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The Use of Orthogonal Projection for System Impulse Response Estimation in the Blind Deconvolution Method

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RÉSUMÉ

Dans [3], on a presenté la nouvelle méthode de déconvolution aveugle, basée sur le modèle d'un type de vibration de propagation d'ondes. La méthode a été projetée pour la déconvolution de signaux seismiques, enregistrés dans les mines. Elle se consiste de deux pas fondamentals: d'un - l'estimation de la rèponse impulsionelle du système et d'autre - la filtration inverse unie avec optimilisation.

Dans l'article presenté on a introdui la méthode d'estimation de la rèponse impulsionelle et la nouvelle définition de la fonction de coût basée sur l'autocorelation du quatrième ordre. L'instrument fondamental de la méthode c'est la projection ortogonale. On a presenté aussi les résultats de déconvolution.

ABSTRACT

In [3], a new blind deconvolution method based on a resonanse model of wave propagation has been outlined. The method has been designed for deconvolution of seismic signals gathered in mines, caused by shocks. It consists of two basic steps: the first one is the estimation of a system impulse response and the second one - the inverse filtering combined with an optimization.

In this paper, a method of system impulse response estimation in which a fundamental tool is orthogonal projection as well as a new cost function definition based on the fourth order autocorrelation function have been introduced. In addition, some deconvolution results has been presented.

1. Introduction

In many cases e.g. processing of seismic signals produced by shocks in mines, neither an excitation $x(t)$ nor a system impulse response $h(t)$ are known but merely a system output $y(t)$. Being interested in $x(t)$ and $h(t)$ we are faced with a problem of blind deconvolution. The deconvolution method, partially presented in [2,3], is designed for seismic signals gathered in the copper mines for which predictive and homomorphic deconvolution methods have failed. It has turned out that signals observed in the copper mines differ essentially from those occurring in seismic exploration, because of differences in environment of seismic waves propagation.

Some assumptions presented below are connected with application of the method. These assumptions allow to design the deconvolution method but also restrict its applicability to a specified class of the signals. Likewise it is a wide class of signals occurring in evaluation of mechanical objects. This class is specified as follows :

1. the input is a short-term impulse,
2. the system is constituted of parallel connection of elementary systems, i.e.

$$h(t) = \sum_{n=1}^{N} h_n(t) \qquad (1)$$

3. the elementary systems (ES) are of vibrating type.

The first step of the blind deconvolution method is estimation of the system impulse response (IR). The second one is the inverse filtering implementing the estimator of the system IR combined with an optimization.

In this paper, mainly the method of estimation of system impulse response as well as the problem of the cost function definition have been presented.

2. The concept of system impulse response estimation

For the short-term impulse input and for the ESs of vibrating type, the system output can be expressed as [2]

$$y(t) \approx \sum_{n=1}^{N} a_x h_n(t - t_x) \tag{2}$$

where a_x and t_x are constants. Equation (2) has a form of approximating sequence

$$y(t) \approx \sum_{l=1}^{L} \alpha_l w_l(t) \tag{3}$$

where the set of coefficient $\{\alpha_l: l=1,...,L\}$ denotes a representation of the $y(t)$ in a basis $\{w_l(t): l=1,...,L\}$.

Similarity of (2) and (3) yields the idea of the system impulse response estimation. Choosing the base $\{w_l(t)\}$ corresponding to impulse responses of ESs it is possible to determine some estimator of IR of the whole system. Since, in practice, the signals are discrete, of finite-energy and of duration T, the problem is actually considered in a unitary space R^T being a subspace of l_2.

Let Θ be an infinite set of functions $\{v_l(t)\}$ with norm $//v_l//=1$ including all possible (according to the model) IRs of ESs. If the discrete signal $y(t)$ duration equals T, then the number of linearly-independent elements which can constitute a basis of the space R^T, whose members will be all signals of that duration, is also equal to T. Thus, the set Θ will also contain linearly-dependent elements and therefore it can not directly constitute a basis of the mentioned space. Hence, an algorithm which allows setting basis $\{w_l\}$ is required. There are many bases of R^T. Having in mind that the basis elements represent IRs of ESs, a condition is that they should assure the fastest convergence of the sequence (3). In this sense the basis should be "matched" to the given signal.

