New MDL based criterion for number of sources determination Application to Synthetic Aperture Radar Imaging

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RESUME

Dans cet article, nous développons un nouveau critère de détection du nombre d'exponentielles complexes en utilisant des principes issus de la théorie de l'information pour la sélection de modèles. Après le développement de ce critère et son extension au cas bi-dimensionnel, nous présentons des résultats de simulation et une application sur des images radar réelles.

ABSTRACT

In this paper, following the information theoretic approach to model selection, we develop a new criterion for number of complex sinusoids detection. After the derivation of this criterion and its extension to two-dimensional case we present some simulation results. Finally we apply our criterion to real radar image.

Introduction

Recently, many methods have been developped in high resolution spectral analysis. Most of them need the knowledge of the number of components in order to make an accurate frequency localisation. Unfortunately, the estimation of this number is not easy when the magnitudes of the sinusoids are not homogeneous, when their frequencies are very close and when the signal to noise ratio is low.
Some criteria [1], [2], [3], [4] has been developped for estimating this number. In the first part of this paper, we derive a new criterion based on the MDL (Minimum Description Length) principle [5] where we consider the probability density function of modified signal eigenvectors. Then, in order to use our criterion in synthetic aperture radar imaging, we propose an extension to the two-dimensional case. We analyse some characteristics of this criterion like frequency resolution and compare it with the most used criterion, MDL. Finally we propose an application of our criterion to radar images.

Problem formulation

Let us consider the sinusoïds-in-noise model :

$$y(n) = \sum_{i=1}^{P} a_i \exp(j2\pi f_i n + j\phi_i) + v(n)$$

where ϕ_i, the initial phase, is supposed to be uniformly distributed in the interval $[0,2\pi]$.

Let the observation vectors $\underline{y}(n)$ be defined as $\underline{y}(n) = [y(n)\cdots y(n+L-1)]^T$, let N be the number of snapshots.

The assumptions made here are the following :

i) v(n) is a cicular ergodic Gaussian process with zero mean and covariance matrix $\sigma_v^2 I$ where I is the identity matrix.
ii) $L \geq M$.

The eigendecomposition of the exact covariance matrix R is :

$$R = E\left[\underline{y}(n)\underline{y}^*(n)\right] = UDU^*$$

where U is an unitary matrix whose colums are the eigenvectors of R, D is a diagonal matrix whose diagonal elements are the eigenvalues of R assumed to be arranged in descending order such that :

$$D = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_L), \text{ and}$$
$$\lambda_1 > \lambda_2 > \ldots > \lambda_p > \lambda_{p+1} = \ldots = \lambda_L = \sigma_v^2$$

The eigendecomposition of the sample covariance matrix is given by :

$$\hat{R} = \frac{1}{N-L+1} YY^* = \hat{U}\hat{D}\hat{U}^*$$

where Y is the hankel data matrix, $\hat{U}$ is a unitary matrix and $\hat{D}$ is a diagonal matrix constructed with the estimated eigenvalues ordered as follows :

$$\hat{\lambda}_1 > \hat{\lambda}_2 > \ldots > \hat{\lambda}_p > \hat{\lambda}_{p+1} > \ldots > \hat{\lambda}_L$$

This decomposition allows to extract two orthogonal subspaces, U_S, the signal one, spanned by the signal eigenvectors $\{u_i\}_{1 \le i \le p}$ and U_n, the noise one, spanned by the noise eigenvectors $\{u_i\}_{p+1 \le i \le L}$. Due to the limited set of data, we can only obtain an estimation of these two subspaces.

$$\hat{R} = \hat{U}_S\hat{D}_S\hat{U}_S^* + \hat{U}_n\hat{D}_n\hat{U}_n^*$$

The new criterion MLM

The new criterion is based on the MDL principle and has the following form:

$$C = -\mathrm{Log}(f(r/\hat{\theta})) + \alpha(N-L+1)$$

The function f is the probability density function of the variable r. $\hat{\theta}$ is the maximum likelihood estimator of the unknown parameter vector θ. α(N-L+1) is a penalty function depending on the number of samples and on the free parameters. This expression is the starting point for the derivation of MDL criteria derived by Wax [1] and by Reddy [4].

