SIRP–Model Based Generation of Image VQ Training Sequences

Yonggang Du

Institute for Communication Engineering, Aachen University of Technology (RWTH)
Melatener Str. 23, W–5100 Aachen, Germany

Résumé
Il est décrit ici un concept novateur pour la génération de blocs pseudo aléatoires bidimensionnels pour mettre au point des quantifieurs vectoriels sans mémoire d'images. Afin d'être utilisable sur un grand nombre d'images réelles, les blocs sont modellés en des processus aléatoires à invariance sphérique (SIRP) basés sur des fonctions gaussiènnes généralisées. Ce faisant, la forme des distributions d'amplitude et les corrélations entre les éléments des blocs peuvent être modifiées indépendament l'une des autres par un paramètre de forme pour la première et une matrice de covariance pour les secondes.

Abstract
A novel concept is described to generate twodimensional pseudo random blocks for training memoryless image vector quantizers. To cover a wide range of real image statistics the blocks are modelled as spherically invariant random processes (SIRP) based on generalized Gaussian functions. In doing so, the form of the univariate amplitude distributions and the correlations between the block components can be changed independently of each other by a form parameter and a covariance matrix, respectively.

1 Introduction

In the past vector quantization (VQ) has found wide applications in image source coding. To increase coding efficiency, the codebook of a VQ has to be adapted to the statistical behavior of image blocks by a training sequence [1]. Theoretically, a VQ training sequence must be infinitely long and must resemble all the statistical characteristics of the signals to be encoded later. Practically, it is impossible to generate such a long sequence from a limited real image data base on the computer. So, in most cases one has to be satisfied with a short, non–representative training sequence drawn from only few real images. For this reason the performance of a so optimized VQ becomes highly sensitive to the kind of images contained in the training sequence, leading to the so called inside/outside–effect.

A possibility to eliminate the inside/outside–effect is to use codebooks trained by image–like pseudo random blocks. Generating Gaussian or uniformly distributed random signals to train a VQ is certainly a simple task, but these distributions usually do not reflect those of real images, which in fact show a strong peak at zero.

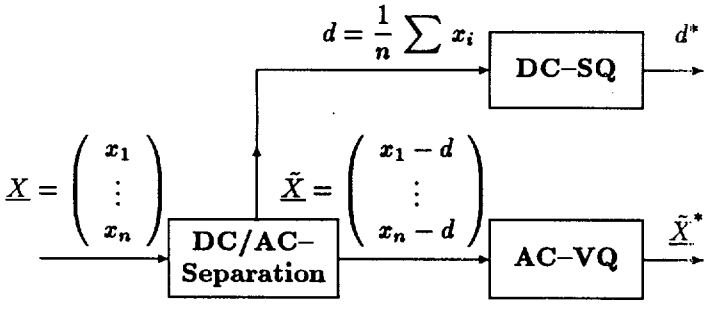

Figure 1: General concept for mean–removed VQ

In this paper we are going to present a new concept for generating pseudo image blocks, which is based on modelling image blocks as samples of spherically invariant random processes (SIRP). We confine ourselves to the generation of image block structures which are described by the AC–terms of the blocks. Each block's DC–term which represents the mean of the block is not taken into account, for in practice VQ is often applied to mean–removed AC–blocks in order to lower coding expenditure.

Fig. 1 shows a general block diagram of a DC/AC–separated block processing, wherein the AC–block $\underline{\tilde{X}}$ is obtained by subtracting the mean d of the input block $\underline{X}$ from each component x_i of $\underline{X}$. The DC–term is then quantized by a scalar quantizer and the AC–block by a vector quantizer. Since d and $\underline{\tilde{X}}$ are nearly uncorrelated, this separation results in only small performance loss, while the realized product code drastically reduces the coding expenditure when compared to directly applying VQ to the input block $\underline{X}$.

Our SIRP–model for the AC–blocks is developed in [2]. The work was first motivated by the fact that ellipse–like contour lines of equal height — a necessary condition for processes to be SIRP — had been measured for bivariate probability density functions (PDF) of the block components. The main advantage of our model is that the form of the univariate PDFs and the covariance matrix of the blocks can be varied independently of each other, so that higher–order statistics are realized. Specifically, we have applied generalized Gaussian functions (GGF) to model univariate PDFs. It turns out that GGF can cover a wide spectrum of real image statistics.

