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A CYCLIC SVD-BASED ALGORITHM FOR MULTIPLE SOURCE LOCALIZATION

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RÉSUMÉ

On propose une nouvelle méthode pour estimer le nombre D de sources ponctuelles à longue distance au moyen d'un réseau de $M > D$ capteurs. La méthode, qui exploite la cyclostationnairité des signaux modulés, est basée sur la Décomposition en Valeurs Singuliers de la matrice de corrélation ciclique des signaux prélevés et se montre supérieure aux approches classiques lorsque le signal utile est très faible et noyé dans un bruit à bande large et/ou dans une interférence à bande étroite et aussi lorsque le bruit est arbitraire (par exemple, non-stationnaire) et inconnu.

ABSTRACT

A new method for detecting the number of radiating sources by a sensor array is presented. The method exploits the cyclostationarity property which is exhibited by all modulated signals. The new technique is based on the rank determination of the cyclic array covariance matrix performed by the Singular Value Decomposition. The main advantage of the method is its immunity to both wideband noise and narrowband interfering signals in weak signal conditions and to arbitrary and unknown interference environments.

1. Introduction

A key issue in the multiple-source localization problem is the detection of the number of signals.

In many physical problems, with radar and sonar as examples, the outputs of an array of sensors are collected over some time interval and used to extract the spatial structure of multiple radiating sources. These are assumed to be located in the far field of the array so that the received wavefronts can be modeled as planewaves.

With reference to narrowband signals impinging on the sensor array, most of the methods [1-4] to estimate the number of sources exploit the eigenstructure of the array covariance matrix (ACM) of the received signal vector by evaluating the multiplicity of the smallest eigenvalue of the ACM. The major drawback of these methods is that the ACM is unknown, and only an estimate from a finite sample size is available. Consequently, the resulting ACM eigenvalues are all different with probability one, and, hence, it can be difficult to determine the number of signals simply by 'observing' the eigenvalues. In [5] it has been proposed a more sophisticated method based on a sequence of hypothesis tests whose threshold levels, however, are very difficult to select. Moreover, a new approach, which applies information theoretic criteria for model selection, has been presented [6, 7] with reference to a zero-mean Gaussian stationary model for both signal wavefronts and sensor noises.

The above-mentioned methods require that the sensor noises be uncorrelated and have the same variance (i.e., spatially white noise). With reference to spatially nonwhite noise with stationary characteristics, the multiple-source localization problem has been solved [8, 9], provided that, however, an estimate of the noise ACM is available.

In adverse noise environments (low signal-to-noise ratios), since a large amount of collect time is required to obtain satisfactory performance, the assumption of stationary noise is not generally reasonable and, therefore, the previous methods cannot be utilized unless some cumbersome procedure for updating noise statistics is performed.

Recently, a new method has been proposed [10] for arbitrary (not necessarily stationary) sensor noises and interferences. It does not require knowledge of the noise and interference covariance matrices to estimate the number of radiating sources and their locations. The approach reasonably assumes that the source signals are cyclostationary. Therefore, it automatically discriminates in favor of the signals of interest (SOI's) against noise and interfering signals on the basis of their known spectral correlation properties [11]. The main advantage of the proposed method is its immunity to arbitrary (stationary or nonstationary, spatially white or coloured) and unknown wideband and narrowband interfering signals in strongly adverse conditions. In particular, it provides good accuracy also when the interfering signals, whose number can be greater than the number of sensors, exhibit an arbitrary degree of correlation amongst themselves and arrive from directions arbitrarily close to those of the SOI's.

Since the method is based on the property that the rank of the cyclic ACM of the received signals is just equal to the number of source signals, in [10] an algorithm is proposed which estimates such a rank on the basis of the magnitude of the determinants of the leading principal submatrices of increasing order. [1]

The present paper is aimed at reducing the quite large sample size required by the method considered in [10] to obtain a satisfactory performance. More specifically, it proposes a new algorithm for the cyclic ACM rank estimation based on the Singular-Value Decomposition (SVD) which, in recent years, has been extensively applied in a number of least squares, spectral estimation and system identification problems for its computational efficiency. Moreover, the performances of the algorithm proposed in [10] and of the new one are evaluated and compared, in terms of average sample size assuring an adeguate error probability (i.e., the probability that the number of source signals is uncorrectly estimated).

[1] It is worthwhile to note that the well-known eigenvalue-based approach for rank determination cannot be adopted in this case since the cyclic ACM is a non-Hermitian matrix.

