



IDENTIFICATION OF SIGNALS CONTAMINATED
WITH IMPULSIVE NOISE

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RÉSUMÉ

Une méthode spécifique d'identification des paramètres des signaux et les problèmes de précision correspondants sont discutés dans le rapport. On suppose que le signal périodique analysé est contaminé par un bruit ayant la forme des impulsions aléatoires. Les amplitudes des composantes sont estimées comme médianes des quotients des valeurs du signal et des fonctions orthogonales (un algorithme "robust"). On analyse les erreurs aléatoires de cette méthode en cas de calcul approximatif. Les valeurs maximales des erreurs sont dérivées des histogrammes des intervalles, qui résultent de la procédure d'estimation.

ABSTRACT

A specific method of signal parameter identification and the corresponding accuracy problems are discussed in the paper. The periodic signal is assumed to be contaminated with random impulsive noise and the amplitudes of components are estimated as medians of quotients of signal values and some reference orthogonal functions (the algorithm is known as "robust"). The problem of accuracy appears, especially if the median is determined by the use of an approximate method. It has been shown in the paper that the maximum errors can be estimated and certain confidence levels can be calculated (for different disturbance distributions).

INTRODUCTION

A specific method of signal analysis in the presence of impulsive noise was described in [1]. According to the method the amplitudes of particular harmonic components of the signal are calculated as medians of quotients of its instantaneous values and the sine wave samples. Some properties of the estimates - especially their standard deviations - were analyzed in [2]. Also a simplified algorithm of estimation, having a limited computational complexity, was there proposed.

Some other features of the algorithm are described in the following contribution. In particular a statistical analysis of maximum errors of the estimate is given. Also certain conclusions concerning the relation between the disturbance distribution and the error value are formulated.

THE ALGORITHM OF ANALYSIS

For a sequence of N samples of a discrete signal $y(n)$ being the sum

$$y(n) = x(n) + z(n) \quad (1)$$

of a periodic component $x(n)$ and a random impulse disturbance $z(n)$, the coefficients a_i of the series

$$\sum_{i=0}^{N-1} a_i \phi_i(n) \quad (2)$$

approximating $x(n)$, are to be estimated. According to the method mentioned in [1] they can be evaluated from the formula

$$a_i = \text{med} \left[\frac{y(n)}{\phi_i(n)} \right] \quad (3)$$

as medians of the given quotients. Usually $n \in [N - 1]$.

As it was mentioned in [2] the precise calculation of medians requires a rather big number of arithmetic operations (N divisions and N^2 comparisons of two numbers for one coefficient of the series). On the other hand approximate values of medians can be determined as the abscissae for which the cumulative histograms equal $1/2$. Obviously, a substantial reduction of the number of operations can be here achieved; it results from the fact that a histogram can be immediately "generated" by sampled values of the quotients - the address is then derived as $\text{int}[\]$ of the quotient values. It results from the corresponding formulae (see [2]) that a median can be thus determined using N multiplications, $(N + M/2)$ additions and $M/2$ comparisons, where $M -$