2 The SIRP multivariate PDF–model

A good generator of pseudo image blocks requires a statistical model whose parameters are used to change the statistical behaviour of the generator outputs. Pseudo random

blocks with Gaussian or uniform distribution are certainly simple to generate, but these two distributions do not resemble the true statistics of image blocks. Statistical measurements have shown that the components of mean–removed AC–blocks have a much greater frequency of occurrence at zero amplitude than assumed by the Gaussian model. Besides, Gaussian and uniform distribution models cannot satisfy our desire to vary the form of the generated PDFs in order to cover a wide spectrum of applications.

An alternative to solve this problem is to model AC–blocks $\underline{\tilde{X}}$ as samples of a SIRP [2]. Each n–variate PDF of such a process can be written as

$$p_{\underline{\xi}}(\underline{\tilde{X}}) = \pi^{-n/2}|\underline{\underline{M}}|^{-1/2} f_n(s) \quad \text{with} \quad |\underline{\underline{M}}| = \det(\underline{\underline{M}}) \,. \quad (1)$$

Here $\underline{\underline{M}}$ is the covariance matrix of the n–dimensional random vector $\underline{\tilde{X}}$ and $f_n(s)$ is a function whose argument s is the quadratic form

$$s = \underline{\tilde{X}}^T \, \underline{\underline{M}}^{-1} \, \underline{\tilde{X}} \quad (2)$$

of $\underline{X}$ built up by the inverse covariance matrix $\underline{\underline{M}}^{-1}$. By this definition, Gaussian processes are apparently special members of the SIRP family, for which $f_n(\cdot)$ is an exponential function.

If in (2) s is held constant, then the equation describes for $n = 2$ the contour lines of equal height for each bivariate PDF. These contour lines are in general elliptic. They become circles if the components of $\underline{\tilde{X}}$ are decorrelated and have equal variances.

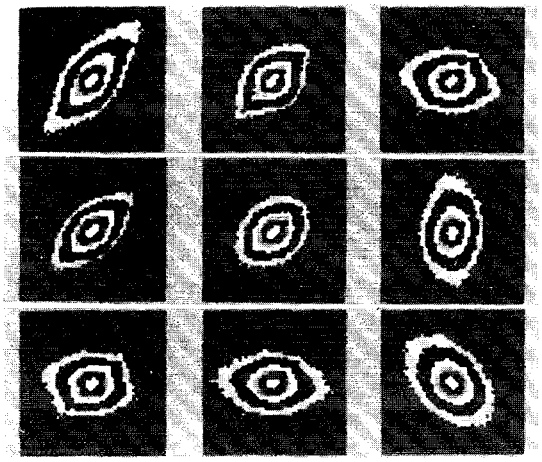

Figure 2: The contour lines of equal height of 9 measured bivariate PDFs

For the purpose of verifying whether AC–blocks of real images can indeed be modelled as SIRPs, Fig. 2 shows the contour lines of equal height for 9 bivariate PDFs measured on pixel pairs chosen from 8×8 AC–blocks. The contour lines were generated by logarithmic quantization of each PDF and visualized by assigning different grey–levels to the quantized values. It is seen that they are approximately ellipses. For a detailed description of our statistical experiments the reader is referred to [2].

The multivariate PDFs of SIRPs can be expressed by [3]

$$p_{\underline{\xi}}^{\text{sirp}}(\underline{\tilde{X}}) = \int_0^\infty p_{\underline{\xi}}^{\text{gauss}}(\underline{\tilde{X}}, r)\, p_\sigma(r)\, dr \,. \quad (3)$$

This expression says that each n–variate PDF $p^{\text{sirp}}(\underline{\tilde{X}})$ of a SIRP with $\underline{\underline{M}}$ as covariance matrix can always be described as a weighted averaging of different Gaussian ones

$$p_{\underline{\xi}}^{\text{gauss}}(\underline{\tilde{X}}, r) = (2\pi)^{-n/2} r^{-n} |\underline{\underline{M}}|^{-1/2} \exp\left(-\frac{\underline{\tilde{X}}^T \underline{\underline{M}}^{-1} \underline{\tilde{X}}}{2\, r^2}\right) \quad (4)$$

which differ only in their covariance matrices. The differences are characterized by a variance scaling factor r for each Gaussian process $p_{\underline{\xi}}^{\text{gauss}}(\underline{\tilde{X}}, r)$. r makes from the covariance matrix $\underline{\underline{M}}$ of the SIRP a scaled version $\underline{\underline{M}}_r = r^2\, \underline{\underline{M}}$ used as the covariance matrix of the corresponding Gaussian process. It is also clear from (3) that the form of a special SIRP distribution can only be influenced by the weighting function $p_\sigma(r)$ which is called Sigma-density. If $p_\sigma(r)$ is a Dirac-impulse at $r = 1$, the averaging results in a Gaussian process.