Having stated the problem in this way, there exists a problem of selection of elements from an initially assigned set of functions. The problem can be solved if a criterion of the selection is specified. In this work the mean-square error has been chosen. Consequently, only such elements of Θ will be selected (as the basis elements), which : a) are linearly-independent, b) assure the least mean-square error.

We notice that the number of elements of Θ used in the approximation is closely related to the assumption that the number N of parallely connected elementary systems is smaller then the signal length T. For $N > T$ the proposed method of estimation of system impulse response is useless. In practice, the task can be effectively solved if $N << T$.

3. Design of the optimum basis

The orthogonal projection is a fundamental tool for choosing of the basis elements from the set Θ. Let S_n denote the subspace spanned by $\{w_1,...,w_n\}$, i.e. $S_n = span\{w_1, . . . , w_n\}$. Let $P(S_n)y$ denote the orthogonal projection of a vector y on the space S_n and $P(S_n^\perp)y$ - the orthogonal projection of y on the orthogonal complement of S_n relative to R^T ($R^T = S_n \oplus S_n^\perp$). Let $P(v)y$ denote the orthogonal projection of a vector y on $span\{v\}$.

In this method, the choice of the subsequent basis elements is made in a recursive manner. Let us assume that $n-th$ step solution is given, i.e. $\{w_1,...,w_n\}$ and $P(S_n^\perp)y$ are known. The task is an optimum selection of an element from Θ, which will be the $(n+1)-th$ basis element. This element must be linearly-independent with respect to the elements $\{w_1,...,w_n\}$, what is satisfied if

$$//P(w_{n+1})P(S_n^\perp)y// > 0 \tag{4}$$

To show (4), observe that $P(S_n^\perp)y$ belongs to the orthogonal complement $S_n^\perp$. If the vector w_{n+1} were expressed as a linear combination of the vectors $\{w_1,...,w_n\}$, then it would belong to the space S_n. Then, the projection of the $P(S_n^\perp)y$ on the w_{n+1} would be a vector with norm equal to zero. Consequently, none of the elements of $S_n^\perp$ (and in particular - w_{n+1}) can be expressed as a linear combination of vectors $\{w_1,...,w_n\}$.

Besides, w_{n+1} must be such an element of Θ, which assures the least approximation error of the vector $P(S_n^\perp)y$, what is satisfied if

$$//P(w_{n+1})P(S_n^\perp)y// > //P(v)P(S_n^\perp)y// \tag{5}$$

where v is any element of the set Θ not equal to w_{n+1}. In an unitary space, the projection norm of the vector $P(S_n^\perp)y$ on $span\{v\}$, where v is a normalized vector, is given by the inner product.

The set $\{w_1,...,w_N\}$ can constitute a basis of the subspace of R^T and this is actually the required set mentioned in Sec. 2. Let us observe that due to established form of the elements of Θ, the basis is nonorthogonal.

4. Models and possible modifications of the estimation method

There are possible various models of IR of ES and therefore various elements sets Θ. The model depends on an object for which the proposed method will be used. In our investigations, the elementary vibrating type systems for which the envelope of IRs can be described by Pearson's curve [1] has been considered. Hence, the set of functions decribing the IRs of ESs constituting the sequence ${}_m h_n(t)$ can be expressed as

$${}_m h_{n(t)} = \beta_n \; 1(t - t_n) \; (t - t_n)^m \, e^{-\gamma_n (t - t_n)} \times \times \sin(2\pi f_n (t - t_n)) \qquad (6)$$

where β_n is a multiplier, $1(t)$ denotes Heaviside step, t_n is a time delay, γ_n decribes vibration dumping and f_n is a vibration frequency. For the given model with m fixed, the IR of ES is determined if the parameters f_n, t_n, γ_n and β_n are known. The presented in Sec. 3 method allows for estimation of the parameters $\{f_n, \gamma_n, t_n, \beta_n \colon n = 1, \ldots, N\}$ and in consequence - the estimator of IR of the whole system.

