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CONSTRUCTION OF CONSTELLATIONS DESIGNED FOR THE RAYLEIGH CHANNEL

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Résumé

Jusqu'à présent, les travaux effectués dans le domaine des réseaux de points ont toujours concerné le canal gaussien. La constellation optimale est dans ce cas celle qui maximise la densité des points du réseau, pour une distance euclidienne donnée minimale entre les points. En dimension 2, la constellation la plus dense pour le canal gaussien est la constellation hexagonale A_2. Dans cet article, nous nous proposons d'élaborer des constellations de points adaptées aux canaux à évanouissements. Le point de départ de notre étude est la définition d'une nouvelle distance entre mots de code. Ceci nous permet d'énoncer le problème de la constellation optimale dans le canal à évanouissements de manière équivalente au canal gaussien. La constellation trouvée en dimension 2 a été simulée dans un canal de Rayleigh, et donne de meilleures performances que la constellation hexagonale

Abstract

Until now, the study of dense lattice points for communication systems was only applied to the Gaussian channel. For this channel, the optimum lattice is the one which maximizes the density of the lattice points for a given minimum Euclidean distance between the points. In two dimensions, the densest lattice for the Gaussian channel is the well known hexagonal lattice A_2. In this paper, we set out to construct a constellation adapted to fading channels. The starting point of our study is the definition of a new distance between the signal points, which we derive from the expression of the error probability. This is to define the optimum constellation for the fading channel in a similar way as for the Gaussian channel. We find a new constellation in two dimensions which give better performance in the Rayleigh channel than the hexagonal constellation.

1. INTRODUCTION

The development of digital communication gives rise to an extending demand for high spectral efficient systems. The TCM codes proposed by Ungerboeck [1] in the early eighties are an efficient way of achieving good performances without spectral efficiency loss. They have been extendly studied for the Gaussian channel in the last decade. Now, a lot of systems are transmitted in a fading channel. For the Rayleigh and the Rice channel, Divsalar and Simon [2] give the design criteria of TCM codes, for MPSK modulation. It consists on maximizing the Hamming distance in symbols between the coded transmitted sequences, and the product Euclidean distance. This is currently applied to QPSK and 8-PSK modulation formats. But is not easily applicable to higher efficient modulation schemes, as the 64QAM.

Our idea is to work on the constellation of points, and search lattice points adapted to the fading channel. Until now, this field has always concerned the Gaussian channel. In this channel, the error probability of the usual bi-dimensional modulations M-PSK and M-QAM decreases exponentionally with the Euclidean distance between the points. From the energy point of vue, an optimum distribution of the points is one which maximizes their density for a given minimum Euclidean distance between the points. In two dimensions, the densest lattice for the Gaussian channel is the well known hexagonal lattice A_2.

The starting point of our study is the definition of a new distance between the signal points, which we derive from the expression of the error probability in a Rayleigh fading channel. This is to define the optimum constellation for the fading channel in a similar way as for the Gaussian case. This part is developped in section 2. In section 3, we use this new distance to construct a lattice in two dimensions adapted to the Rayleigh channel. The resulting constellation is simulated in section 4, and compared to an hexagonal and a square constellation.

2. A NEW DISTANCE

Let $\vec{x}$ denotes a sequence of N transmitted channel symbols $(x_1,\ldots,x_N)$. Assuming perfect interleaving and a Rayleigh channel, a Chernoff bound of the pairwise error probability i.e., the probability of detecting the sequence $\vec{t}$ in place of the transmitted sequence $\vec{x}$, is given by [2]

$$P(\vec{x}\rightarrow\vec{t}) \leq \prod_{i=1}^{N} \frac{1}{1+\frac{\gamma^2 \|x_i-t_i\|^2}{4N_0}} \quad (1)$$

where γ^2 is the fading variance, and $N_0/2$ is the noise dsp. Noting $K=\gamma^2/4N_0$, we define a new distance between the transmitted channel symbols by

$$dist_P(\vec{x},\vec{t}) = \mathrm{Ln}\left(\prod_{i=1}^{N} 1+K\|x_i-t_i\|^2\right) \quad (2)$$

So that the expression of the pairwise error probability (1) can be written as

$$P(\vec{x}\rightarrow\vec{t}) \leq e^{-dist_P(\vec{x},\vec{t})} \quad (3)$$

Remark: $dist_P(.,.)$ is a distance. It is not a norm since it does not preserve the multiplication by a scalar ($dist_P(\lambda.\vec{x}, \lambda.\vec{t}) \neq |\lambda|.dist_P(\vec{x},\vec{t})$). It does not either preserve the rotation ($dist_P(\mathrm{rot}(\vec{x}), \mathrm{rot}(\vec{t})) \neq dist_P(\vec{x},\vec{t})$).