The averaging character of (3) enables us to think of a SIRP as a random mixture of various Gaussian processes with differently scaled covariance matrices. Since Gaussian processes with a given covarinace matrix $\underline{\underline{M}}_r$ can easily be generated, the task to realize a n–dimensional SIRP is reduced to switching from one Gaussian process to another under the condition that the occurrence of these Gaussian processes is controlled by the Sigma–density $p_\sigma(r)$. Thus, the key to SIRP-block generation is the evaluation of $p_\sigma(r)$ for a desired PDF form.

Fortunately, the spherical symmetry provides a possibility to derive $p_\sigma(r)$ analytically from the univariate PDF with normalized variance. In [4] it is shown that all multivariate PDFs of a SIRP are related to the univariate PDF by a differential equation. From this differential equation a relationship can be derived between the Sigma–density and the univariate PDF. Eventually, the form of each multivariate PDF of a SIRP is completely determined by the normalized univariate PDF. Hence, to develop a SIRP–model one only needs to find a model function for the univariate PDF and to make the Sigma–density computable for every value of model parameters.

For our SIRP-model to be practicable we have used generalized Gaussian functions (GGF)

$$p_\xi(x) = \frac{\nu\alpha(\nu)}{2\,\Gamma(1/\nu)} \cdot \exp\{-[\alpha(\nu)\,|x|]^\nu\} \quad (5)$$

with

$$\alpha(\nu) = \sqrt{\frac{\Gamma(3/\nu)}{\Gamma(1/\nu)}}$$

to model the normalized univariate PDF of each AC–block component. Though GGFs have only one parameter ν, called form parameter, to vary their form, this parameter turns out to be sufficient to fit a wide range of real image distribution forms. In [2] it is proved that GGFs can be a SIRP–model function as long as the form parameter ν is less than or equal to 2.

Because of the functional speciality of GGFs we have applied Mellin–transform to the differential equations between the multivariate PDFs and the univariate one and can get in the Mellin–domain an easy expression for the Sigma–density

$p_\sigma(r)$ [2]. The inverse Mellin–tranform of this expression, which is defined by a Barnes–integral

$$p_\sigma(r) = K \frac{1}{2\pi i} \int_{C-i\infty}^{C+i\infty} \left[2\sqrt{\alpha(\nu)}\, r^2\right]^{-z} \frac{\Gamma(\frac{2z}{\nu})}{\Gamma(z)}\, dz \qquad (6)$$

with

$$K = \frac{\sqrt{\pi\Gamma(3/\nu)}}{[\Gamma(1/\nu)/2]^{3/2}}$$

leads finally to a possibility to numerically evaluate the Sigma–density for every valid value of the form parameter ν. In [2] the integral problem (6) is solved by asymptotic expansions for either small or large r.

While the form parameter ν of the GGFs changes the distribution form of the pseudo image blocks, the generated random structures of the blocks are determined by the covariance matrix $\underline{\underline{M}}$. It is to note that the covariance matrix of mean–removed AC–blocks — unlike that of original blocks — can no longer be assumed as stationary, that means as shift–invariant. Considering the lowpass characteristics of image signals, Du and Fischer have shown a mathematical proof of this effect and derived non–stationary covariance model functions for AC–blocks of limited dimensions (up to 8×8) [5][6].

3 The generator concept

The interpretation of SIRPs as mixtures of Gaussian processes has first been utilized by Brehm to generate spherically invariant speech–model signals [4]. Unfortunately, his generator concept cannot directly be taken over for the generation of SIRPs as image VQ training sequences. So, our concept differs from his in the following aspects:

- Brehm generates continuous one–dimensional spherically invariant variables as pseudo speech signals; we generate two–dimensional ones as pseudo image blocks without considering continuity from block to block.
- Brehm uses Meijer's G–functions to model the normalized univariate PDF of the samples; we use GGFs, thus considerably easing the fitting problem of model parameters.
- Brehm uses the Laplace–transform to get the Sigma–density from the normalized univariate PDF, which is essential for the generation; we use the Mellin–transform, because no Laplace–table is known for GGFs.
- Brehm uses autocorrelation functions to embed correlations in the samples; we use covariance matrices which need not be shift–invariant to embed correlations in the block components.

