

## RADAR CROSS SECTION OF FINITE WIRE ARRAYS

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RÉSUMÉ

ABSTRACT

The scattering behavior of a finite two-dimensional array of thin wire structures excited by an incident plane wave is analyzed. Based on the moment method the analysis takes into account the mutual coupling effects of the scatterers as well as different incident angles or polarization angles of the exciting plane wave. The frequency selective and polarization selective behavior of single as well as of coupled straight, circular and quadratic wire scatterers is analyzed by calculating the radar cross-section (RCS) or the scattered electrical field strength, respectively. Utilizing the additionally calculated radiation patterns of such arrays for different frequencies and angles of incidence, it is possible to obtain reliable information on the complicated spatial scattering behavior of coupled wire scatterers. Moreover, optimum incident and receiving angles may be assessed for a possible application as improved frequency selective surfaces (FSS). The theory is verified by comparison with available measured results at an array of 4 x 4 scatterers.

### 1. INTRODUCTION

The electromagnetic scattering behavior of wire scatterers is of considerable interest for many applications [1] - [5], such as for identifying the radar cross-section (RCS) of antenna structures or complicated other conducting bodies, which can be modeled by wire grid elements, and for estimating the behavior of antennas in the presence of scatterers. Moreover, frequency or polarization selective scattering structures (FSS or PSS) composed of wire scatterers may be used as subreflector elements in related antenna systems which increases the capacity of communication links by utilizing simultaneously several channels of different frequency or polarization angles within the same link.

Previous analyses on wire scatterers are hitherto confined to single wires [1], [5], to straight coupled wires [3], or infinite wire arrays [2]. However, for a reliable investigation of realistic scattering structures, finite arrays of coupled wire scatterers of arbitrary shape are of considerable importance. The purpose of this paper is, therefore, to describe a rigorous method for such scattering structures of finite extend and to discuss their spatial scattering behavior at typical examples, such as arrays of coupled quadratic and of circular wire scatterers.

### 2. THEORY

The theory is based on the electric field integral equation (EFIE) derived from Maxwell's equations [6]. Enforcing the boundary condition  $\vec{n} \times \vec{E} = 0$  over the wire surfaces yields Pocklington's equation [7] for curved wires in the following form:

$$\vec{e}_s \cdot \vec{E}_i(s) = \frac{j}{4\pi\omega\epsilon} \cdot \int_{s'} \left[ \frac{d^2 \Psi(s, s')}{ds'^2} + k^2 \Psi(s, s') \right] I(s') \vec{e}_s \cdot \vec{e}_{s'} ds', \quad (1)$$

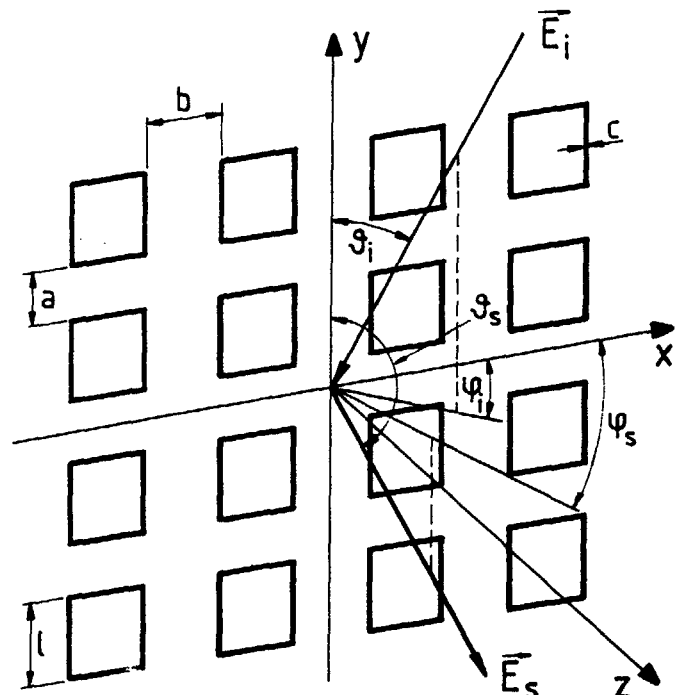


Fig. 1: Finite 4 x 4 wire element array with elements of arbitrary shape  
Example: quadratic scattering elements



where  $I$  is the unknown induced current on the wire axis of the scatterers due to the incident field  $\vec{E}_i$ ,  $k$  is the free space wave number,  $\Psi$  is the free space Green's function, and  $\vec{e}_s$  is the unit vector of the wire axis.