3. CONSTRUCTION OF A CONSTELLATION OF POINTS ADAPTED TO THE RAYLEIGH CHANNEL

Given the above definition, we can now define the optimum constellation for the fading channel in a similar way as for the Gaussian case. An optimum distribution of the points is one which minimizes the average energy, for a given minimum distance $dist_P$ between the points called R i.e, a given maximum error probability. A constellation in N dimensions can be obtained through the following iterative algorithm. Let $\vec{O}_1=(0,0,\ldots 0)$ be the starting point, and $\vec{O}_2, \vec{O}_3, \ldots, \vec{O}_m$ the points obtained at step m. At the next step, find the point $\vec{O}_{m+1}$ such that

$dist_P(\vec{O}_1,\vec{O}_{m+1}) = R$

$dist_P(\vec{O}_2,\vec{O}_{m+1}) \geq R$

...

$dist_P(\vec{O}_m,\vec{O}_{m+1}) \geq R$

and $\vec{O}_{m+1}$ minimizes the average energy of the set of points $\{\vec{O}_1, \vec{O}_2, \vec{O}_3, \ldots, \vec{O}_{m+1}\}$

In two dimensions, the resolution of the corresponding equations leads to a regular constellation i.e, the set of points forms an additive group. A basis of this lattice, denoted R_2, is

$$\vec{v}_1 = \begin{pmatrix} \alpha/\sqrt{2} \\ \alpha/\sqrt{2} \end{pmatrix} \qquad \vec{v}_2 = \begin{pmatrix} \frac{\alpha-\sqrt{3}\beta}{2\sqrt{2}} \\ \frac{\alpha+\sqrt{3}\beta}{2\sqrt{2}} \end{pmatrix} \quad (4)$$

where α and β are two scalars which depend on the ratio K defined in section 2, and on the minimum given distance R between the points

$$\alpha = \sqrt{\frac{2\,(e^{R/2}-1)}{K}} \quad (5.a)$$

$$\beta = \sqrt{\frac{-10+2\,e^{R/2}+8\sqrt{e^R-4\,e^{R/2}+4}}{3K}} \quad (5.b)$$

The generator matrix of this lattice is

$$M = \begin{pmatrix} \alpha/\sqrt{2} & \frac{\alpha-\sqrt{3}\beta}{2\sqrt{2}} \\ \alpha/\sqrt{2} & \frac{\alpha+\sqrt{3}\beta}{2\sqrt{2}} \end{pmatrix} \quad (6)$$

and the points of the lattice consist on all points $M\vec{\varepsilon}$, with $\vec{\varepsilon}=(\varepsilon_1,\varepsilon_2)$ a vector of integer components. The fundamental region is formed of six points round $\vec{O}_1=(0,0)$, and is represented in Figure 1, next to the densest hexagonal constellation for the Gaussian channel. It is to notice that the constellation found for the Rayleigh channel can be obtained from the hexagonal constellation, expanded according to each axis respectively of α and β, and rotated by 45°. Stated in another way

$$R_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} \alpha & 0 \\ 0 & \beta \end{pmatrix} A_2 \quad (7)$$

