DELAY-AND-MULTIPLY DETECTORS FOR SIGNAL INTERCEPTION IN NON-GAUSSIAN NOISE

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RÉSUMÉ

Dans cet article, on traite le problème de la détection d'un signal aléatoire faible noyé dans un bruit non gaussien en utilisant un système de préfiltrage avec retard et multiplication, suivi d'un détecteur de raies spectrales. La déflexion de cette structure d'interception est estimée et maximisée suivant la fonction de transfert de préfiltrage et suivant le temps de retard.

ABSTRACT

The problem of detecting the presence of a weak random signal embedded in additive non-Gaussian noise by a prefilter-delay-and-multiply device followed by a spectral line detector is addressed. The deflection of the considered interception structure is evaluated and optimized with respect to the prefilter transfer function and the delay value.

1. INTRODUCTION

Signal interception, which is attempted for a variety of reasons, such as reconnaissance and surveillance, has recently received a great deal of attention, also owing to the growing use of systems employing direct-sequence and frequency-hopped spread-spectrum signals.

In interception problems, the knowledge of the signal to be detected is limited to a few characteristics, such as frequency band, modulation format, and modulation characteristics (e.g., hop rate, keying rate, etc.). Then, the signal is suitably modeled as a random process.

A possible approach to the detection task for signal interception is based on likelihood ratio testing and leads to locally optimum (LO) (i.e., optimum under low signal-to-noise ratio (SNR) conditions) interceptors whose structure can result quite difficult to implement when a non-Gaussian noise environment is considered. In particular, even if the simplifying assumption of independent noise samples is made, the detection structures include zero-memory nonlinearities that depend on the univariate probability density function (PDF) of the noise [1].

A different approach to the interception problem, which is called feature detection, is based on a scenario in which the signal presence is established according to whether some signal characteristics (features) are detected. Possible features depend on the model adopted for the signal to be intercepted and include zero-crossing rate, power spectral density, carrier frequency, bit rate, etc. In particular, carrier frequency and bit rate have been widely used in interception problems of digitally modulated signals.

Recently [2,3], the problem of detecting the presence of weak binary-phase-shift-keyed (BPSK) signals or BPSK direct-sequence spread-spectrum signals embedded in additive Gaussian noise by means of a prefilter-delay-and-multiply (PFDM) device followed by a spectral line detector has been addressed. In particular, both prefilter response and delay value have been optimized with respect to the criterion of the maximum output SNR. Moreover, the robustness of the interceptor, as measured by the degree of tolerance to errors in pulse rate and carrier frequency, has been investigated.

Although the Gaussian noise model is usually adopted, it is not always justified because there is a wide range of the communication spectrum for which the background noise exhibits highly non-Gaussian characteristics. Therefore, since the synthesis of the LO interception structure in non-Gaussian noise assumes perfect knowledge of at least the univariate noise PDF [1], it is interesting to consider the very simple PFDM device for feature extraction in non-Gaussian noise.

The present paper deals with feature detection for interception of weak signals embedded in additive white non-Gaussian noise. Specifically, the PFDM interception structure is analyzed under the assumption that the signal to be detected is modeled as an arbitrary cyclostationary or almost cyclostationary random process. At first, the deflection of the PFDM detection statistic is evaluated as a function of the prefilter transfer function and the delay value. Then, a necessary and sufficient condition that allows to achieve the maximum deflection is found. Finally, the performance of the PFDM detector optimized against non-Gaussian noise is compared with those of two suboptimum PFDM interception structures.

2. INTERCEPTION STRUCTURE AND DEFLECTION EVALUATION

The interception structure considered here consists of a linear time-invariant filter followed by a delay-and-multiply device that multiplies the filtered received signal by a delayed replica of itself. The output signal is then sent to a spectral line detector which measures the strength of the regenerated spectral line at frequency α. Therefore, under the assumption of large sample size, the decision statistic of the PFDM detector is well approximated by

$$|T| = \left| \int_{-1/2}^{1/2} H(\nu+\alpha/2)H^*(\nu-\alpha/2)F(\nu,\alpha)\cos(2\pi\nu k)d\nu \right|. \tag{1}$$

In (1), the filter transfer function $H(\nu)$ is assumed to be Hermitian, k is the delay value, $*$ denotes complex conjugation, and

$$F(\nu,\alpha) \triangleq N^{-1} \sum_{i,p=1}^{N} r_i r_p e^{-j2\pi[\nu(i-p)+(\alpha/2)(i+p)]}, \tag{2}$$

where N is the sample size and r_i is the ith sample of the received signal. When the signal to be detected is present, $r_i = s_i + n_i$ where s_i is the ith sample of the zero-mean cyclostationary (or almost cyclostationary) real random signal to be intercepted and n_i is the ith noise sample. The random variables n_i are assumed to be mutually independent, identically distributed, and statistically independent of s_i.

