A STATISTICAL ESTIMATOR OF TIME DELAY AND DOPPLER SHIFT FROM MULTI-SAMPLED RANDOM SIGNALS

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RÉSUMÉ

L'estimation du retard et du coefficient doppler est un sujet important en plusieurs applications du traitement du signal. L'effet du doppler pour les signaux à bande large peut être regardé comme une compression temporelle. Après une estime primaire basée sur les signaux discrets en temps et doppler, nous considérons ici une approximation parabolique pour l'estimation fine. Les résultats numériques ont été référé à signaux aléatoires gaussiens perturbés par bruits gaussiens independents. La performance obtenu montre les capacités potentielles de cette méthode.

ABSTRACT

Time delay and doppler shift estimation is an important issue in many signal processing areas. Doppler in wide-band signals acts as a time scaling. Starting from a coarse estimate from sampled (both in time and doppler) signals, a parabolic approximation is here employed for fine estimation. The numerical results have been obtained for random Gaussian signals corrupted by independent Gaussian noises. The achieved performance shows the potential capability of such method.

1. INTRODUCTION

Time delay and doppler shift estimation is an important issue in many signal processing areas [1-2]. These include the direction of arrival and trajectory in underwater acoustics, sonar and radar range and speed estimation in a multisensor environment, inter-satellite communications, timing acquisition in a spread spectrum communication system, motion detection and compensation in moving images, stereo vision, etc..

A model for dopplered narrow-band signal consists of a frequency shift of the spectrum [3]. This simple model unfortunately does not apply in the presence of wide-band signals, where doppler is regarded as an instantaneous time scale compression or expansion (or, for duality, an expansion or compression of the frequency scale).

Minimization of the ambiguity function based on generalized cross-correlation allows to find ML-optimal estimates of the unknown parameters [1]. Efficient estimation methods are based on a two-steps algorithm [4]: a *coarse* estimate is obtained from some unambiguous smoothed function; a subsequent *fine* estimate works on a wide-band ambiguity function starting from the coarse estimate. This allows to estimate the absolute minimum of the ambiguity function avoiding the wrong convergence on a relative one. In a recent paper [5], a parabolic approximation was employed for fine (sub-sample) estimation of the time delay from sampled signals. The method is herein extended to doppler estimation.

2. CHOICE OF A SUITABLE MODEL

One preliminary question arises, namely whether it is possible to separate the estimation procedures of time delay and doppler shift. The answer provided by the theory is that one may use two separate estimators if the relative estimation errors are not correlated. In fact, this fact actually depends on the particular model chosen to represent a doppplered signal. Starting from a reference signal s(t) defined for sake of simplicity in the time interval [-W/2,W/2], we have assumed for the scaled, delayed and dopplered received signal r(t) a model [6] for which the above condition applies, i.e.:

$$r(t) = a \cdot s\left(\frac{t-D}{1+F}\right) \quad (1)$$

where D and F are the actual time delay and the doppler velocity, respectively.

Moreover, the time delay error is usually less relevant than the doppler error. In practice [2], 30-60 Nyquist samples are enough for a good estimation performance of time delay, while at least 2000 samples need to be employed to achieve a similar performance for the doppler shift. As a consequence, working on the largest window, we will focus on the estimation of the doppler coefficient.

A very simple and efficient way to obtain dopplered versions of a received signal is to sample it with different rates. For our purposes, only three measurements of the dopplered signal are needed, other than the reference one. The only requirement is that the actual doppler belongs to an interval determined by the lowest and the highest employed sampling rates. This can be implemented by driving the reference rate with some a priori value of the doppler coefficient (for example, obtained by a past measurement), while the other two rates depends on a given maximum of acceleration of the moving object. If no a priori information is available, an alternative scheme uses a parallel grid of samplers, tuned at different speeds. This is equivalent to sample the ambiguity function in the doppler domain.

In practice, after defining A(d,f) as the ambiguity function of time delay (d) and doppler shift (f), we must estimate it on a proper discretized grid in the {d,f} space. The coarse estimate is implemented by searching for the minimum value of $A(d_i,f_j)$, say $A(d_I,f_J)$. In order to find a fine (sub-sample) estimate of (D,F), we interpolate A(d,f) around $A(d_I,f_J)$ by a two-dimensional Taylor expansion after retaining the only terms up to the second order. Since the estimation errors are not correlated for the assumed model, such interpolation reduces to two separable one-dimensional ones. In other words, two distinct parabolic interpolations, based on other four measurements $A(d_i,f_j)$ placed in a cross around $A(d_I,f_J)$, need for estimating time delay and doppler shift:

$$d = d_I - \frac{\Delta_d}{2} \cdot \frac{A(d_I+\Delta_d, f_J) - A(d_I-\Delta_d, f_J)}{A(d_I+\Delta_d, f_J) - 2A(d_I, f_J) + A(d_I-\Delta_d, f_J)} \quad (2)$$

$$f = f_J - \frac{\Delta_f}{2} \cdot \frac{A(d_I, f_J+\Delta_f) - A(d_I, f_J-\Delta_f)}{A(d_I, f_J+\Delta_f) - 2A(d_I, f_J) + A(d_I, f_J-\Delta_f)} \quad (3)$$

where Δ_d and Δ_f are the difference between two subsequent values of the quantized parameters d_i and f_j.