The integral equation (1) is solved by the moment method [7] by utilizing cubic spline functions [8] per segment  $n$  along the curved wire axes with  $N$  segments as basis functions for the unknown induced currents

$$I(s') = \sum_{n=1}^N I_n(s'), \quad (2)$$

which yields the advantage of requiring only a relatively low number of segments along the wires. The coefficients of the cubic spline function

$$I_n(s') = a_n + b_n(s'-s'_n) + c_n(s'-s'_n)^2 + d_n(s'-s'_n)^3 \quad (3)$$

are related to each other by the continuity condition along the wire segments which yields the matrix relations [8]

$$\vec{b} = \vec{B}_n^T \vec{a}, \quad \vec{c} = \vec{C}_n^T \vec{a}, \quad \vec{d} = \vec{D}_n^T \vec{a}. \quad (4)$$

The well-known moment method [7] is applied for solving the integral equation (1) and dirac delta functions are used for the weighting functions (point matching). This yields the following equation for the unknown coefficients  $a_n$  in (2)

$$[\vec{a}_n] = (-j4\pi\omega\epsilon) [Z_{mn}]^{-1} [\vec{e}_{sm} \vec{E}_i(s_m)], \quad (5)$$

where  $Z_{mn}$  is the impedance matrix given by the relations (1) to (4).

The farfield of the scatterers is then calculated by

$$\vec{E}_f = -\frac{j\omega\mu}{4\pi} \sum_{n=1}^N \vec{e}_{s'_n} \int_{s'_n}^{s'_{n+1}} I_n(s') \Psi(s, s') ds'. \quad (6)$$

### 3. RESULTS

Convergence investigations at quadratic and circular  $\lambda/2$  and  $\lambda$  structures demonstrate that already about 40 segments show sufficient asymptotic behavior for calculating the radiation pattern. This is illustrated at the example of a single quadratic wire scatterer with oblique wave incidence, Fig. 2.

The frequency selective behavior of wire scatterers is investigated by calculating the radar cross section (RCS)  $\sigma$  [9] as a function of frequency for some typical structures of interest. It is demonstrated that the RCS shows a significant resonance characteristic dependent from the structure under investigation.

Fig. 3 shows the result for a straight wire of variable length. Excellent agreement with the calculated and measured results of [10] may be stated. In Fig. 4, the RCS of a  $4 \times 4$  finite wire array is compared with the results of an infinite array according to Chen [2] with the same wire lengths and distances. The slight frequency shift and the deviations for higher frequencies of the finite structure is due to the finite number of resonators as compared with the infinite array.

An array of  $2 \times 4$  quadratic scatterers is shown in Fig. 5. The maximum RCS of this example is  $446 \lambda^2$ . The resonance frequency of the RCS as a function of frequency may be observed if the side length of the quadratic scatterer is about  $\lambda/2$ .

