0, -1, 0)	4	1/2	1
(1, 1, -1, -1)	4	1	$\sqrt{2}$
(1, 1, 0, -1, -1, 0)	6	2/3	$2/\sqrt{3}$

M_1 – number of elements ± 1 ; P_q – sequence pseudopower

Proof

We first make some observations:

1. distances between two elements 0 are equal and greater than zero,
2. at most two elements +1 or two -1 may be consecutive to each other and if the first pair is present then the second also exists,
3. if the element just before a pair of elements ± 1 is 0 then the next element is also 0.

Now we will prove that no sequence exists with length $M > 6$. Assume that q is a sequence with $M > 6$. Assume first that this sequence contains only elements ± 1 . It must contain a pair of elements +1, otherwise it would contain at least two neighbouring sequences (1, -1), i.e., more than one period of a sinusoid. From observation 2 we conclude that q contains also a pair of elements -1. It is, however, not possible because it would contain a sequence (1, 1, -1, -1), i.e., again more than one period of a sinusoid.

Thus, q must contain at least one element 0. It is, however, also not possible because it would contain a sequence (1, -1, 0) or a sequence (1, 1, 0, -1, -1, 0), i.e., in both cases it would contain more than one period of a sinusoid.

Constructing now, on the basis of the above observations, sequences with lengths $1 \leq M \leq 6$, we arrive at the sequences listed in Table 1.

q.e.d.

3. Losses and recovery of the effective pseudopower

Definition 1: Let $x(t_n)$ be an arbitrary real discrete-time signal. The quantities

$$p_x(t_n) = x^2(t_n), \quad (2)$$

$$\varepsilon_x = \lim_{n \rightarrow \infty} \sum_{\nu=-\infty}^n p_x(t_\nu) \quad (3)$$

are said to be the pseudopower at the instant t_n and the pseudoenergy of the signal $x(t_n)$, respectively.

Definition 2: A set of frequency bands $\{\mathcal{P}_{-L}, \mathcal{P}_{1-L}, \dots, \mathcal{P}_{-1}, \mathcal{P}_1, \mathcal{P}_2, \dots, \mathcal{P}_L\}$

$$\mathcal{P}_l = [\omega_{l1}, \omega_{l2}], \quad \mathcal{P}_{-l} = [-\omega_{l2}, -\omega_{l1}], \quad (4)$$

$$l = 1, 2, \dots, L,$$

$$\omega_{l1} = m_l \Omega_o + \omega_{l-1} \quad \text{and} \quad \omega_{l2} = m_l \Omega_o + \omega_l \quad (5)$$

where m_1, m_2, \dots, m_L are arbitrary integers and

$$0 = \omega_0 \leq \omega_1 \leq \dots \leq \omega_L = \Omega_o/2 \quad (6)$$

is called the integer set of bands with respect to the frequency Ω_o .

Definition 3: A signal $x_c(t)$ with spectrum lying in a set of bands $\{\mathcal{P}_{-L}, \mathcal{P}_{1-L}, \dots, \mathcal{P}_{-1}, \mathcal{P}_1, \mathcal{P}_2, \dots, \mathcal{P}_L\}$ which is integer with respect to the frequency Ω_o , and containing no Dirac pulses at the ends of all bands, is called the integer L -band signal with respect to the frequency Ω_o .

On the basis of the above considerations, the following theorem may be proven:

Theorem 2: If the input signal of the product modulator in Fig. 1 corresponds to an integer L -band signal with respect to the frequency

$$\Omega_o = 2\pi/T_o, \quad T_o = MT \quad (7)$$

and the M -element multiplier-free modulation sequence q contains M_1 elements ± 1 then the pseudoenergies ε_u and ε_q defined according to (3) for the signals $x_u(nT)$ and $x_q(nT)$ at the input and at the output of the modulator, respectively, are related by the expression

$$\varepsilon_q = (M_1/M)\varepsilon_u = P_q\varepsilon_u. \quad (8)$$

From this theorem we immediately conclude that the existence of elements equal to 0 in the carrier signal $q(nT)$ is the cause for losses of the effective pseudopower. In the same time, however, these zeros may be exploited for the recovery of the lost pseudopower, namely, by changing up to 1 the value of the time-varying gain coefficient $\gamma(nT)$ in the feedback loops shown in Fig. 2 [5], i.e.