In weak-signal interception problems, an appropriate performance measure is the deflection [1,4], which is defined by

$$D(Y) \triangleq \frac{|E_1(Y) - E_0(Y)|^2}{VAR_0(Y)}, \tag{3}$$

where $E_0(\cdot)$, $E_1(\cdot)$, and $VAR_0(\cdot)$ denote the expectations conditioned to the signal absence hypothesis H_0 and the signal presence hypothesis H_1, and the variance under H_0 of the decision variable Y.

To ensure mathematical tractability, one evaluates here the deflection of T instead of that of $|T|$. This renders the resulting deflection formulas less useful than desired since they are based on a statistic that differs from the true detection statistic by the nonlinear operation $|\cdot|$. However, since for $N \gg 1$ the deflection of the statistic $|T|$ is always larger than the deflection of T, the evaluated deflection is a conservative measure of the performance of the PFDM detector [4]. Moreover, it can be easily shown [5] that, for $\alpha \neq 0$, $D(T)$ turns out to be coincident with the output SNR (frequently assumed as performance measure [2,3]) defined as the ratio of the power of the regenerated spectral line at frequency α to the power of the output noise term in a very narrow band centered at frequency α.

The evaluation of the deflection $D(T)$ can be carried out if one assumes that the sample size N satisfies the following conditions:

$$N \gg 1/(\alpha+\beta) \quad \forall\, \beta \neq -\alpha, \tag{4}$$

$$N \gg \max_{\beta}(\tau_\beta), \tag{5}$$

where β represents all cycle frequencies associated with the signal to be intercepted and τ_β is the width of the cyclic autocorrelation function $K_s^\beta(\cdot)$ [6]. On these assumptions, it results that

$$D(T) = \frac{N}{2m_2^2} \frac{\left| \int_{-1/2}^{1/2} S_s^\alpha(\nu)W(\nu)d\nu \right|^2}{\gamma(W)}, \tag{6}$$

where

$$W(\nu) \triangleq H(\nu+\alpha/2)H^*(\nu-\alpha/2)\cos(2\pi\nu k), \tag{7}$$

$$\gamma(W) \triangleq \chi/2 \left| \int_{-1/2}^{1/2} W(\nu)d\nu \right|^2 + \int_{-1/2}^{1/2} |W(\nu)|^2 d\nu, \tag{8}$$

$$\chi \triangleq \frac{m_4}{m_2^2} - 3, \tag{9}$$

$$m_l \triangleq E[n_i^l], \quad l = 2,4\ , \tag{10}$$

and $S_s^\alpha(\cdot)$ denotes the spectral correlation function at frequency α of the signal to be intercepted. Note that $m_4 \geq m_2^2$, which implies $\chi \geq -2$. Moreover, relative to the Gaussian PDF ($\chi = 0$), positive values of χ pertain (apart from pathological distributions) to PDF's that are more peaked, whereas negative values of χ correspond to less peaked PDF's.

In the particular case of Gaussian noise, the maximum-deflection interception structure can be easily obtained applying the Schwarz inequality to (6) [2,3]. This leads to the condition

$$H(\nu+\alpha/2)H^*(\nu-\alpha/2)\cos(2\pi\nu k) = cS_s^\alpha(\nu)^*, \tag{11}$$

where c is an arbitrary constant not equal to zero. By substituting (11) in (1), one obtains the maximum-deflection statistic for the Gaussian noise case:

$$|T_G| = \left| \int_{-1/2}^{1/2} S_s^\alpha(\nu)^* F(\nu,\alpha)d\nu \right|. \tag{12}$$