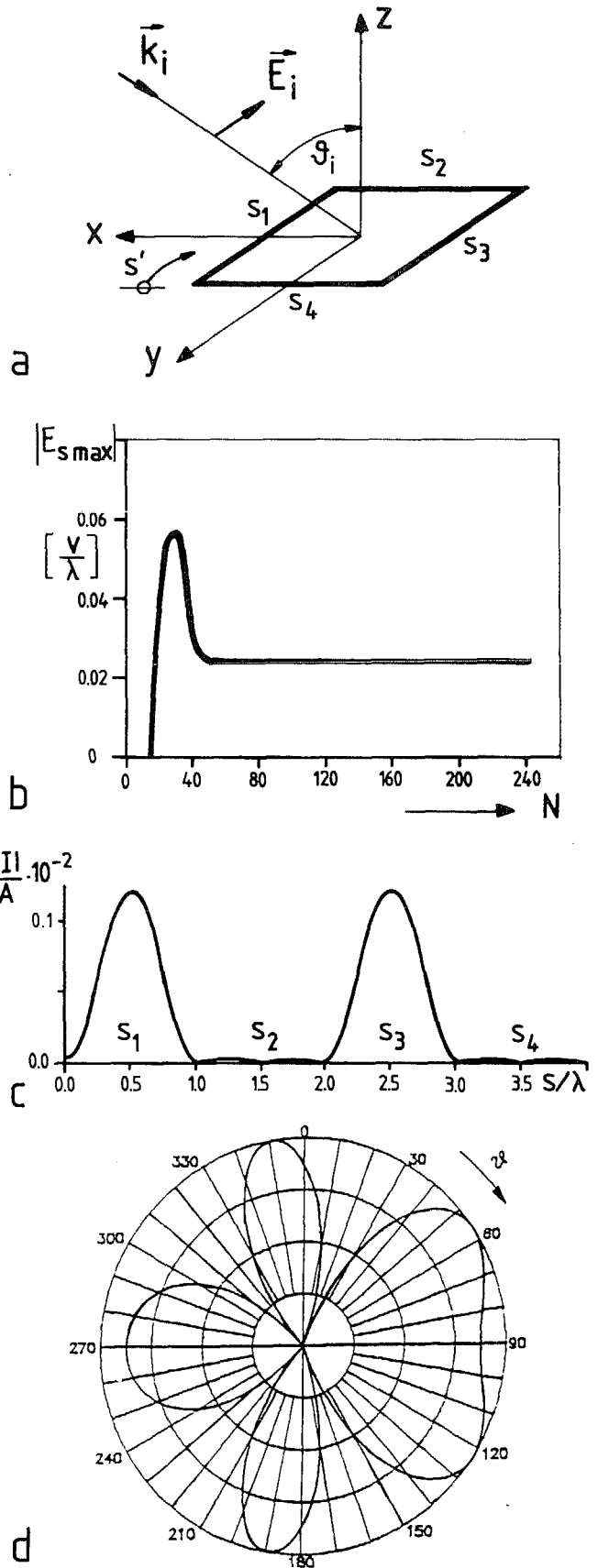
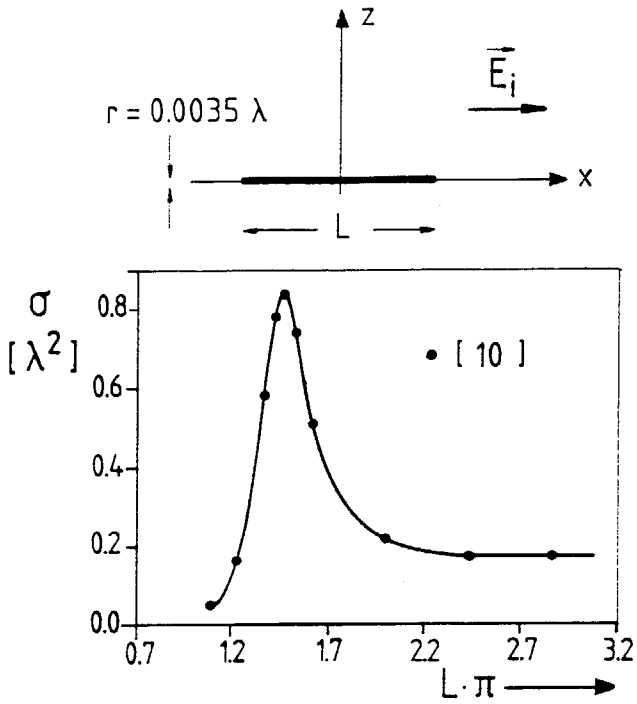
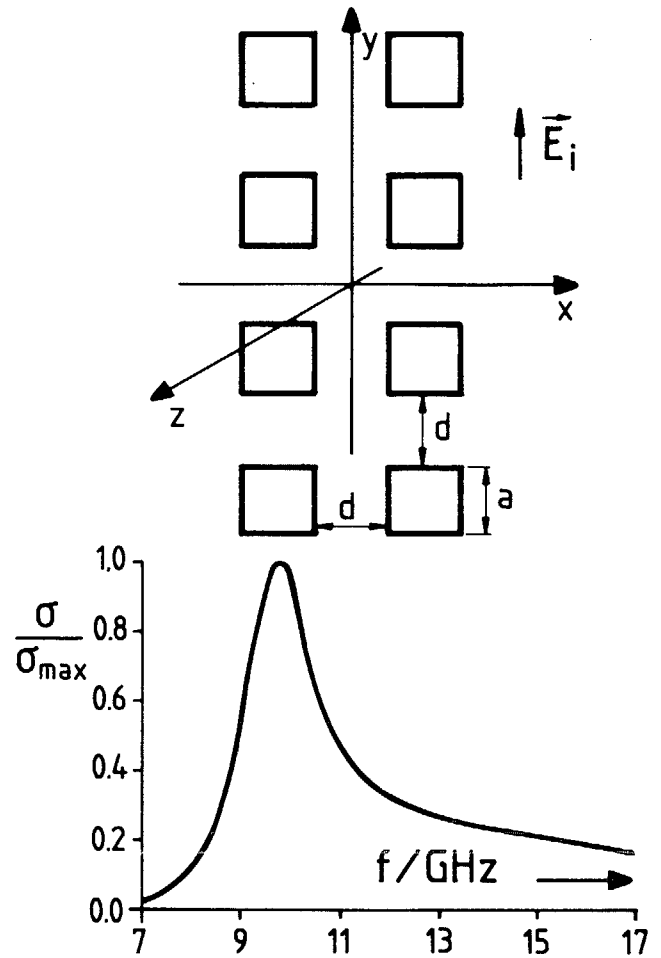


