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A NEW METHOD FOR THE ESTIMATION OF THE DIRECTIONS OF ARRIVAL OF MULTIPLE WIDEBAND SOURCES

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RÉSUMÉ

This paper presents a new method for the high resolution estimation of the Directions Of Arrival (DOA) of multiple wide band plane waves impinging on a linear array of equispaced sensors. The proposed method does not require preliminary estimates of the DOA and possesses the interesting feature of providing asymptotically unbiased estimates when the Cross Spectral Matrices tend to their exact values. Furthermore, simulation results show that this approach leads to performances comparable to narrow band MUSIC with same amount of data at the central frequency, both in terms of resolution and mean square error of the estimated DOA .

ABSTRACT

Cet article présente une nouvelle méthode haute résolution d'estimation des gisements de sources à large bande dans le cas d'une antenne linéaire à capteurs équidistants. La mise en oeuvre de cette méthode ne nécessite pas d'estimées préliminaires des gisements. Des simulations montrent que, pour un produit BT donné, les performances de la méthode proposée en large bande sont comparables à celle du goniomètre à vecteur propre en bande étroite.

I- Introduction

The estimation of the DOA of multiple wide band plane waves from the Fourier transform of an array output requires the joint processing of all the narrow band components available in the frequency band of interest. In this context, several solutions based on extensions of the narrow band MUSIC scheme have been proposed.

The simplest one is the incoherent MUSIC algorithm [1] in which narrow band signal subspace processing of each narrow band component is followed by an averaging of the spatial spectra over the frequency band. This way of proceeding is known to reduce the fluctuations of the spatial spectrum at each frequency bin, but it does not increase the resolving power which is essentially limited by the number of observations available at each narrow band component.

Later, Wang and Kaveh proposed the so-called Coherent Signal Subspace Method (CSSM) [2]. Their method requires preliminary estimates of the DOA that may be obtained by conventional wide band beamforming. At each frequency, the estimated DOA are used to determine focussing matrices that map the associated frequency dependent steering vectors to the corresponding steering vectors computed at a reference frequency f_0. The focussing matrices allow the linear transformation of the narrow band components into a fictitious observation that would be measured if the sources were narrow band with center frequency f_0, and from which the DOA can be obtained by applying the MUSIC algorithm. Given a set of preliminary estimates of the DOA, there exists an infinity of possible choices for the focussing matrices -several examples have been studied in [2,3,4]- and it has been shown that good focussing matrices are unitary [4]. However, this method has the major drawback of requiring preliminary estimates of the DOA.

To overcome this problem, a new class of focussing matrices has been developped in the case of linear arrays of equispaced sensors [5,8]. These focussing matrices are designed to interpolate the spatially sampled wave field measured by the sensors. The aim of this interpolation is to create at each frequency fictitious sensors such that the product of the inter-element spacing by the frequency is kept constant over all the wide frequency band. Given two frequencies f and f_0, this procedure is an attempt to map all the steering vectors at f to the corresponding steering vectors at f_0 by a linear transformation that depends only on f and f_0. Though this method performs well, it must be pointed out that these ideal focussing matrices depending only on f and f_0 do **not** exist, so that this mapping is approximate. A similar idea which avoided the computation of the focussing matrices was independently derived in [6] using in the interpolation step both AR and windowed periodogram spectrum estimation techniques for the spatial spectrum.

We present in this paper an alternative method in the case of linear arrays of equispaced sensors. It has the advantage of not requiring preliminary estimates of the DOA and provides asymptotically unbiased estimates when the estimated Cross Spectral Matrices (CSM) at each frequency bin tend to their exact values. Section 2 derives the proposed algorithm for a spatially white noise, though the algorithm is easily extended to deal with a spatially colored noise with known CSM. Section 3 presents some simulations and comparisons with the incoherent wide band MUSIC algorithm and the CSSM.