In Fig. 3 the block diagram of our SIRP–generator is shown. The generator consists — like that of Brehm's — of two paths. The top path realizes a random variable r for each block to be generated, which has the Sigma–density $p_\sigma(r)$ as PDF. To obtain this distribution a non–linearity is used which transforms the uniform distribution of a PN–generator (W) into the Sigma–density. For each desired form

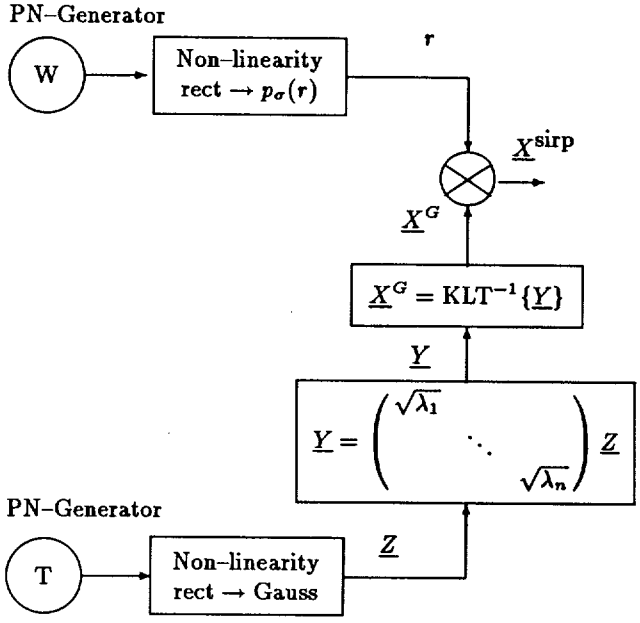

Figure 3: The block diagram of the SIRP–block generator

parameter ν the Sigma–density is evaluated by asymptotic expansions of (6).

The bottom path realizes a Gaussian distributed block $\underline{X}^G$ with the covariance matrix $\underline{\underline{M}}$ which is the covariance matrix of the SIRP-blocks to be generated. To get this correlated Gaussian block, firstly a n–dimensional decorrelated Gaussian block $\underline{Z}$ with unit variances is formed after a non–linearity transforming uniformly distributed random variables into these Gaussian ones. The n components of $\underline{Z}$ are then multiplied by the square roots of the eigenvalues λ_i of the covariance matrix $\underline{\underline{M}}$. This results in a decorrelated block $\underline{Y}$ with components having variances equal to the eigenvalues of the covariance matrix $\underline{\underline{M}}$. This feature indicates that one can think of $\underline{Y}$ as the Karhunen–Loève–Transform (KLT) of the correlated Gaussian block $\underline{X}^G$. Finally, an inverse KLT (KLT^{-1}) whose matrix is to be calculated from the covariance matrix $\underline{\underline{M}}$ transforms the Gaussian block $\underline{Y}$ in the KLT–domain back into the original domain, delivering the correlated block $\underline{X}^G$.

The last step is to randomly scale the variances of the Gaussian block $\underline{X}^G$, whereby the scaling factor r for each block obeys the Sigma–density. In Fig. 3 this is done by multiplying each component of $\underline{X}^G$ with the very same variable r coming from the top path. Since r is random from block to block and has the Sigma–density as PDF, the ensemble of the generator output blocks $\underline{X}^{sirp}$ can be regarded as a random mixture of differently variance–scaled Gaussian processes. As a result, these output blocks form a spherically invariant random process with the desired Sigma–density.

4 The generation results

Using the proposed generator concept pseudo image blocks with given covariance matrix and normalized univariate PDF were generated. To show the effect of the covariance matrix $\underline{\underline{M}}$ on the realized structures of the SIRP–blocks,

the generation has been carried out for four different covariance matrices. Each covariance matrix corresponds to a certain preferential orientation of the structures. They were obtained by first classifying a long sequence of real 8×8 image AC–blocks into the orientations of 0°, 22°, 45° and blocks of mixed orientations (mixture) and then measuring for each class of blocks the corresponding covariance matrix. A modified version of the classifier from [7] was used for this purpose.