Fig. 2: Single quadratic wire scatterer  $s_1 = \lambda$  with oblique wave incidence

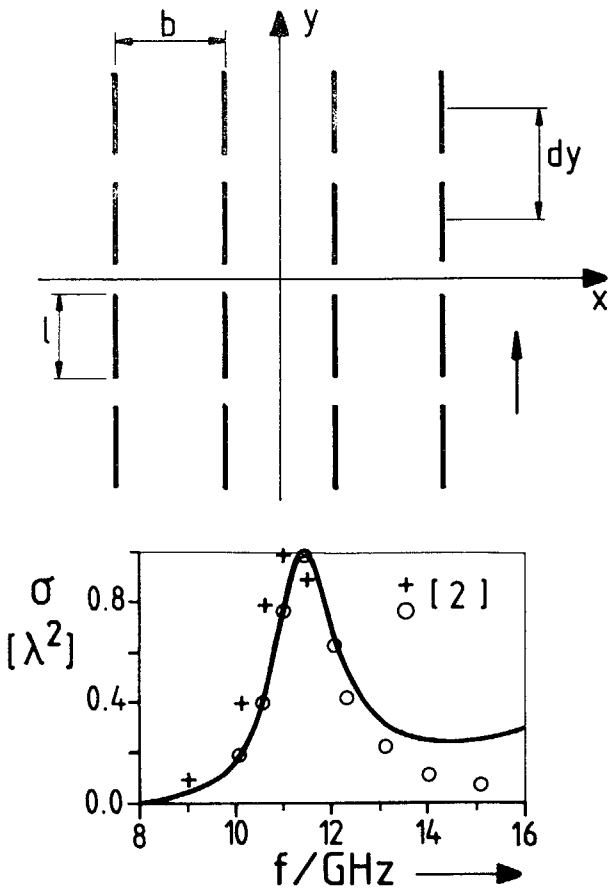
- a) Geometry
- b) Convergence behavior as a function of the number  $N$  of total segments
- c) Current distribution along the wire axes
- d) Radiation pattern



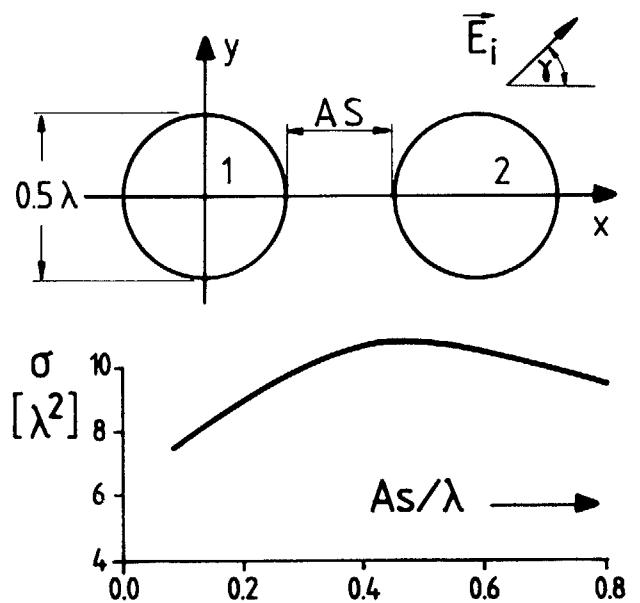
**Fig. 3:** Radar cross section of a straight wire of radius  $r = 0.0035 \lambda$  as a function of wire length  $L$  (Comparison with the results of [10] · · ·)



**Fig. 5:** Array of  $2 \times 4$  quadratic scatterers  $d = a = 15\text{mm}$ , wire radius  $r = 0.2\text{mm}$   
RCS  $\sigma$  as a function of frequency  
Maximum RCS  $\sigma_{\max} = 446 \lambda^2$