3. DISCUSSION OF NUMERICAL RESULTS

The numerical results have been obtained for random Gaussian signals with a Gaussian-shaped auto-correlation function, i.e.:

$$R_{ss}(\tau) = e^{-\frac{\tau^2}{2a^2}} \quad (4)$$

corrupted by uncorrelated Gaussian white noises with several Signal-to-Noise Ratios (SNRs).

We assume to know the reference signal $x(t)=s(t)+n_1(t)$ and a given number of delayed and dopplered versions of the received signal $y(t;d_i,f_j)=r[(1+f_j)t+d_i]+n_2(t)$.

The doppler compensation can be implemented at low cost by sampling the received signal at several rates $(1+f_j)T$. In practice, while we know the maximum value of acceleration of the object, we can state the number of the rates to be considered. On the other hand, we can use only three rates by chosing the current speed estimate and the two limit future ones.

Different discrete-time correlators have been used for estimating the sampled ambiguity function $A(d_i,f_j)$. For our purposes, three discrete-time cost functions have been used. The first one seeks to maximize the direct cross-correlation:

$$D_{xy}(d_i,f_j) = \frac{1}{N}\sum_{k=1}^{N} x(kT)\, y(kT;d_i,f_j) \quad (5)$$

while the ASDF method [5] minimizes:

$$S_{xy}(d_i,f_j) = \frac{1}{N}\sum_{k=1}^{N} [x(kT)-y(kT;d_i,f_j)]^2 \quad (6)$$

as far as the digitally faster AMDF method [5]:

$$M^{1/2}_{xy}(d_i,f_j) = \frac{1}{N}\sum_{k=1}^{N} | x(kT)-y(kT;d_i,f_j)| \quad (7)$$

In practical applications, the available a priori information on timing and speed may be very different. In fact, in the presence of a good prediction, we can make the estimation by multi-sampling the received signal with a central value of the timing and doppler coefficient (d_I,f_J) very close to their actual values.

Conversely, if the movement of the object is unpredictable, we do not know where the sampling grid should be placed. The achievable performance is then a periodical function of the distance from the ideal sampling rate of the closest parameter values in the actual sampling grid.

As a consequence, both the two limit situations of perfect and wrong sampling have been analysed, namely the ideal case of perfect sampling and an intermediate situation of wrong sampling.

The practical example considered in our analysis refers (just like in [2]) to an underwater object moving with a radial speed of 7 knots (corresponding to a time scaling factor on the order of $F=2.4\ 10^{-3}$) and uses an observation time of 0.1 sec to obtain the timing and doppler estimates. The central sampling time is $T=5\ 10^{-5}$ sec, while the autocorrelation standard deviation in eq. (4) has been assumed $a=T$. The sampling grid resolution is one sampling period for the timing and 2 knots for the speed.

In particular, since the bias is strongly dependent on the autocorrelation assumed for the signals, the variances of the speed estimator (3) using the three discrete-time correlators (5)-(7) has been evaluated by 1000 independent runs of computer simulations and is reported in the figures versus the SNR in the range [0,40] dB. They refer to the case of perfect sampling (Fig. 1) and an intermediate wrong sampling (viz: 0.25 T of delay and 0.5 knots of speed) for the actual parameter values (Fig. 2).

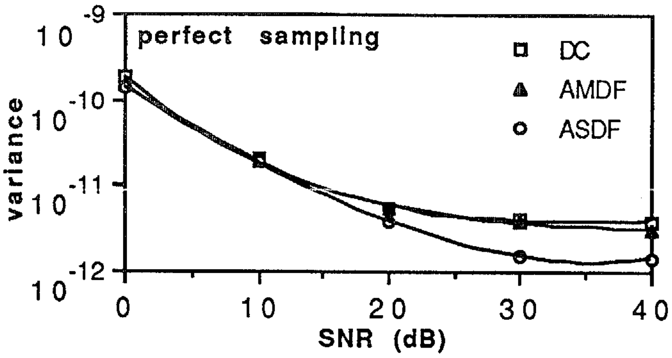

Fig.1. Variance of the doppler ($F=2.4\ 10^{-3}$) estimates in the ideal case of a perfect sampling of the two-dimensional ambiguity function.

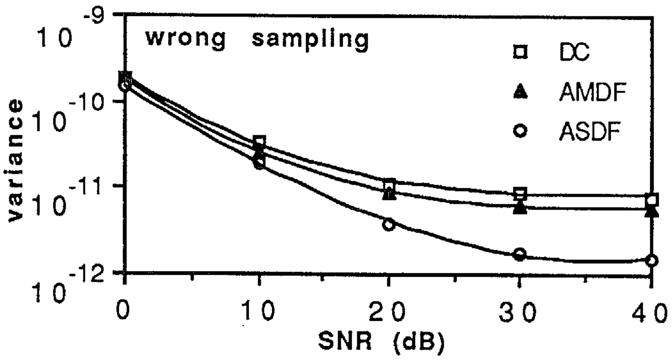

Fig.2. Variance of the doppler ($F=2.4\ 10^{-3}$) estimates in the intermediate case of wrong sampling of the two-dimensional ambiguity function.

The achieved performance shows the potential capability of such an open-loop algorithm and suggests useful guidelines for further investigations.

References

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