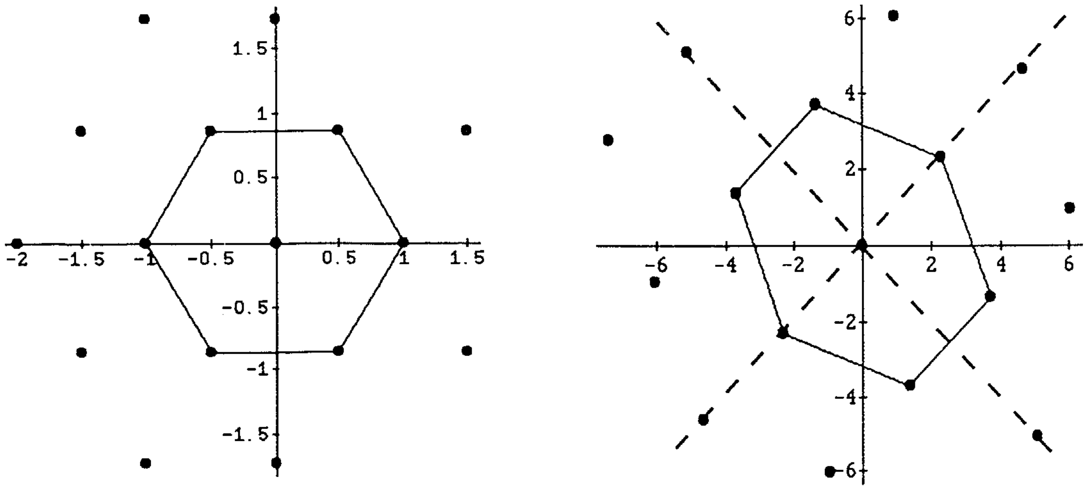
4. SIMULATION

The general transmission model is represented in Figure 2. The input bits are first mapped into a channel symbol (x,y) of R_2. Assuming perfect interleaving of the quadrature components x and y, and coherent detection at the reception, the received signals z_1 and z_2 are given by

$$z_1 = a_1.x + w_1$$

$$z_2 = a_2.y + w_2$$

where w_i (i=1,2) is an additive white Gaussian noise with zero mean and dsp $N_0/2$, and a_1 and a_2 are two Rayleigh uncorrelated fading variates with zero mean and variance γ^2.



Gaussian channel

Rayleigh channel

Figure 1. Densest constellations in two-dimensions.

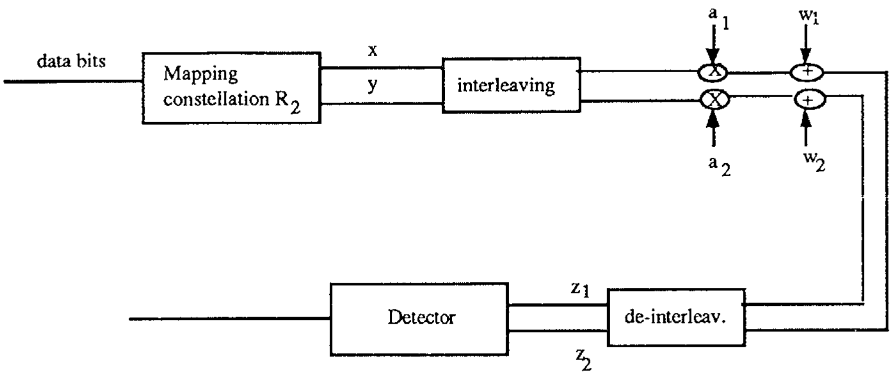


Figure 2. General transmission model

The signal to noise ratio is given by

$$\Gamma = \frac{\gamma^2 . E\left(x^2+y^2\right)}{2N_0} \qquad (8)$$

The detection is made following a Gaussian decoding metric, provided with ideal channel state information (CSI). In other words, the detector selects the point $(\hat{x}, \hat{y})$ of R_2 which minimizes the metric $\| z_1 - a_1 \hat{x} \|^2 + \| z_2 - a_2 . \hat{y} \|^2$.

We simulate the two constellations of M=16 points, shown in Figure 1, as well as the 16-QAM constellation. The results are shown in Figure 3. For the conventional squared 16-QAM and hexagonal 16-A_2 constellations, the rate of descent of the symbol error probability P_S is inversely proportional with the bit energy signal to noise ratio E_b/N_0. For the new R_2 constellation, the slope is of order two i.e, the constellation achieves a second order diversity. The gain is 4 dB at $P_S = 10^{-3}$, compared to the hexagonal lattice.