**Fig. 4:**  $4 \times 4$  wire array.  
Length  $l = 1.27 \text{ cm}$ , distances  $b = dy = 1.78 \text{ cm}$   
RCS as a function of frequency  
Comparison with the infinite array of Chen [2]  
(+ + + measured, o o o calculated [2])



**Fig. 6:** Two circular wire scatterers with a diameter of  $\lambda/2$ .  
RCS  $\sigma$  as a function of the mutual distance  $AS$

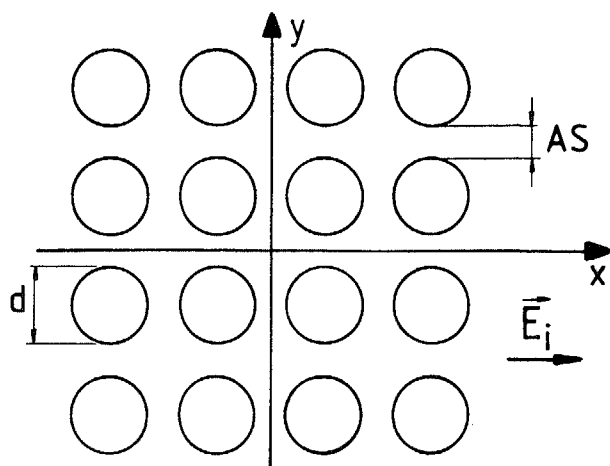
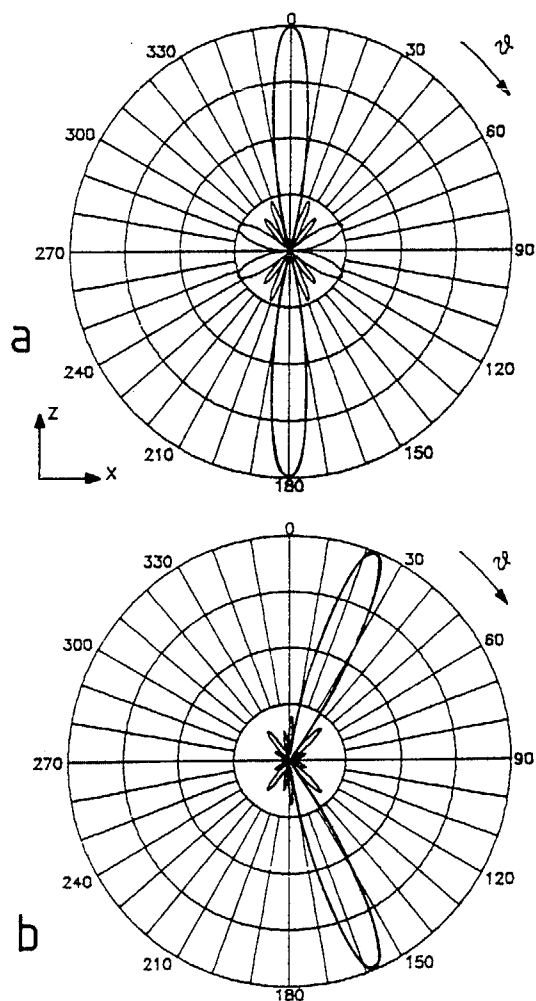


Fig. 6 shows the calculated RCS of two circular wire scatterers (Fig. 6a) with a diameter of  $\lambda/2$  as a function of the mutual distance  $AS$ . The maximum RCS is obtained for about  $AS = \lambda/2$ . This is valid for horizontal polarization  $\gamma = 0^\circ$  of the incident field (Fig. 6). Investigations for vertical polarization  $\gamma = 90^\circ$  (i.e. parallel to the  $y$ -axis) give as a result that the maximum RCS is then obtained for about  $0.6 \cdot \lambda$ .

In Fig. 7, the radiation patterns of a  $4 \times 4$  array of circular wire scatterers are presented for different diameters, distances and incident angles. The maximum RCS for varying incident angle  $\delta_i$  is obtained for  $\delta_i = 22.5^\circ$  (horizontal polarization).



**Fig. 7:**  $4 \times 4$  array of circular wire scatterers  
Radiation patterns in the  $x$ - $z$ -plane at  
different incident angles  $\delta_i$

a)  $d = \lambda/2$ ,  $AS = \lambda/2$ ,  $\delta_i = 0^\circ$

b)  $d = \lambda$ ,  $AS = 0.6 \cdot \lambda$ ,  $\delta_i = 22.5^\circ$

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