Let us consider the transformed signal eigenvectors r_i introduced by Viberg, Ottersten & Kailath [6] and defined by the expression:

$$r_i = Q^* \hat{u}_i$$

where Q verifies:

$$P_S^{\oplus} = QQ^* \quad \text{and} \quad Q^*Q = I$$

The matrix $P_S^{\oplus}$ is the orthogonal projection operator based on the Vandermonde matrix:

$$P_S^{\oplus} = (I - S(S^*S)^{-1}S^*)$$

Let us consider in the asymptotic distribution (pdf) of these vectors. We can show that the vectors r_i are independant and also asymptotically gaussian distributed. The mean and the variance of the transformed vectors are:

$$E\{r_i\} = 0 + O(N^{-1})$$

$$C_{ij} = E\{r_i r_j^*\} = Q^* E\{\hat{u}_i \hat{u}_j^*\} Q = \delta_{ij} \frac{D_i}{N-L+1} I + O(N^{-1})$$

where δ_{ij} is the kronecker operator,

$$D_i = \frac{\lambda_i \sigma_v^2}{(\lambda_i - \sigma_v^2)^2}$$

and we have used the fact that: $Q^* U_N U_N^* Q = I$.

Let k be the estimated number of components, then we obtain the pdf of the transformed vectors:

$$f(r_1, \ldots, r_k | \hat{u}_1, \ldots, \hat{u}_k)$$
$$= (\pi)^{-k(L-k)} (\prod_{i=1}^{k} \det(C_{ii}))^{-1} \exp(-\sum_{i=1}^{k} r_i^* C_{ii}^{-1} r_i)$$

The penalty function is given by:

$$\alpha(N-L+1) = kL\log(N-L+1)$$

According to these results, the function MLM is given by the following expression:

$$\boxed{\begin{aligned} MLM(k) = {} & k(L-k)\log(\pi) + (L-k)\log(\prod_{i=1}^{k} D_i) \\ & + k^2 \log(N-L+1) \end{aligned}}$$

Then, the estimated number of sinusoids $\hat{p}$ is the following one :

$$\hat{p} = \arg\min_k(MLM(k))$$

Extension to two-dimensional case

In many applications, such as synthetic aperture radar imaging, it is often desired to estimate two-dimensional (2D) frequencies from a 2D data set. Like in 1D case, if the data set is relatively small the classical correlogram method can't be satisfactory. So 2D high resolution techniques has been developped. These techniques need an estimation of the number of 2D sinusoids. For this reason, we present now an extension of our MLM criterion to the 2D case.

We consider the following model which consists of p 2D sinusoids:

$$y(m,n) = \sum_{i=1}^{p} a_i \exp(j2\pi(f_{1i}m + f_{2i}n) + j\phi_i) + v(m,n)$$

where $0 \le m \le M-1$ and $0 \le n \le N-1$, v(m,n) is a 2D noise sequence assuming to be white, zero man, gaussian, circular with variance σ_v^2.

Let us consider the samples covariance matrix:

$$\hat{R} = \frac{1}{(M-L+1)(N-K+1)} YY^*$$

$$\text{where } Y = \begin{bmatrix} Y_0 & Y_1 & \cdots & Y_{M-K} \\ Y_1 & Y_2 & \cdots & Y_{M-K+1} \\ & & \cdots & \\ Y_{K-1} & Y_K & \cdots & Y_{M-1} \end{bmatrix} \text{ and}$$

$$Y_m = \begin{bmatrix} y(m,0) & y(m,1) & \cdots & y(m,N-L) \\ y(m,1) & y(m,2) & \cdots & y(m,N-L+1) \\ & & \cdots & \\ y(m,L-1) & y(m,L) & \cdots & y(m,N-1) \end{bmatrix}$$

with $L \ge p$ and $K \ge p$.

