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ROBUST IDENTIFICATION OF A FREQUENCY DRIFT OF THE HIGH QUALITY OSCILLATOR

Andrzej DOBROGOWSKI

Politechnika Poznańska, Instytut Elektroniki i Telekomunikacji
ul. Piotrowo 3A, 60-965 Poznań, Poland

RÉSUMÉ

Une expérimentation de l'identification du dérive de l'oscillateur est présentée. On donne des arguments motivant le choix d'un modéle de regression linéaire pour le dérive de fréquence d'un VCXO de haute qualité. On nomme aussi des raisons pour lesquelles une estimation "robuste" du dérive est nécessaire. Un algorithme résultant de la méthode des moindres carrés (adoptée) est introduit. L'auteur utilise les fonctions de Huber et de Tukey pour assurer la "robustness" de cette méthode. Enfin les résultats de l'estimation "robuste" et "non-robuste" sont présentés.

ABSTRACT

The oscillator's drift identification experiment is reported. Argumentation is given for the choice of linear regression model for frequency drift of high quality VCXO. The reasons for robust estimation of the drift are enumerated. Algorithm of the adopted least squares method is presented. Huber's function and Tukey's biweight were used to introduce the robustness into least squares method. The results of nonrobust and robust drift estimation are discussed.

Introduction

A high quality crystal oscillator provides its environment with time scale. When environments of several oscillators collaborate with each other involving time scales these time scales have to be coordinated. This is the case of the synchronized digital communications network. High stability oven-controlled voltage-controlled crystal oscillator (OCVCXO) is used as the node clock which is phase locked to the master clock (e.g. higher quality quartz crystal or caesium beam). Sometimes, however, because of the network failure the node clock does not recive a synchronization signal from the master. During the period of losing the synchronization signal the free-running frequency of the oscillator changes causing non-cumulative and cumulative time interval error (TIE). Non-cumulative TIE (caused by jitter-type or/and wander-type phenomena) can be reduced by proper filtering and by means of using the elastic buffer of a proper size.

The oscillator's free-running frequency drift results in cumulative TIE. Knowing the drift it is possible to compensate its undesirable effect on the oscillator's frequency by setting suitable signal at the control input of the VCXO. This is the reason of doing identification study presented in this paper. The robust version of experimental data processing algorithms was used. Robust statistics exploits very effectivelly the information which is contained in an ensemble of data. It represents information by means of parametric models but uses procedures for which the dependence on the model's premises is not fatal. To process the data obtained as the result of identification experiment linear regression model was applied and two robust versions of the least-squares method were considered.

Table 1

Day	Result	Day	Result	Day	Result	Day	Result
1	143	36	147	78	152	114	155
2	143	37	145	79	151	115	154
3	143	38	145	80	150	116	154
4	143	39	146	81	151	117	154
5	143	40	146	82	150	118	153
6	143	41	147	83	150	119	153
7	143	42	148	84	151	120	154
8	143	43	146	85	150	121	154
9	143	44	147	86	151	122	155
10	144	45	146	87	151	123	154
11	144	47	146	88	151	124	154
12	144	48	148	89	150	125	155
15	144	49	147	90	150	126	155
16	145	50	147	91	151	127	155
17	144	51	147	92	151	128	155
18	144	52	148	93	153	129	154
19	144	53	149	94	152	130	155
20	145	54	149	95	152	131	155
21	146	55	149	96	153	132	156
22	144	56	147	97	153	133	156
23	144	57	148	98	153	134	156
24	145	58	148	99	153	135	155
25	145	59	150	100	153		
26	145	60	149	101	153		
27	145	61	149	102	153		
28	146	62	149	103	153		
29	145	63	149	104	154		
30	146	64	149	105	154		
31	146	65	149	106	153		
32	145	66	148	107	152		
33	146	67	148	108	153		
		68	150				
		69	149				
		70	149				
		71	150				
		72	149				
		73	149				
		74	149				

Table 2

	1	2	3	4	5
a^o	9.711	10.000	11.290	9.091	9.761
Huber					
a^1	9.664	10.041	11.511	8.999	9.771
a^f	9.652	10.051	11.567	8.975	9.774
Tukey					
a^4	9.655	10.085	11.431	8.895	9.767
a^f	9.655	10.085	11.431	8.895	9.767

The identification experiment

In the experiment [1] the identified oscillator being a constituent of digital processing phase-locked loop (DP-PLL) was synchronized to the reference oscillator (Fig.1). The oven-controlled voltage-controlled crystal quartz oscillators OCXO-5 and OCXO-10 manufactured by OMIG-Warszawa were correspondingly the identification object and the reference. When the PLL is locked the variations of free-running frequency of the identified oscillator are mapped into proportinal changes (neglecting quantization effect) of the signal controlling VCXO assuming stable reference. Thus, the output of D/A converter (Fig.1) contains information relative to frequency drift of free-running frequency of the oscillator inside the phase-locked loop. In many applications we are not directly interested in frequency drift of the loop oscillator but rather we are interested in possible values of the oscillator's controlling signal which compensate this drift. The identification experiment enabled us to gain such knowledge. PLL's microcomputer processed the output of the phase comparator (PI control was adopted) supplied the D/A converter with controlling word and stored the controlling signal.

The experiment lasted 135 days. Every one, at midnight, the reading of the actual VCXO controlling signal was taken. These readings are given in Table 1. During the experiment four times we were forced to suspend the reading schedule for several days. The itemes in Table 1 constitute the set of data for subsequent robust processing.