The system IR of type (6) can be described by means of several parameters. Though orthogonal projection is a fundamental tool of the estimation, because of computational efficiency, the use of some supporting procedures is useful. Effectiveness of these procedures depends on such signal parameters like e.g. signal length and the number of parallely connected ESs. The frequencies $\{f_n\}$ of vibrations can be estimated more effectively by means of one of many spectral analysis algorithms as well as the time delays $\{t_n\}$ - by means of one of time delay estimation algorithms [4].

5. Optimization and cost function definition

In the presented method of deconvolution merely the estimator $\hat{h}(t)$ is known. $\hat{h}(t)$ is the form of $h(t)$, besides that the exact value of parameters of the system IR are unknown but merely their estimators. Consequently, the inverse filtering results in the estimator of the input, i.e.

$$z(t) = \hat{x}(t) = y(t) * {}^{-1}\hat{h}(t) \qquad (7)$$

where $*$ denotes convolution and ${}^{-1}\hat{h}(t)$ is an inverse filter.

In [2] it has been shown that the errors of determination of $\{\beta_n\}$ and $\{t_n\}$ result in some additional terms in the output of the inverse filter. Those terms are actually the echoes of the input with various amplitudes and delays.

In order to minimize the estimation errors of the parameters, one can employ an optimization process [2,3]. It needs determination of a cost function. The cost function is defined on the base of the inverse filter output. It is comprehensible, that the optimization result is sensitive to the cost function definition. We have examined several ways of determinig the cost function [2..4], among which those based on the power cepstrum and the second order autocorrelation allows one to obtain quite satisfactory results. Unfortunately, in some situations the optimization with the cost function defined in this way has failed. One of the reason is that second order characteristics do not contain information about the signal phase and that the signals and the system IR are nonminimum phase. To remove this inconvenience the cost function definition based on the fourth order autocorrelation has been introduced. The cost function M_{32} has been defined as

$$M_{32} = \sum_{\tau_1 = p}^{P} \sum_{\tau_2 = p}^{P} \frac{R_z^2(0, \tau_1, \tau_2)}{R_z^2(0,0,0)} \qquad (8)$$

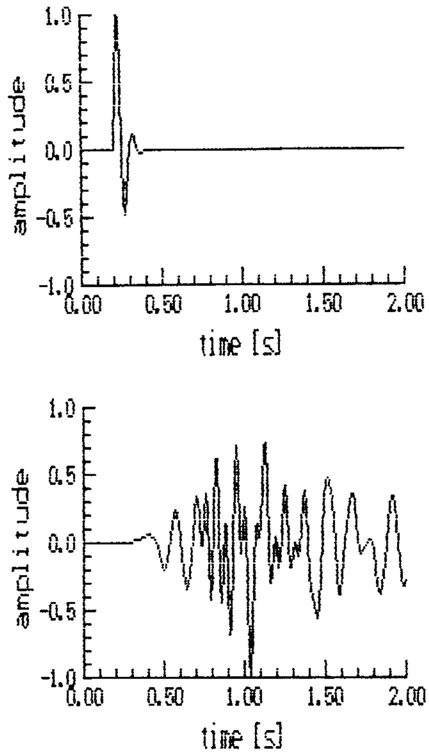

Fig.1. Exemplary input and output test-signals

where p and P describe this fragment of the autocorrelation function which serves for calculation of the cost function value.

6. Exemplary deconvolution results

In Fig.1, exemplary input and output test-signals have been shown. The system consists of four ESs. In Fig.2 are shown : the result of deconvolution of the signal shown in Fig.1 with the application of the cost function M_{32}, the corresponding second order autocorrelation function and the section of the corresponding fourth order autocorrelation.