5. FUTURE PROSPECTS

In this paper, we present a two-dimensions regular lattice which give better performances in the Rayleigh channel than the conventional squared or hexagonal lattices. This lattice provides a 2nd-order diversity without encoding. The future prospects will concern the search of high-dimensional lattice, in order to obtain high diversity gain.

REFERENCES

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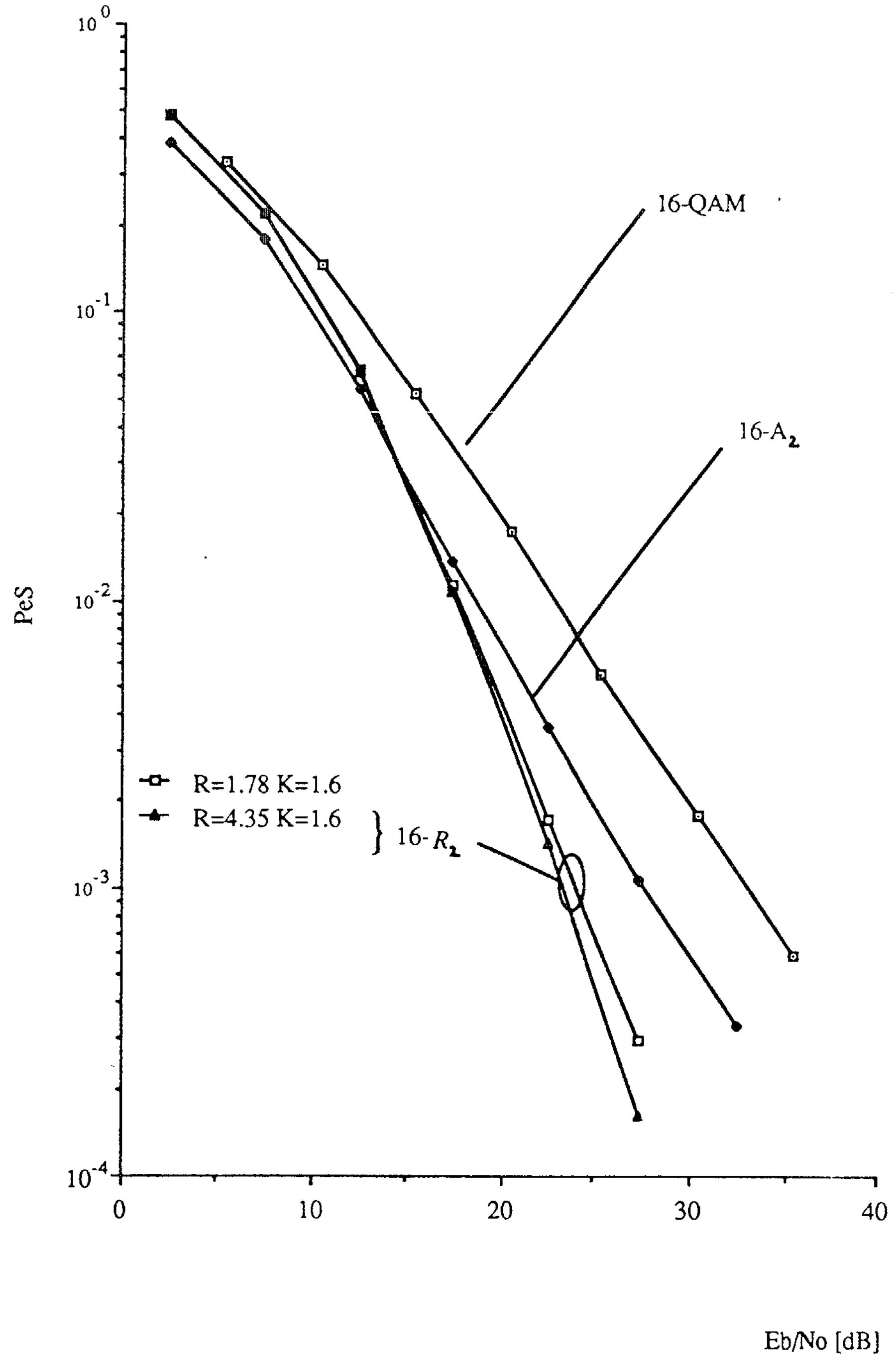

Figure 3. Simulation results of the Rayleigh channel adapted constellation