II- The proposed method

Let us consider a linear array of N equispaced sensors with spacing d which receive the plane waves generated by P uncorrelated sources in a medium with velocity c. The source signals and the noise measured by the sensors are assumed to be spatially stationary band limited processes with flat spectra in $(f_0-B/2, f_0+B/2)$. The noise is supposed to be spatially white. The estimation problem of the DOA is referred to as a wide band problem when the propagation time of any source signal across the array is not very small compared to the reciprocal of the bandwidth B. In a practical point of view, this means that:

$$10(N-1)d/c > 1/B, \quad (1)$$

and we consider in the sequel that this inequality holds.

The CSM of the array output is given in $(f_0-B/2, f_0+B/2)$ by:

$$R(f)=\sum_{p=1}^{P}\gamma_p \mathbf{D}(f\sin\theta_p d/c)\mathbf{D}^*(f\sin\theta_p d/c)+\sigma I, \quad (2)$$

where:

- $\mathbf{D}(\nu)=[1,\exp(2i\pi\nu),\ldots,\exp(2i\pi(N-1)\nu)]^t$;
- $\theta_1,\ldots,\theta_P$ are the DOA measured from broadside;
- $\gamma_1,\ldots,\gamma_P$ are the source PSD's;
- σ is the noise PSD.

Due to the spatial stationarity hypothesis, $R(f)$ is hermitian Toeplitz, and this structure will be used both in the estimation of the CSM's and in the derivation of the wide band algorithm.

We assume that the sensors outputs have been Fast Fourier transformed over T consecutive temporal intervals, so that the observation consists in a set of TK complex vectors

$$X_t(f_k)=[x_{1,t}(f_k),\dots,x_{N,t}(f_k)]^t, \quad (3)$$

where:

- $x_{n,t}(f_k)$ denotes the DFT of the n-th sensor on the t-th temporal interval at frequency f_k;
- $1\le k\le K$ and $f_0-B/2=f_1<f_2<\dots<f_K=f_0+B/2$;
- $1\le t\le T$.

An estimate of the CSM $R(f)$ at frequency f_k is readily obtained by averaging the periodogram and is denoted by $R_{PER}(f_k)$:

$$R_{PER}(f_k)=T^{-1}\sum_{t=1}^{T} X_t(f_k)X_t^*(f_k). \quad (4)$$

Knowing that the exact CSM $R(f)$ is toeplitz, we force the estimate $R_{PER}(f_k)$ to be toeplitz by simply averaging along its diagonals. This usual way of taking the CSM Toeplitz structure into account increases the estimation precision; it is also known to improve the resolution capability of the MUSIC algorithm. This estimate is denoted by $\hat{R}(f_k)$ and will be used in the remaining of this section.

The proposed method is based on the following direct consequence of Caratheodory's theorem [7]: let $A=(a_{ij})$ be a NxN Hermitian Toeplitz matrix whose minimum eigenvalue is simple, equal to zero, with associated eigenvector $u=[u_1,\dots,u_N]^t$. Then, the roots $z_1,\dots,z_{N-1}$ of the polynomial $P(z)=u_1+u_2z+\dots+u_Nz^{N-1}$ are located on the unit circle and A admits the decomposition:

$$A=\sum_{p=1}^{N-1}\alpha_p D(\nu_p)D^*(\nu_p), \quad (5)$$

where:

- $D(\nu)$ is defined in equation (2);
- $z_p=\exp(2i\pi\nu_p)$ with $-1/2\le\nu_p\le 1/2$;
- the α_p's are strictly positive and can be determined by equating the first column of each member of equation (5), leading to:

$$\begin{bmatrix}\alpha_1\\ \cdot\\ \cdot\\ \cdot\\ \alpha_{N-1}\end{bmatrix}=\begin{bmatrix}z_1 & \dots z_{N-1}\\ \cdot & \cdot\\ \cdot & \cdot\\ \cdot & \cdot\\ z_1^{N-1} & \dots z_{N-1}^{N-1}\end{bmatrix}^{-1}\begin{bmatrix}a_{21}\\ \cdot\\ \cdot\\ \cdot\\ a_{N1}\end{bmatrix}$$

Since the roots $z_1,\dots,z_{N-1}$ of the polynomial $P(z)$ are on the unit circle, they can be determined by searching for the zeros of the Fourier transform of the components of u. We now derive the proposed method in the case of spatially white noise ,from simple considerations about the CSSM.