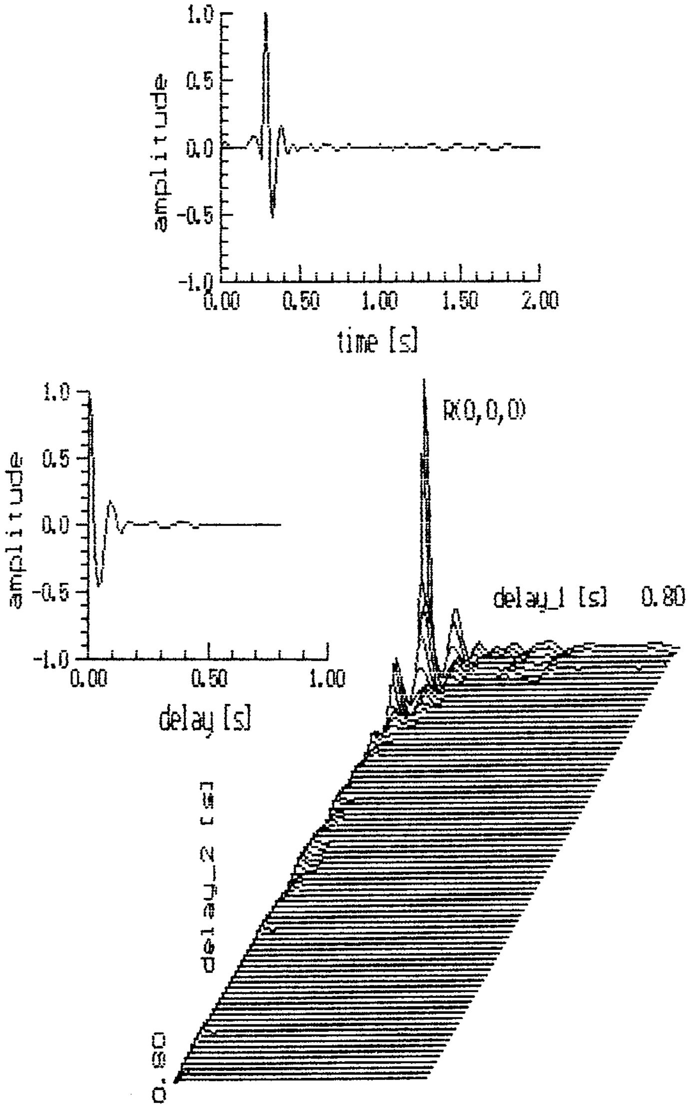

Fig.2. The result of deconvolution of the signal shown in Fig.1 with the application of the cost function M_{32} and the section of the corresponding fourth order autocorrelation function

For comparison, in Fig.3 are shown : the result of deconvolution of the same signal with the application of the cost function based on the second order autocorrelation and corresponding autocorrelation.

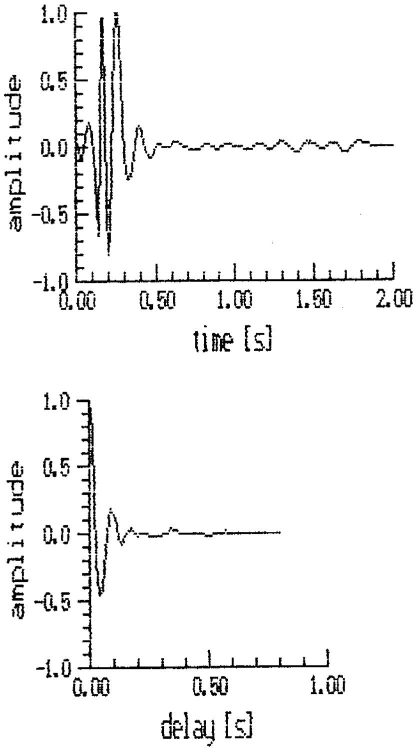

Fig.3 The result of deconvolution of the signal shown in Fig.1 with the application of the cost function based on the second order autocorrelation and the corresponding second order autocorrelation function

7. Conclusions

The method of the system impulse response estimation, for the class of signals specified in Introduction, in which a fundamental tool is orthogonal projection, has been presented. Also, a new cost function definition based on the fourth order autocorrelation function has been proposed. These novelty allows to improve results of the blind deconvolution.

References

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