The deflection of T_G in a Gaussian noise environment results to be

$$D(T_G) = \frac{N}{2m_2^2} \int_{-1/2}^{1/2} |S_s^\alpha(\nu)|^2 d\nu. \tag{13}$$

Moreover, the deflection of T_G in a non-Gaussian noise environment, taking into account (6), turns out to be

$$D^{NG}(T_G) = D(T_G) \frac{\int_{-1/2}^{1/2} |S_s^\alpha(\nu)|^2 d\nu}{\frac{\chi}{2} |K_s^\alpha(0)|^2 + \int_{-1/2}^{1/2} |S_s^\alpha(\nu)|^2 d\nu}. \tag{14}$$

Then, the deflection of T_G in a non-Gaussian noise characterized by $\chi > 0$ is never greater than the maximum value achievable in Gaussian noise. Moreover, when $K_s^\alpha(0) = 0$ both deflections are equal with each other, independently of the noise environment (i.e., for any value of χ).

Finally, note that a general solution to the nonlinear functional equation (11) has not been found. Only some cases, where the delay value and the signaling format are fixed, have been treated [2,3]. For example, for a BPSK signal, when the delay is equal to zero and the bit rate is the feature to be extracted, the solution to (11) leads to the so-called matched-filter squarer.

3. DEFLECTION MAXIMIZATION

The aim of the present section is to find a condition (involving both prefilter transfer function and delay value) assuring the deflection maximization.

From (6)-(8) it follows that $D(T)$ is invariant to scaling (i.e., $D(T) = D(cT), c \neq 0$) so that maximizing $D(T)$ is equivalent to maximizing $Re\{\int_{-1/2}^{1/2} S_s^{\alpha}(\nu) W(\nu) d\nu\}$ ($Re\{\cdot\}$ denotes the real part of the quantity in the brackets) under the constraint that $\gamma(W)$ is equal to a constant. Thus, the optimum $W(\nu)$, say $W_0(\nu)$, is given by

$$W_0(\nu) = \arg\{\max_W \Phi(W)\}, \qquad (15)$$

where

$$\Phi(W) \triangleq Re\{\int_{-1/2}^{1/2} S_s^{\alpha}(\nu) W(\nu) d\nu\} - \lambda \gamma(W) \qquad (16)$$

and λ is a Lagrange multiplier.

Defining $J_W(\epsilon) = \Phi(W + \epsilon\, \delta W)$, where δW is an arbitrary variation in W, a necessary condition for W_0 to solve (15) is

$$J'_{W_0}(0) = 0, \qquad \forall\ \delta W, \qquad (17)$$

where prime denotes the derivative operation. After some manipulations, one has

$$J'_W(0) = \frac{1}{2} \int_{-1/2}^{1/2} \delta W(\nu) \left[S_s^{\alpha}(\nu) - \zeta - 2\lambda W^*(\nu)\right] d\nu, \quad (18)$$

where

$$\zeta \triangleq \lambda\, \chi \int_{-1/2}^{1/2} W^*(\nu) d\nu. \qquad (19)$$

Since $\delta W(\nu)$ is arbitrary, $J'_W(0)$ will be zero if and only if

$$S_s^{\alpha}(\nu) - \zeta - 2\lambda W^*(\nu) = 0, \qquad \nu \epsilon (-1/2, 1/2], \qquad (20)$$

which leads to

$$2\lambda W_0(\nu) = S_s^{\alpha}(\nu)^* - \frac{\chi}{\chi + 2} K_s^{\alpha}(0)^*, \qquad \chi \neq -2. \qquad (21)$$

A sufficient condition for W_0 to maximize the deflection is that $J_{W_0}(\epsilon) \leq J_{W_0}(0)$ for arbitrary δW and ϵ. Since

$$J_{W_0}(\epsilon) = J_{W_0}(0) - \lambda \epsilon^2 \gamma(\delta W) \qquad (22)$$

for all ϵ and δW, (21) with a positive λ is a necessary and sufficient condition for W_0 to maximize the deflection. Note that, as expected, in the Gaussian noise case (21) coincides with (11). Moreover, it is worthwhile to emphasize that if $K_s^{\alpha}(0) = 0$ the PFDM optimized against Gaussian noise retains its optimality properties also in a non-Gaussian noise environment. The condition $K_s^{\alpha}(0) = 0$ is verified, for example, when the signal to be detected is a full-duty-cycle rectangular envelope BPSK signal, and the considered cycle frequency α is an integer multiple of the bit rate or the chip rate for a spread-spectrum signal. Note that, for spread-spectrum signals, the baseband pulse has large excess bandwidth (bandwidth in excess of the minimum bandwidth for zero intersymbol interference) so that it can be usefully approximated in the time domain as a rectangular pulse.