2. The new approach

If one considers a passive array consisting of M sensors and assume that D narrowband sources ($D < M$), centered around a known frequency f_0, impinge on the array from the directions of arrival $\theta_1, \theta_2, \ldots, \theta_D$, the M-vector $\mathbf{x}(t)$ of the received analytic signals can be expressed as

$$\mathbf{x}(t) = \mathbf{A}(\theta)\mathbf{s}(t) + \mathbf{n}(t) \tag{1}$$

where $\mathbf{s}(t)$ is the D-vector of the analytic SOI's, $\mathbf{n}(t)$ is the M-vector modeling the noise and interference signals and $\mathbf{A}(\theta)$ is the $M \times D$ matrix of the steering vectors.

The source-localization problem is solved in [10] by exploiting the cyclostationarity exhibited by all modulated signals [11], but which is ignored in the traditional methods. The approach is based on the properties of the cyclic conjugate cross-correlation function (CCCF) defined by:

$$R^{\alpha}_{x_i x_j^*}(\tau) \triangleq < E[x_i(t+\tau/2)x_j(t-\tau/2)] \exp(-j2\pi\alpha t) > \tag{2}$$

where $< \cdot >$ and $E[\cdot]$ denote the time and ensemble (respectively) average and α is the cycle frequency parameter. For stationary processes the CCCF is zero for any $\alpha \neq 0$. Moreover, for many carrier-modulated cyclostationary processes, $R^{\alpha}_{x_i x_j^*}(\tau) \neq 0$ for some values of α and τ. [2]

By assuming that all SOI's exhibit cyclostationarity at a same known value of the cycle frequency α but that the noises and interferences do not, the cyclic conjugate cross-correlation matrix (CCCM) of the received vector is given by:

$$\mathbf{R}^{\alpha}_{\mathbf{xx}^*}(\tau) = \mathbf{A}(\theta)\mathbf{R}^{\alpha}_{\mathbf{ss}^*}(\tau)\mathbf{A}^T(\theta) \tag{3}$$

where T denotes the transpose.

Equation (3) shows that, by an appropriate selection of α, the contributions to the CCCM from arbitrary (not necessarily stationary and/or spatially white) noises and interferences vanish. Such a result allows us to predict for this approach, based on the properties of an estimate of the CCCM, satisfactory performance even in strongly adverse interference environments. In particular, the method can provide good accuracy also when the interfering signals, whose number can be greater than the number of sensors, exhibit an arbitrary degree of correlation amongst themselves and arrive from directions arbitrarily close to those of the SOI's. Moreover, the solution to the problem does not require the knowledge of the noise statistics which is essential in the traditional methods.

Assuming that the matrix $\mathbf{A}(\theta)$ has full column rank (i.e., the direction vectors are linearly independent with one another) and the signal CCCM $\mathbf{R}^{\alpha}_{\mathbf{ss}^*}(\tau)$ is nonsingular for a known value of the lag parameter τ, the rank of the matrix $\mathbf{R}^{\alpha}_{\mathbf{xx}^*}(\tau)$ is equal to the number D of source signals, provided that $M \geq D$.

Finally, let us note that the matrix $\mathbf{R}^{\alpha}_{\mathbf{xx}^*}(\cdot)$ is unknown and, therefore, it is necessary to estimate it from a finite sample of size $2K+1$:

$$\hat{\mathbf{R}}^{\alpha}_{\mathbf{xx}^*}(m) = \frac{1}{2K+1} \sum_{l=-K}^{K} \mathbf{x}(l+m)\mathbf{x}^T(l)\, e^{-j2\pi\alpha(l+\frac{m}{2})} \tag{4}$$

[2] The choice of the CCCF is appropriate when α is related to the carrier frequency. On the other hand, if α is related only to possible periodicities of the modulating signals, such as a keying rate, this approach can be modified by using the cyclic cross-correlation function $R^{\alpha}_{x_i x_j}(\tau)$.

3. Algorithms for CCCM rank estimation

Determining an appropriate statistical test for the estimation of the rank of the matrix (4) is a challenging task due to the nonstationarity and non-Gaussian behavior of most signals of interest. Hence, it is necessary to carry out some "ad hoc" procedures whose characteristic parameters can be fixed on the basis of experimental evidence. In [10] the rank estimation has been performed on the basis of the magnitude $\mathcal{D}_i$ $(i = 1, 2, \ldots, M)$ of the determinants of the leading principal submatrices of increasing order. More specifically, a two-step procedure, which requires the setting of just one meaningful parameter, has been adopted. In the first step one evaluates the $\mathcal{D}_i's$ for increasing values of the sample size in order to single out a value assuring a satisfactory noise rejection, i.e., such that any increase of the sample size does not significantly affect the values of $\mathcal{D}_i's$. The procedure is described in detail by the flowchart of Fig.1, where the notation $\mathcal{D}_i(n)$ is introduced to denote $\mathcal{D}_i$ evaluated for a sample size n. Let us note that, although the parameters k_0, n_0, γ and T_C have to be fixed, the only one playing a key role in the whole procedure is the convergence threshold T_C: its value can be selected taking into account the trade-off between reliability of the estimates and observation time.