Robust linear regression

The main application aspect of this study is keeping the high quality timing for digital communication network when the node does not recive timing information from the master clock. It is expected in digital communication network that the node fails to recive synchronization signal from the reference clock during the period not greater than several weeks. For this reason linear regression was considered as an adequate model for frequency drift of the high quality VCXO. The least squares method was used to estimate parameters of the model.

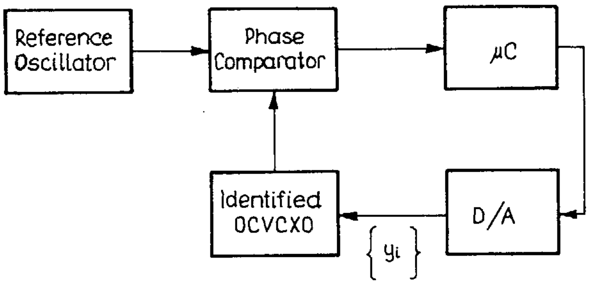

Fig.1 Block diagram of the identification system

Classical method of least squares are very sensitive to outlying observation by giving them excessive importance. Rey [2] explains that "robust regression is a field where the concern is more on getting a regression certainly right in "the bulk of the data" than taking the risk of being misled by some odd observation". The robust least squares method is required in the oscillator's drift estimation for the following reasons:

1. Ocassionally remarkable deviations from linear model are expected to appear because of quartz oscillator's physics.
2. The experiment supplies an investigator with quantized data.
3. Rather long-term experiment carried over rather common conditions suffers from the environmental changes.

Classical least square method can be "robustified" by minimizing the sum of the values of the function of residuals when this function grows not so fast as the square of residuals [3].

Adopted robust least squares method is based on called H-algorithm for Huber's estimator [4]. We want to find the line of regression $a \cdot x+b$. We can enter the procedure using the ordinary (nonrobust) least square method. Then, using the obtained values of parameters a^{o} and b^{o} we calculate fitted values $a^{o} \cdot x_i + b^{o}$ and residuals $r_i^{o}=y_i-(a^{o} \cdot x_i+b^{o})$ where y_i are observed values of a signal controlling VCXO and x_i are the values of time coordinate. After that applying the function $w(\cdot)$ we transform residuals into

$z_i^o=w(r_i^o)$ and calculate pseudoobservations $y_i^1=a^o\cdot x_i+b^o+z_i^o$. Using again the least square method we receive new estimates a^1 and b^1 of the parameters. We follow this procedure until convergence is achived.

The crucial role in supplying estimation with robustness plays w(·) function. The choise of w(·) function is rather the matter of "art" estimation. Probably the best known w(·) functions were proposed by Huber, Hampel, Andrews and Tukey [3,4]. Among them Huber's proposal seems to be best motivated. Investigating robust estimation of the location parameter for contaminated normal distribution Huber's function leads to the minimax solution of this problem [2,3,4].

For our study we used a variant of Huber's function having the form

$$w(r)=\min(2\cdot k\cdot s,\max(2\cdot r,-2\cdot k\cdot s))$$

with $k=0.1$.

The function that avoids any incidence of the outliers is Tukey's biweight. We used a variant of the form

$$w(r)=r\cdot\left[a^2-\left(\frac{r}{s}\right)^2\right]^2\cdot\mathrm{rect}\left(\frac{r}{2\cdot a\cdot s}\right)$$

with $a=1$.

For both w(r) functions the scale parameter s was estimated as the median of the residuals in absolute values

$$s=\mathrm{median}(|r_1|,\dots,|r_n|)$$

Let us remind that median is the robust estimator. The scale parameter s was computed in each step of iterative procedure.

Estimation results

Estimation results are given in Table 2. Events that forced to suspend the readings naturally divided the whole set of observed data into four subsets. For succeeding subsets the estimation results are given i successive colums in Table 2. The last column contains the results for whole set of data. All numbers in Table 2 must be multiplied by 10^{-2}. The items of Table 2 have the dimension 1/day. In Table 2 a^o means the estimates resulted from ordinary least squares. For Huber's function the results of first iteration (a^1) are given, but for Tukey's biweight-after fourth one (a^4). For both, the final results are shown.

We observed that both robust procedures move the estimate in the same direction: for two subsets-up, and for two others-down. The largest differences between nonrobust and robust estimation appear in the third subset of data. The relative frequency drift (item of Table 2 x gain of digitally controlled oscillator) for this subset in the case of ordinary least squares estimation equals $3.161\cdot10^{-10}$, for Huber's-$3.29\cdot10^{-10}$ and for Tukey's-$3.201\cdot10^{-10}$. The time difference after 30 days resulting from the difference between least squares and Huber's assessment of the drift equals 302 μs, that means two slips of PCM 30/32 frame (and the third slips after several days more). In the same period the smallest time difference (for the whole set of data) resulting from the robust and nonrobust estimation of the drift equals 6.6 μs.

Robust estimation may be advantageous for preparation the signal for control of digital network node clock. The synchronization signal may be severely contaminated along transmission path and the robust processing seems to be the reasonable solution.

References

1. Borowiak K.: Predykcja Częstotliwości Drgań Własnych Generatora, D.Sc.Thesis, Technical University of Poznań, 1988.
2. Rey W.J.J.: Robust Statistical Methods, Springer-Verlag 1978.
3. Huber P.J.: Robust Statistics, Wiley, 1981.
4. Hampel F.R. at al.: Robust Statistics, The Approach Based on Influence Functions, Wiley, 1986