By substituting (21) in (1), it results that the PFDM decision statistic optimized against non-Gaussian noise can be written as

$$|T_{NG}| = \left| T_G - \frac{\chi}{\chi + 2} \frac{K_s^{\alpha}(0)^*}{N} \sum_{i=1}^{N} r_i^2 e^{-j2\pi\alpha i} \right|, \ \chi \neq -2. \qquad (23)$$

In other words, the statistic of the PFDM detector optimized against non-Gaussian noise can be viewed as a modified version of the one optimized against Gaussian noise. Specifically, a term is subtracted to T_G that depends on the signaling format through $K_s^{\alpha}(0)^*$ and on the noise through the second- and fourth-order moments. Note that, unlike the LO interception structure synthesized in [1], the detection statistic (23) does not require knowledge of the univariate noise PDF.

In regard to the deflection of T_{NG}, by substituting the optimum solution $W_0(\nu)$ (given by (21)) into (6) one obtains that

$$D(T_{NG}) = D(T_G) - \frac{N}{2m_2^2} \frac{\chi}{\chi + 2} \mid K_s^{\alpha}(0) \mid^2, \ \chi \neq -2. \ (24)$$

Then, the deflection achievable in a non-Gaussian noise characterized by $\chi > 0$ by using the optimized decision statistic is never greater than the maximum deflection achievable by the decision statistic optimized against Gaussian noise.

From (14) and (24), for $K_s^{\alpha}(0) \neq 0$ and $\chi \neq -2$, it results that

$$\rho \triangleq \frac{D(T_{NG})}{D^{NG}(T_G)} = \frac{2d + \chi}{2d} \frac{2d + \chi(d-1)}{d(\chi + 2)}, \qquad (25)$$

where

$$d \triangleq \frac{\int_{-1/2}^{1/2} \mid S_s^{\alpha}(\nu) \mid^2 d\nu}{\mid K_s^{\alpha}(0) \mid^2} \geq 1 \qquad (26)$$

is the width parameter of the cyclic autocorrelation function $K_s^{\alpha}(\cdot)$.

As a function of d, the ratio ρ reaches the maximum value $\rho_{\max} = (4 + \chi)^2/[8(\chi + 2)]$ for $d = 2$. Moreover, when $d \gg 1$, the advantage that can be achieved by using the PFDM detector optimized against non-Gaussian noise rather than the one optimized under the Gaussian noise assumption is limited (see Fig.1). The deflection of T_{NG} can be significantly superior to that of T_G in highly non-Gaussian noise environments.

Note that $D(T_G)$, $D(T_{NG})$, and $D^{NG}(T_G)$ do not depend on the delay value k, although the filter transfer functions satifying (11) or (21) change as the delay value changes. To examine a detection structure whose performance depends on the delay value, let us now consider the PFDM