Let $\hat{\sigma}$ be the smallest eigenvalue of the estimated CSM $\hat{R}(f_k)$. This eigenvalue is simple with probability 1, so that $\hat{R}(f_k)-\hat{\sigma}I$ satisfies the requirements of Caratheodory's theorem. Therefore, $\hat{R}(f_k)$ admits the decomposition:

$$\hat{R}(f_k)=\sum_{p=1}^{N-1}\hat{\gamma}_p D(\nu_p)D^*(\nu_p)+\hat{\sigma}I. \quad (6)$$

Following the CSSM idea, our aim is to transform $\hat{R}(f_k)$ into an hypothetical CSM measured at the center frequency f_0. Since it was shown in [4] that good focussing matrices are unitary, let us assume temporarily that there exist ideal unitary transformation matrices $T(f_k,f_0)$ that map any steering vector at frequency f_k to the corresponding steering vector at frequency f_0:

$$\text{For all } \theta:\ T(f_k,f_0)D(f_k\sin\theta\, d/c)=D(f_0\sin\theta\, d/c). \quad (7)$$

As pointed out before, such transformations cannot exist, for this would imply for any θ_1 and θ_2:

$$D^*(f_k\sin\theta_1 d/c)D(f_k\sin\theta_2 d/c) = D^*(f_0\sin\theta_1 d/c)D(f_0\sin\theta_2 d/c) \quad (8)$$

which is easily shown to be false by expanding the two scalar products. However, if relation (7) was possible, by setting $\nu=f_k\sin\theta\, d/c$ and making no particular assumption about the sensor spacing so that ν may vary over the entire period $(-1/2,1/2)$ of $D(\nu)$, we should have:

$$\text{For all } \nu\in(-1/2,1/2):\ T(f_k,f_0)D(\nu)=D(\nu f_0/f_k). \quad (9)$$

Then it would follow from relations (6) and (9) that the terms $T(f_k,f_0)\hat{R}(f_k)T^*(f_k,f_0)$ of the coherently averaged correlation matrix $\sum_k T(f_k,f_0)\hat{R}(f_k)T^*(f_k,f_0)$ in the CSSM would be given by:

$$T(f_k,f_0)\hat{R}(f_k)T^*(f_k,f_0) = R_k, \quad (10)$$

where:

$$R_k=\sum_{p=1}^{N-1}\hat{\gamma}_p D(\nu_p f_0/f_k)D^*(\nu_p f_0/f_k)+\hat{\sigma}I. \quad (11)$$

Though R_k defined by (11) cannot be obtained through the focussing matrices $T(f_k,f_0)$, it is however very easy to get it from the decomposition (6) of $\hat{R}(f_k)$. Then, proceeding as in the CSSM, the DOA can be obtained by applying the MUSIC scheme-or any other high resolution method- to the coherently averaged CSM at the center frequency:

$$R=K^{-1}\sum_{k=1}^{K}R_k. \quad (12)$$

The aim of the averaging defined by (12) is to increase the estimation precision of the CSM at the reference frequency f_0, as will be checked on simulations in Section (3).

The steps of the resulting method are given here below:

- For each frequency bin f_k:
  1. Find the parameters $\hat{\sigma},\hat{\gamma}_p,\nu_p$ of the decomposition (6) of $\hat{R}(f_k)$:
     - Compute the smallest eigenvalue $\hat{\sigma}$ of $\hat{R}(f_k)$ and the associated eigenvector $u=[u_1,.,u_N]^t$
     - Find the N-1 zeros $\nu_1,\dots,\nu_{N-1}$ lying in $(-1/2,1/2)$ of the Fourier transform $u_1+u_2\exp(2i\pi\nu)+\dots+u_N\exp(2i\pi(N-1)\nu)$ of the components of u.
     - Determine $\hat{\gamma}_1,\dots,\hat{\gamma}_{N-1}$ by:

$$\begin{bmatrix}\hat{\gamma}_1\\ \cdot\\ \cdot\\ \cdot\\ \hat{\gamma}_{N-1}\end{bmatrix}=\begin{bmatrix}\exp(2i\pi\nu_1) & \dots\dots\exp(2i\pi\nu_{N-1})\\ \cdot & \cdot\\ \cdot & \cdot\\ \cdot & \cdot\\ \exp(2i\pi(N-1)\nu_1) & ..\exp(2i\pi(N-1)\nu_{N-1})\end{bmatrix}^{-1}\begin{bmatrix}\hat{R}_{21}(f_k)\\ \cdot\\ \cdot\\ \cdot\\ \hat{R}_{N1}(f_k)\end{bmatrix}$$

  2. Compute R_k according to equation (11).
- Apply the MUSIC scheme to the coherently averaged CSM at the center frequency defined by (12).

Due to numerical errors in the determination of the ν_p's, the $\hat{\gamma}_p$'s computed here above will fail to be real. Since they should be positive, we replaced them with their absolute value in the simulations.

It can be shown that R_k tends to the exact CSM $R(f_0)$ at the center frequency when the estimated CSM $\hat{R}(f_k)$ tends to its exact value $R(f_k)$; thus, the averaged CSM R defined by (12) tends also to $R(f_0)$. Therefore, the estimated DOA, obtained by applying the MUSIC scheme to R, are asymptotically unbiased. Another consequence is that the N-P smallest eigenvalues of R are asymptotically equal, so that the number of sources could be evaluated for finite K by looking at the distribution of the eigenvalues of R. However, testing the equality of the smallest eigenvalues of R would require a statistical analysis of R (which is not Wishart distributed) that has not yet been done.

III- Simulations

In this section, we present some examples which illustrate the behavior of the proposed method. The array is linear with N=8 equally spaced sensors, and a spacing between sensors equal to a half wave length at the normalized frequency 0.5. The sources signals and the noise have bandwidth B=0.1 and central frequency $f_0=0.45$. For this array, the beamwidth BW is then about 16 deg at f_0. The estimated CSM's $\hat{R}(f_k)$ are measured at K=20 frequency bins in the frequency interval (0.4,0.5), which means that f_k varies from $f_1=0.4$ to $f_{20}=0.5$ with a step of 0.1/19. The observation consists in a set of T K independent zero-mean random complex vectors $X_t(f_k)$ defined by (3), distributed according a complex circular Gaussian distribution, with covariance $E[X_t(f_k)X_t^*(f_k)]=R(f_k)$. The number T of observations per frequency bin varies from 50 to 100, depending on the example. The estimated CSM's $\hat{R}(f_k)$ are computed from the vectors $X_t(f_k)$ as described at the beginning of Section 2.

In the first example, there are P=3 uncorrelated sources with DOA -25 deg, -2 deg, 2 deg and respective SNR's -10 dB, 0 dB, 0 dB. The number T of observations at each frequency bin is equal to 50. All methods compared below are applied to the same data.

Figure 1 gives the response of the Incoherent Wide Band MUSIC estimator to 10 independent trials. This method clearly fails to resolve the two sources at -2 deg and 2 deg. This is due to the poor results given by the MUSIC estimator at each frequency bin.

Figure 2 shows the results obtained by the CSSM, using the unitary focussing matrices described in [4]. Taking for preliminary estimates of the group angles -25 deg and 0 deg, the unitary focussing are designed as recommended in [4] by choosing the following focussing angles: -25 deg-BW/4 = -29 deg, -25 deg, -25 deg+BW/4 = -21 deg, -BW/4 = -4 deg, 0 deg, BW/4 = 4 deg. The two sources at -2 deg and 2 deg are now well resolved.

Figure 3 displays the results given by our method. These are comparable to those obtained on figure 2 by the CSSM, without the need of preliminary estimates of the DOA.