As result of the first step, one obtains that the magnitudes $\mathcal{D}_i's$ of the determinants are weakly affected by noise and interference. Moreover, if a normalization of $\hat{\mathbf{R}}^{\alpha}_{\mathbf{xx}^*}(m)$ is performed by scaling all terms of the matrix by the factor

$$1/M^2 \sum_{i,j=1}^{M} \mid \hat{R}^{\alpha}_{x_i x_j^*}(m) \mid \tag{5}$$

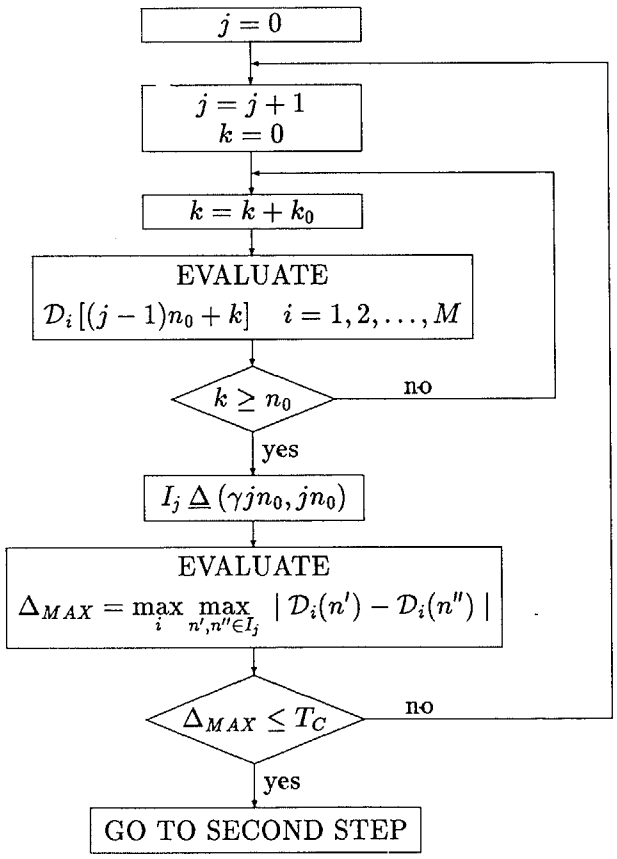

Fig.1

the $\mathcal{D}_i's$ for $i = 1, 2, \ldots, D$ (in the following referred to as "signal determinants") are of the same order of magnitude of each other. The $\mathcal{D}_i's$ for $i = D+1, \ldots, M$ ("noise determinants") are, on the contrary, very small compared with the other ones and strongly decrease as the index i increases, in that they refer to submatrices in which $i - D$ rows are linear combinations of the other ones, except for very small noisy terms. On the basis of these considerations, in the second step of the algorithm one determines the value of the index i which maximizes

$$\mathcal{E}_i = \frac{\mathcal{D}_i^2}{\mathcal{D}_{i-1}\mathcal{D}_{i+1}} \qquad \mathcal{D}_0 \triangleq 1; \quad i = 1, 2, \ldots, M-1 \quad (6)$$

and assumes the number of source signals equal to this value of i. In other words, such procedure evaluates the number of sources by detecting the "edge" between the different behaviors of signal and noise determinants. It is worthwhile to emphasize that in this second step no threshold test is performed, avoiding so any parameter setting problem.

The algorithm for rank determination proposed in this paper is based on the SVD, which in recent years has received a great attention because of its high computational efficiency. More specifically, with reference to the CCCM (3) (i.e., in the ideal case of infinite-time conditions) one has:

$$s_1 \geq s_2 \geq \ldots \geq s_D > s_{D+1} = \ldots = s_M = 0 \quad (7)$$

with s_i $(i = 1, 2 \ldots, M)$ denoting the i-th singular value (SV) of $\mathbf{R}^{\alpha}_{\mathbf{xx}^*}(\cdot)$. In practice, the SV's of the matrix (4), say $\hat{s}_i$, do not satisfy the relation (7); in particular $\hat{s}_{D+1}, \ldots, \hat{s}_M$, say the noise SV's, will be small (provided that $\hat{\mathbf{R}}^{\alpha}_{\mathbf{xx}^*}(\cdot)$ is a good estimate of $\mathbf{R}^{\alpha}_{\mathbf{xx}^*}(\cdot)$) but not necessarily zero. Therefore, the problem arising in rank evaluation is to determine the number of SV's significantly different from zero (i.e., the number of signal SV's), or equivalently, the number of SV's very close to zero (i.e., the number of noise SV's). According to the previous mentioned difficulties to derive a statistical test, again an "ad hoc" decision strategy is proposed. Each step of the procedure consists of:

i) evaluating for the $M-1$ subsets of SV's

$$G_i \triangleq \{\hat{s}_i, \hat{s}_{i+1}, \ldots, \hat{s}_M\} \qquad i = 1, 2, \ldots, M-1 \quad (8)$$

the dispersion coefficients

$$d_i \triangleq \frac{\sigma_i^2}{m_i^2} \quad (9)$$

where

$$m_i \triangleq \frac{1}{M-i+1} \sum_{k=i}^{M} \hat{s}_k \quad (10)$$

and

$$\sigma_i^2 \triangleq \frac{1}{M-i+1} \sum_{k=i}^{M} (\hat{s}_k - m_i)^2 \quad (11)$$

are mean and variance of G_i;

ii) performing the decision according to the rule

$$\hat{D} = i^* \quad (12)$$

with $d_{i^*} = \max_i d_i$.

Such a rule can be justified by considering that the set of SV's can be partitioned in two classes: the noise SV class and the signal SV one. The former consists of the elements close to zero; the latter is formed by the remaining ones. It is not difficult to intuitively accept that the maximum dispersion coefficient will very likely occur in correspondence of the subset G_D containing all the noise SV's plus the smallest signal SV, provided that the separation between the classes is sufficiently large and, within each class, the SV's are of the same order of magnitude.

Figure 2 shows the implemented procedure: once evaluated the decision variables $d_i(n)$, with n again denoting the actual sample size and, hence, estimated the number of signals, the magnitude of the difference between the two largest decision variables, say ϵ, is determined to update the variable Γ which controls the end of the procedure by a test whose threshold sets the reliability degree. The choice of the updating rule of the variable Γ is motivated by the need to state the end of the procedure not only on the basis of a large maximum but also on the ground of a sufficiently large number of steps characterized by the same estimate with, however, small values of ϵ.

We present now simulation results to evaluate and compare the performance of the SVD-based method with that of the determinant-based one. The examples assume a passive uniform linear array with interelement spacing $\Delta = c/2f_0$ (c is the propagation velocity of the wavefront) and consider amplitude-modulated (AM) source and interfering signals with modulating signals modeled as independent zero-mean stationary processes, obtained by filtering white Gaussian processes by a Butterworth filter of first order with a fractional bandwidth of 0.04. The SOI's have carrier frequency f_0, and the interfering signals have