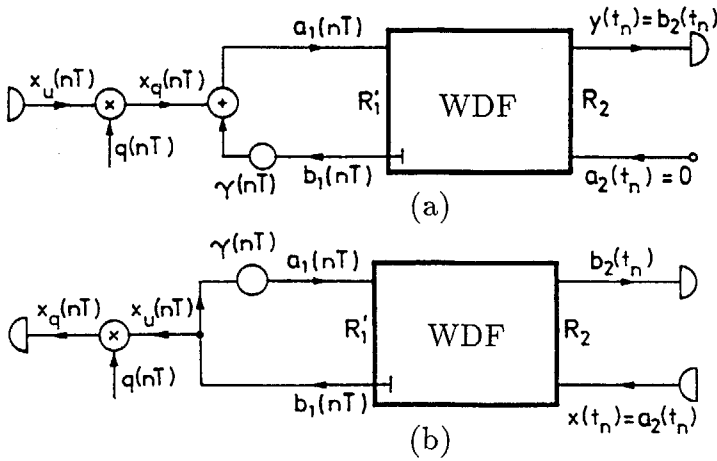


Fig. 2. WDF's with pseudopower recovery:
 (a) postmodulation filter,
 (b) premodulation filter

$$\gamma(nT) = \begin{cases} \gamma_0 & \text{for } q(nT) \neq 0 \\ 1 & \text{for } q(nT) = 0 \end{cases} \quad (9)$$

$$-1 < \gamma_0 < 1.$$

4. Construction of frequency characteristics

The premodulation filter and the postmodulation filter in Fig. 1 can be described by the transmittances $H_1(e^{j\omega T})$ and $H_2(e^{j\omega T})$ only if they are time-invariant systems. However, since the filters with pseudopower recovery (Fig. 2) contain a periodically time-varying feedback loop, their description is more complicated and may be based on a set of M functions $K_0(e^{j\omega T}), K_1(e^{j\omega T}), \dots, K_{M-1}(e^{j\omega T})$ called conversion functions [2, 4]. Considering, e.g., the postmodulation filter, we can express the output signal as

$$y(t_{2n}) = \sum_{m=0}^{M-1} y_m(t_{2n}), \quad (10)$$

$$y_m(t_{2n}) = \frac{1}{\Omega} \int_{-\Omega/2}^{\Omega/2} Y_m(e^{j\omega T}) e^{j\omega t_{2n}} d\omega, \quad (11)$$

$$Y_m(e^{j(\omega+m\Omega_o)T}) = K_m(e^{j\omega T}) X_u(e^{j\omega T}), \quad (12)$$

$$X_u(e^{j\omega T}) = \sum_{n=-\infty}^{\infty} x_u(nT) (e^{j\omega T})^{-n}. \quad (13)$$

The frequency ω in (12) corresponds to the input signal $x_u(nT)$ while the frequencies $\omega + m\Omega_o$, $m = 0, 1, \dots, M-1$, correspond to the respective components $y_m(t_{2n})$ of the output signal $y(t_{2n})$ given by (10). Assuming that the signal $x_u(nT)$ corresponds to an integer L -band signal with respect to the frequency Ω_o and defining new, the so-called band-fitted conversion functions

$$\check{T}_m(e^{j\omega T}) = \check{K}_m(e^{j(\omega-m\Omega_o)T}), \quad (14)$$

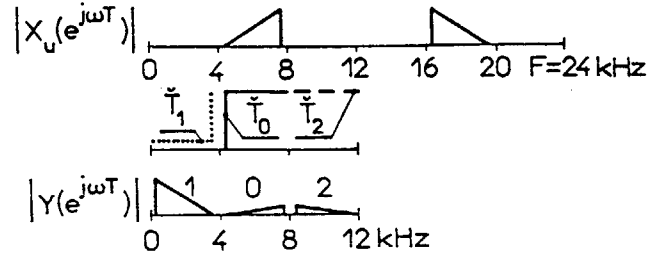


Fig. 3. Construction of the frequency characteristic

$$m = 0, 1, \dots, M-1,$$

$$\check{K}_m(e^{j\omega T}) =$$

$$\begin{cases} K_m(e^{j\omega T}) & \text{for } \omega \in \bigcup_{k=-\infty}^{\infty} \bigcup_{l=1}^L [\omega_{l1} + k\Omega, \omega_{l2} + k\Omega] \\ & \cup [-\omega_{l2} + k\Omega, -\omega_{l1} + k\Omega] \\ 0 & \text{for other bands} \end{cases}$$

we can fit the frequencies of the conversion functions to the frequencies of the output signal components.