We can check, on this example, the improvement in the estimation of the CSM's due to the coherent averaging. Let us denote by ||.|| the Frobenuis norm. Then, the average over the frequency bins and the trials of the relative error of estimation of the CSM's $||R(f_k)-R(f_k)||/||R(f_k)||$ is 13.56 %. After coherent averaging at the center frequency, it is just 3.42 % for our method and 3.65 % for the CSSM.

Figure 4 presents the resolving power of the different wide band methods as a function of the sources SNR's. There are P=2 uncorrelated sources with equal powers located at -2 deg and 2 deg, the number T of observations per frequency bin is 100. For each value of the SNR, 100 trials were performed and the two sources were considered as resolved when the spatial spectrum presented two maxima between -BW/2=-8 deg and BW/2=8 deg. In the CSSM, we used diagonal focussing matrices with 0 deg as preliminary estimate of the DAO, as recommended in [4] in the case of a single group of sources. Also included as a reference are the results of narrow band MUSIC with KT=2000 snapshots at the center frequency 0.45.

Narrow band MUSIC with 2000 snapshots, the CSSM and our method give about the same results. As expected, the CSSM and the proposed method have a much greater resolving power than the Incoherent Wide Band MUSIC algorithm.

Finally, a Root Mean Square Error (RMSE) comparison of the estimated DOA's, based on trials for which the two sources were resolved, is provided in table 1. It clearly shows that our method performs as well as the CSSM.

SNR	Proposed Method	CSSM	Narrow band MUSIC with 2000 snapshots
-6dB	RMSE=1.08 deg	RMSE=1.09 deg	RMSE=0.97 deg
-5dB	RMSE=1.07 deg	RMSE=1.06 deg	RMSE=0.99 deg
-4dB	RMSE=0.88 deg	RMSE=0.87 deg	RMSE=0.79 deg
-3dB	RMSE=0.71 deg	RMSE=0.65 deg	RMSE=0.72 deg
-2dB	RMSE=0.69 deg	RMSE=0.64 deg	RMSE=0.62 deg
-1dB	RMSE=0.55 deg	RMSE=0.53 deg	RMSE=0.54 deg
0dB	RMSE=0.46 deg	RMSE=0.46 deg	RMSE=0.44 deg

Table 1: RMSE of estimated DOA's

IV- Conclusion

Following the same idea as the CSSM, we have shown that the estimation of the DOA's of multiple wide band sources can be handled without the need of preliminary estimates when the array is linear with equispaced sensors. The proposed algorithm is an alternative method to the one presented in [5,8]. Simulations have shown that the performances of the proposed method are about the same as the CSSM and narrow-band MUSIC with the same amount of data at the central frequency. Though the DOA were estimated in the paper by applying the MUSIC scheme to the coherently averaged CSM, they could also be determined by applying to it any other narrow band high resolution method.

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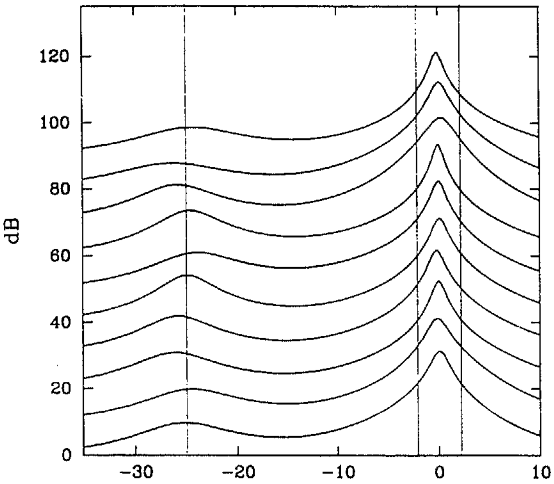

Figure 1
Incoherent wideband MUSIC.

8 sensors at λ/2 at frequency 0.5 .
3 wide band sources in the frequency band (0.4,0.5) .
Source location and power: −25'(−10 dB) ,−2'(0 dB) ,2'(0 dB) .
20 frequency bins, 50 snapshots.
Background noise is spatially white with power 0.DB .