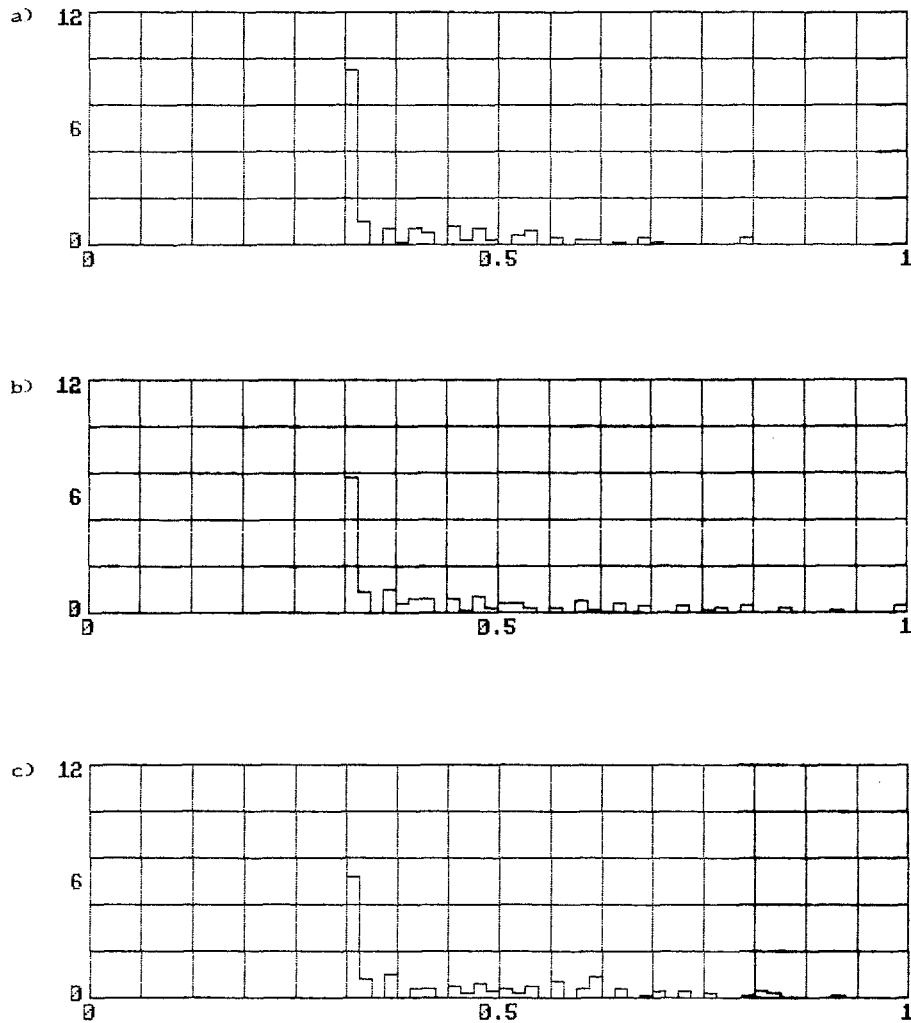


Fig.1 - Histograms of the subinterval widths
(gaussian impulsive noise)

a) $\sigma_z = 8$ b) $\sigma_z = 11$ c) $\sigma_z = 14$

denotes the number of subintervals into which the total range of quotients was divided.

Obviously, the random error in the approximate method results from the fact that the abscissa, corresponding to cumulative probability equal $1/2$, can be determined with a limited accuracy - the possible error may equal half the width of the subinterval. This effect can be seen in examples described in [3].

PROPERTIES OF THE ESTIMATES

Pdf's of the quotients. The quotients $y(n)/\phi(n)$ in Equ.3 are generally random variables because of the disturbing component $z(n)$ of the analyzed signal. For typical signals, including several harmonics, the appearance of an impulse disturbance results in particular values of the quotient

exceeding substantially all other values. The shape of the histogram will be thus strongly influenced, assuming a predetermined number M of the subintervals and an automatic adjusting of the analyzed range to the maximum and minimum values of the quotients.

Pdf's of the subinterval widths. As it was already mentioned above, the estimates of medians, determined by the use of the approximate method, are not error-free. The maximum error results from the width of subintervals on the horizontal axis of the histogram. It seems therefore interesting to analyze the influence of the parameters of the impulsive disturbance on subinterval widths, which can be treated as random variables, nonlinearly dependent on this disturbance. The maximum error of the estimate can be easily derived from the