Thus, we can write

$$Y_m(e^{j\omega T}) = \check{T}_m(e^{j\omega T}) X_u(e^{j(\omega-m\Omega_o)T}), \quad (15)$$

$$m = 0, 1, \dots, M-1.$$

Definition 4: A function

$$T(e^{j\omega T}) = \sum_{m=0}^{M-1} \check{T}_m(e^{j\omega T}) \quad (16)$$

where $\check{T}_m(e^{j\omega T})$, $m = 0, 1, \dots, M-1$, are defined by (14), is called the resultant conversion function of the system described by the conversion functions $K_0(e^{j\omega T}), K_1(e^{j\omega T}), \dots, K_{M-1}(e^{j\omega T})$.

Notice, that in (16) no real summing takes place because in a specific frequency only one of the band-fitted conversion functions can be different from zero.

The resultant conversion function can serve as a frequency characteristic of the system under consideration. The attenuation α , the phase β , and the group delay τ can be defined as

$$\alpha = 20 \log |T^{-1}(e^{j\omega T})| \quad [\text{dB}], \quad (17)$$

$$\beta = \arg T(e^{j\omega T}) \quad [\text{rad}], \quad (18)$$

$$\tau = \frac{d\beta}{d\omega} \quad [\text{s}], \quad (19)$$

respectively.

An example of the construction of the frequency characteristic for the modulation sequence $(1, -1, 0)$ and a lowpass postmodulation filter op-



erating with the rate $F = 24$ kHz is shown in Fig. 3. The input signal spectrum has been restricted to the band $\omega \in [4, 8]$ kHz and its aliases.

5. Design of filters with pseudopower recovery

First of all, the tolerance scheme for the attenuation of the filter must properly be defined. We should take into account that the products of modulation would occur in more bands in the case if a filter with pseudopower recovery were used in comparison with those for a classical time-invariant filter. Thus, the selectivity requirements for the former may be much stronger.

Second, the filter coefficients must be properly optimized. Two different optimization strategies are reasonable:

- Strategy 1: minimization of the absolute attenuation (the absolute loss) in the passband
- Strategy 2: optimization with flat loss in the passband, i.e., merely the passband ripple minimization.

By the first strategy filters with lower passband sensitivity but also with lower stopband attenuation can be obtained than by the second strategy.

As an illustration of the design of the considered arrangements we continue the example in Fig. 3. We have assumed that the filter should have a passband edge at 3.402 kHz, a stopband edge at 4.598 kHz, a passband ripple of at most 0.1 dB, and a stopband attenuation of at least 50 dB. As a reference filter the fifth-degree Cauer filter C051544 [10] has been chosen. The results of optimization are shown in Fig. 4.

6. Conclusions

We have shown how the technique of recovery of the effective pseudopower, previously applied to wave digital multirate arrangements, can also be adopted for systems with product modulation. Among advantages thus obtainable are, above all, improved stability properties in a loop (formed, e.g., by a telecommunication link) [11]. Increased stopband attenuation for a given filter structure is certainly also advantageous (e.g., the minimum stopband attenuation of the optimized illustrative filter is greater by 12.1 dB for Strategy 1 and by 14.6 dB for Strategy 2 than the stopband attenuation of the Cauer filter C051544) but this advantage may in some applications occur to be of minor importance because of possibly stronger selectivity requirements for filters with pseudopower recovery.

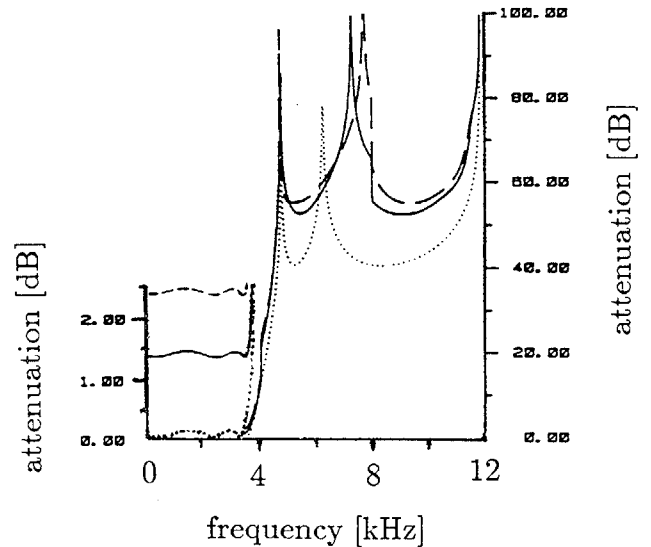


Fig. 4. Attenuation characteristics: of the Cauer filter C051544 (...) and of filters with pseudopower recovery having coefficient values optimized by Strategy 1 (—) and by Strategy 2 (- -)

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