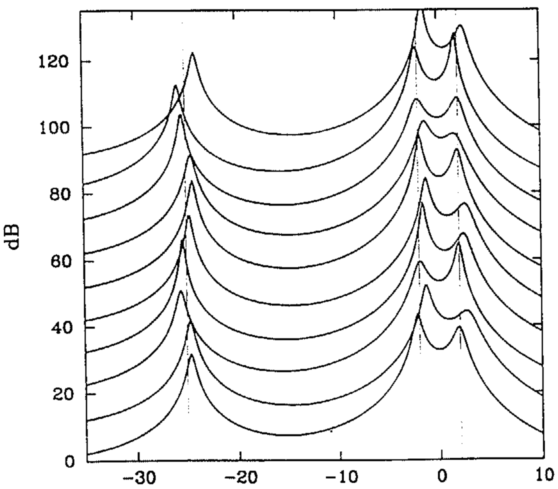

Figure 2
CSSM.

8 sensors at λ/2 at frequency 0.5 .
3 wide band sources in the frequency band (0.4,0.5) .
Source location and power: −25'(−10 dB) ,−2'(0 dB) ,2'(0 dB) .
20 frequency bins, 50 snapshots.
Background noise is spatially white with power 0.DB .
6 focussing angles: −29, −25, −21, −4, −2, 4.

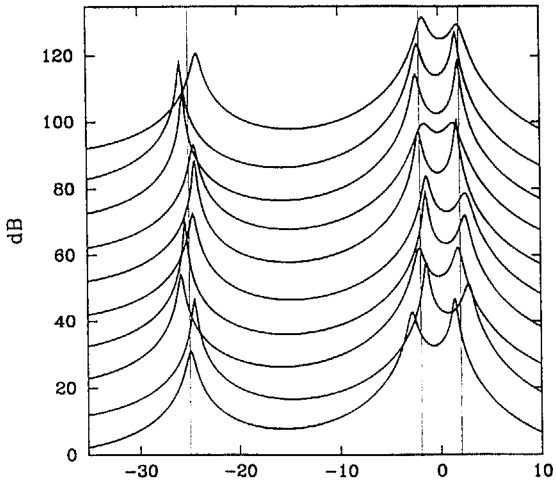

Figure 3
Proposed method.

8 sensors at λ/2 at frequency 0.5 .
3 wide band sources in the frequency band (0.4,0.5) .
Source location and power: −25'(−10 dB) ,−2'(0 dB) ,2'(0 dB) .
20 frequency bins, 50 snapshots.
Backgound noise is spatially white with power 0.DB .

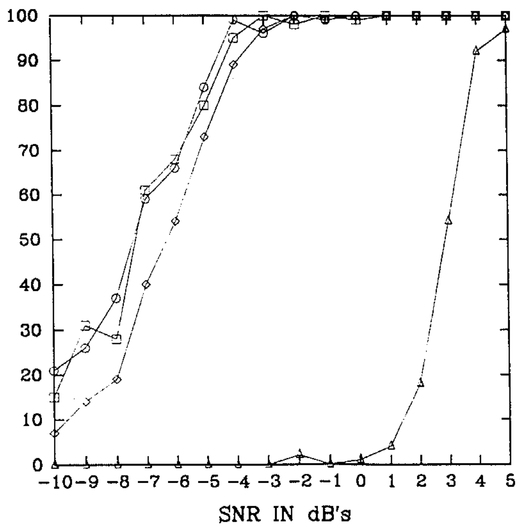

Figure 4
Resolving power.

8 sensors at λ/2 at frequency 0.5 , 2 sources at −2' and 2' .
100 trials per value of the SNR.
O : Narrow Band MUSIC with 2000 snaphots.
◇ : Wide Band method with 20 freq. bins in (0.4,0.5) and 100 snapshots per bin.
□ : CSM with 20 freq. bins in (0.4,0.5) and 100 snapshots per bin.
Δ : Wide Band MUSIC with 20 freq. bins in (0.4,0.5) and 100 snapshots per bin.