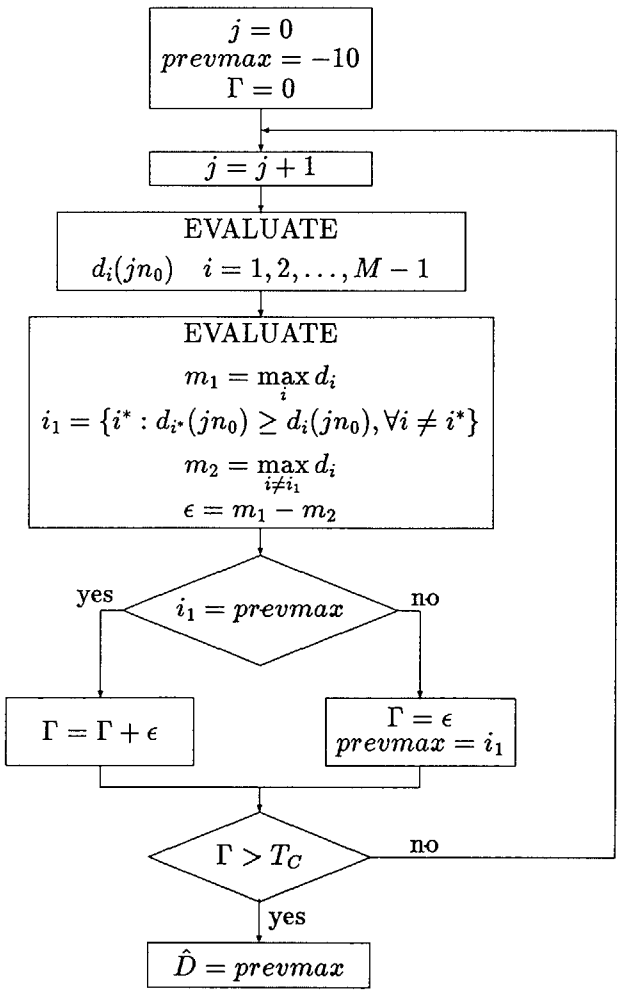

Fig.2

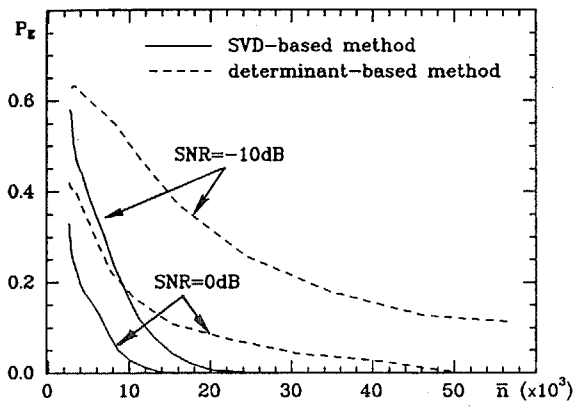

Fig.3

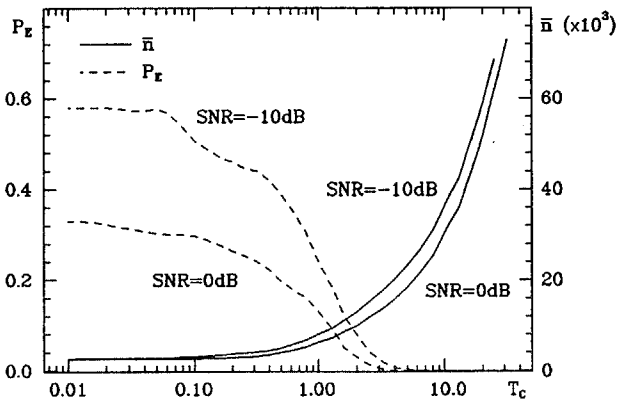

Fig.4

carrier frequencies unequal to f_0. It follows that the SOI's are purely cyclostationary processes with period $1/2f_0$ [11]. Moreover, the only value of α such that $\mathbf{R}^{\alpha}_{ss^*}(\tau) \neq 0$ for some values of τ is $\alpha = 2f_0$. Therefore, the choice of AM signals leads one to consider the CCCF at $\alpha = 2f_0$ whose magnitude peaks at $\tau = 0$, which becomes the obviously selected value for the lag parameter. The wideband noises are modeled as white Gaussian processes uncorrelated from sensor to sensor.

The numerical results refer to 100 trials of the following experiment: four sensors, two SOI's, five narrowband interferers, wideband sensor noises. The power of all desired and interfering signals is fixed at 1 unit. The DOA's of source and interfering signals are fixed at: $\theta_1 = -0.3rad.$, $\theta_2 = 0.2rad.$, $\theta_{I_1} = \theta_{I_2} = \theta_{I_3} = -0.3rad.$, $\theta_{I_4} = \theta_{I_5} = 0.2rad.$ The carrier frequencies of the interfering signals are chosen as follows: $0.80f_0$, $0.85f_0$, $0.90f_0$, $0.95f_0$ and $0.98f_0$. It is worthwhile to note that in this particularly severe noise environment, characterized by a number of interferers greater than the number of sensors, the conventional methods fail. For the determinant-based procedure the values $k_0 = 80$, $n_0 = 32k_0 = 2560$ and $\gamma = 0.75$ (see flowchart of Fig.1) have been chosen for the parameters, whereas for the SVD-based procedure the unique parameter n_0 (see flowchart of Fig.2) has been fixed at $n_0 = 2560$.

Figure 3 shows the probability P_E that the number of SOI's is uncorrectly estimated as a function of the required average sample size $\overline{n}$ for both procedures and for two values of SNR. The results show that the SVD-based method largely outperforms the determinant-based one. For example, to assure an error probability of 10% when SNR=0dB, the SVD-based method requires about 7000 samples, whereas, the determinant-based one needs about 17000 samples. Moreover, a better improvement is achieved in correspondence of lower values of P_E.

With reference to the SVD-based procedure, and for the same noise and interference environment, Figure 4 shows P_E and $\overline{n}$ as a function of the threshold T_C. For example, the results show that, for SNR=0dB, values of T_C belonging to the interval $(1, 3)$ assure a good compromise between observation time and reliability of the estimates.

Finally, we note that, although a quite large number of samples is required in order for interferences and noises to decorrelate in the CCCM estimates, the performance is fully satisfactory even in these adverse operative conditions where the DOA's of the desired and interfering signals are the same.

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