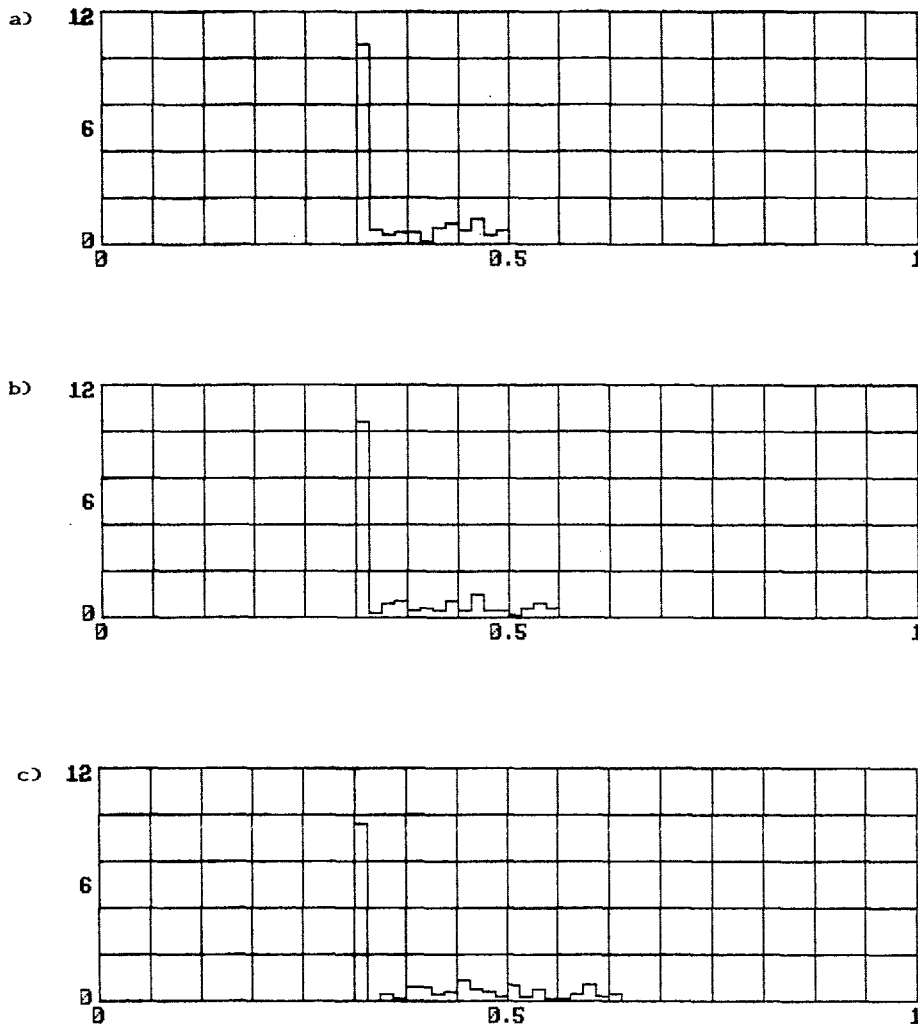


Fig.2 - Histograms of the subinterval widths
(uniformly distributed noise)

a) $\sigma_z = 8$ b) $\sigma_z = 11$ c) $\sigma_z = 14$

probability density curve of the subinterval width, for a given probability of its occurrence (or vice versa). Certain "confidence levels" can be thus defined.

In order to analyze the above presented connections a simulation program was prepared, enabling us to determine the distributions and probabilities for different signal and disturbance parameters as well as for different parameters of the algorithm itself. The program includes:

- a signal generator with adjustable parameters,
- a generator of impulsive disturbance having a predetermined distribution,
- a maximum and minimum detector, calculating the quotients, their extreme values and the subinterval width,
- a pdf analyzer.

The following example was considered in order to illustrate the above presented problem. For a given periodic signal

$$x(n) = 50 \sin(2\pi/N)n + 15 \sin(2\pi/N)2n + 10 \sin(2\pi/N)3n$$

where N denotes the number of samples corresponding to the period of the signal, the analysis was done for an impulsive disturbance, having gaussian or uniform distributions (zero-mean, $\sigma = 8, 11$ and 14). The resulting histograms are presented in Figs. 1 and 2.

The following conclusions can be formulated:

- a) all histograms show the same minimum subinterval, depending on the relation between the amplitude of the basic component and the sum of all other harmonics;
- b) the probability of taking this



minimum value by the subinterval - also the probability of not-overshooting $1/2$ of this value by the estimate of the median - equals about 50% for the uniform distribution and about 30% for the gaussian distribution and does not depend on the parameters of the disturbance;

c) in case of a uniform distribution of the disturbance the maximum width of the subinterval depends explicitly on the standard deviation σ_z ; for the gaussian distribution there is no evident relation between these values (the maximum width can result from the maximum disturbances limited by technical circumstances);

d) the maximum subinterval width is larger for gaussian disturbances than for the uniform ones, for the same values of σ .

It must be noted that if a system of rectangular orthogonal functions is used

instead of harmonic functions [3], for the same signal being analyzed, the effect of increasing dispersion of the quotients can be observed. It results from the fact that functions which are used as reference, "are not adapted" to the signal.

FINAL REMARKS

The above described properties of the median treated as a parameter estimate enabled us to determine some relations between the disturbance parameters and the accuracy of the estimation procedure. It seems that the presented method could be applied also in other procedures, where the confidence levels are to be estimated. Some theoretical aspects of the method should be investigated in the future, also some further simulation are necessary.

References

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