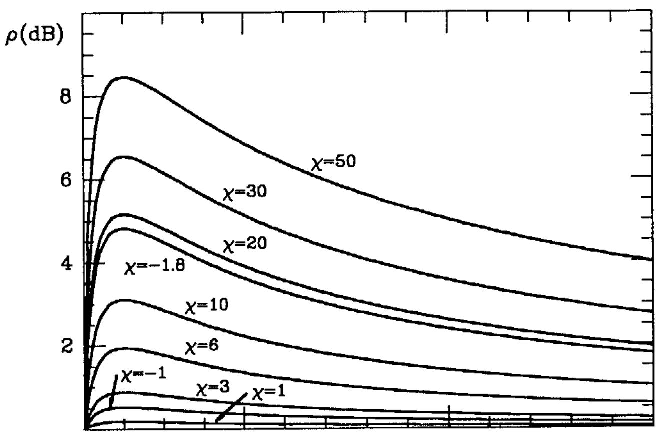

Fig.1

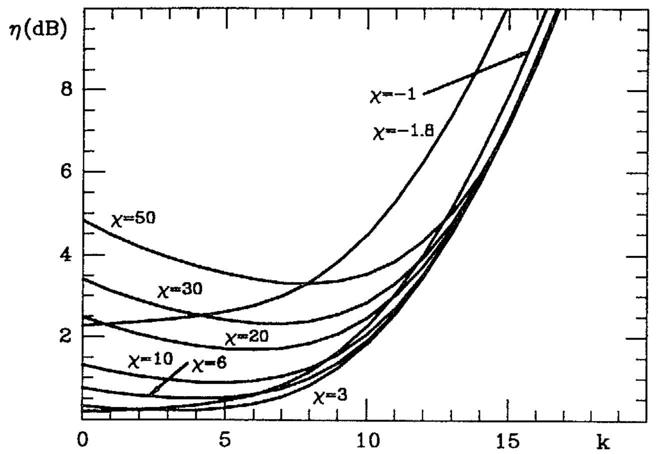

Fig.2

detection structure obtained by adopting as filter transfer function the solution of (11) for $k = 0$. The resulting decision statistic is

$$|T_G^0| = \left| \int_{-1/2}^{1/2} S_s^\alpha(\nu)^* \cos(2\pi\nu k) F(\nu,\alpha) d\nu \right|. \tag{27}$$

Moreover, it results that

$$D^{NG}(T_G^0) = \frac{N}{m_2^2} \frac{[g(k)]^2}{\chi \mid K_s^\alpha(k) \mid^2 + g(0) + g(2k)}, \tag{28}$$

where

$$g(m) \triangleq \int_{-1/2}^{1/2} \mid S_s^\alpha(\nu) \mid^2 \cos(2\pi\nu m) d\nu. \tag{29}$$

Since T_G^0 is the optimum statistic for $k = 0$ and Gaussian noise, the consideration of $k \neq 0$ when $\chi = 0$ is a suboptimum choice. However, if this interception structure operates in a non-Gaussian noise environement, it can exist a delay value, not equal to zero, that maximizes $D(T_G^0)$.

Figure 2 presents the ratio

$$\eta \triangleq \frac{D(T_{NG})}{D^{NG}(T_G^0)} \tag{30}$$

as a function of the delay value k and for different values of the noise parameter χ. Specifically, it refers to the case where the signal to be intercepted is a full-duty-cycle rectangular envelope BPSK signal sampled at rate f_s. The sampling rate has been fixed at $f_s = 16/T_0$, with T_0 the bit duration, and the selected cycle frequency α is twice the carrier frequency. For $k = 0$ it results that $\eta = \rho$, since in such a case $D^{NG}(T_G^0) = D^{NG}(T_G)$. For a fixed $k \neq 0$, when η assumes a value less than the one corresponding to $k = 0$, one obtains that $D^{NG}(T_G^0) > D^{NG}(T_G)$. With regard to this, Fig.2 shows that in highly impulsive noise ($\chi \gg 1$) the choice $k = 8$, that is, a delay equal to one-half of the bit duration, is appropriate.

REFERENCES

[1] L.Izzo, L.Paura, and M.Tanda, "Signal interception in non-Gaussian noise," *IEEE Trans. Commun.*, vol. 40, pp. 1030-1037, June 1992.

[2] J.H. Kuehls and E. Geraniotis, "Presence detection of binary-phase-shift-keyed and direct-sequence spread-spectrum signals using a prefilter-delay-and-multiply device," *IEEE J. Selected Areas Commun.*, vol. 8, pp. 915-933, June 1990.

[3] C.M. Spooner and W.A. Gardner, "Robust signal detection for signal interception," *IEEE Trans. Commun.*, vol. 42, pp. 2165-2173, May 1994.

[4] W.A. Gardner and C.M. Spooner, "Signal interception: Performance advantages of cyclic-feature detectors," *IEEE Trans. Commun.*, vol. 40, pp. 149-159, January 1992.

[5] W.A. Gardner, "Signal interception: A unifying theoretical framework for feature detection," *IEEE Trans. Commun.*, vol. 36, pp. 897-906, August 1988.

[6] W.A. Gardner, "An introduction to cyclostationary signals," in *Cyclostationarity in Communications and Signal Processing,* (W.A. Gardner Ed.), pp. 1-90, New York, IEEE Press, 1994.