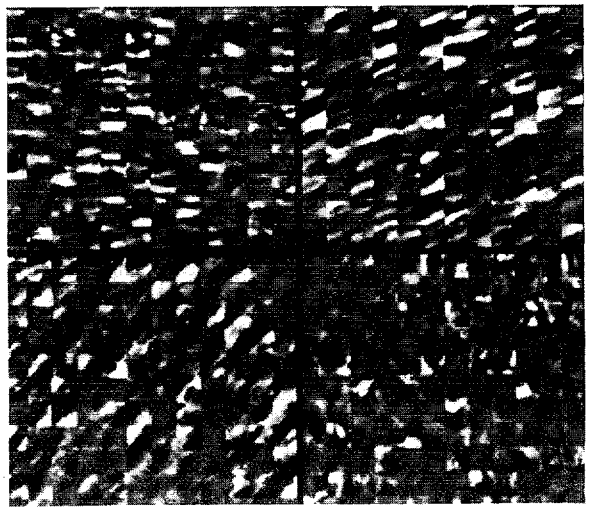

Figure 4: The generated SIRP–blocks for 4 structure classes

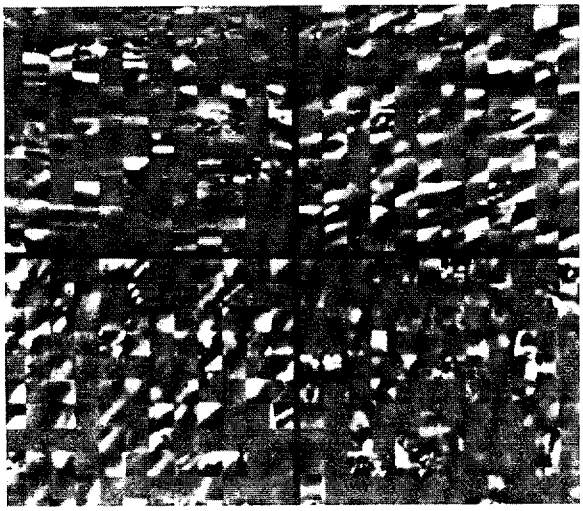

Figure 5: The corresponding real image AC–blocks

The results of the SIRP–generation for the four structure classes are shown in Fig. 4. The figure is partitioned into four quadrants, each containing generated 8×8 SIRP–blocks for a class. Starting from the left top quadrant the classes are arranged clockwise as 0°, 22°, mixture and 45°. For each class the form parameter ν was fitted to the univariate PDF of the real image AC–blocks which the covariance matrix was measured from. ν varies from 0.7 to 0.9 for the four cases of the generation. To verify that these generation results do reflect the real statistics, for each orientation a randomly chosen part of the real image AC–blocks which the model parameters are fitted to is shown in Fig. 5. A comparison shows quite a similar appearance of the model–generated and the corresponding real blocks.

For n–dimensional decorrelated SIRP–blocks $\underline{X} = (x_1, x_2, \ldots, x_n)$ with λ_i as variance of x_i the n–variate PDF

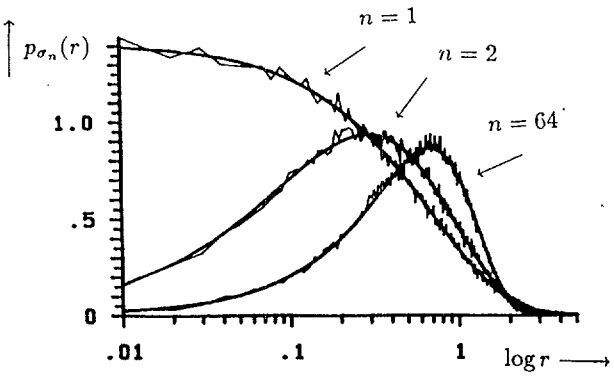

Figure 6: Comparison of the measured and computed radius density $p_{\sigma_n}(r)$ for $\nu = 1.0$

depends only on the normalized radius

$$r = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \frac{x_i^2}{\lambda_i}} \,. \qquad (7$$

x_i are random and so is r. For a given form parameter ν the PDF $p_{\sigma_n}(r)$ of r can numerically be exactly evaluated [2]. But it also can be measured from the generated SIRP–blocks. In Fig. 6 the computed and the measured PDFs of so defined r are compared to each other for $\nu = 1$ and $n = 1$, 2, 64, where the zigzag curves correspond to the measured PDFs. It is seen that the measured PDFs well approach the corresponding PDFs computed based on the SIRP–theory. This again confirms the performance of the generator.

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