Let us call $\{\lambda_i\}_{1 \le i \le L*K}$ the eigenvalues of $\hat{R}$ arranged in descending order. According to these notations, the MLM2D criterion has the following form:

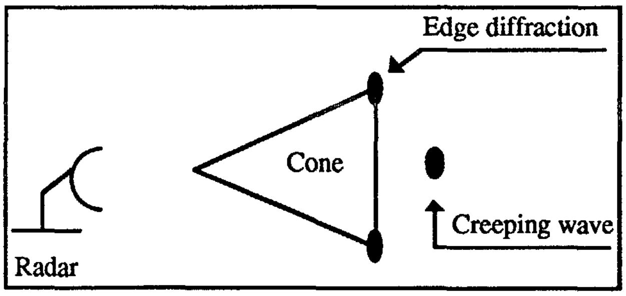
$$MLM2D = k(LK-k)\log(\pi) + (LK-k)\log\left(\prod_{i=1}^{k} D_i\right) + k^2\log((M-K+1)(N-L+1))$$

Simulations and application to radar imaging

In this section we present simulation results and a application of MLM on a real radar signal.

-*Simulation results*: To demonstrate the performances of our MLM criterion, we evaluate its frequency resolution and compare it to MDL one. The simulations conditions are listed in Table n°1. MLM results are plotted on fig.1, MDL one on fig.2. We note that the detection zone (ie the zone where the detection rate is upper than 50%) is the same for the two criteria. However MLM has a better stability in this zone.

- *Application on real radar signal:* The target is a metallic cone represented below. Its radar signature, is mainly composed of three major points contributing to the Radar Cross Section (RCS): two edge diffractions and a creeping wave.



The 2D FFT is plotted on fig.3. We note that it can't separate the three contributors. So it's necessary using high resolution methods. Before using these methods, we estimate the number of contributors by MLM2D. **The result given by MLM is 3, which is the theoretical order for such a target.** Results given by 2D MUSIC are plotted on fig.4.

Conclusion

In this paper, we present a new MDL based criterion to estimate the number of sinusoids. We apply this principle to the transformed eigenvectors introduced by Viberg [6]. Then, in order to use our criterion in synthetic aperture radar imaging, we propose a 2D extension. The empiric detection performances of our criterion are compared with Wax's criterion ones. We note that the stability of MLM is better than MDL one. Finally we present a real radar image for which the result given by MLM is good. However, for more complicated targets for which contributors may be very close and with a very large dynamic, it will be very difficult to easily use such criteria.

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number of observation vectors (*N*) : 30	size of the covariance matrix (*L*): 30
number of sinusoïds : 3	monte carlo runs : 100
magnitudes : *a(1)=a(2)=a(3)=1*	SNR : 0-30 dB by step of 2,5 dB
frequencies : f1= 0.2 , f2=0.3 and f3=0.2+df by step of 1/10	

Table n°1: Conditions of simulations

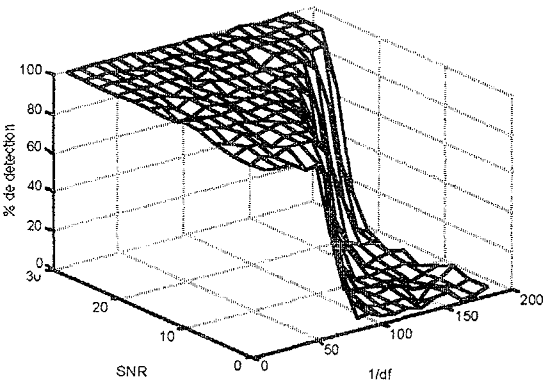

Figure n°1 : MLM

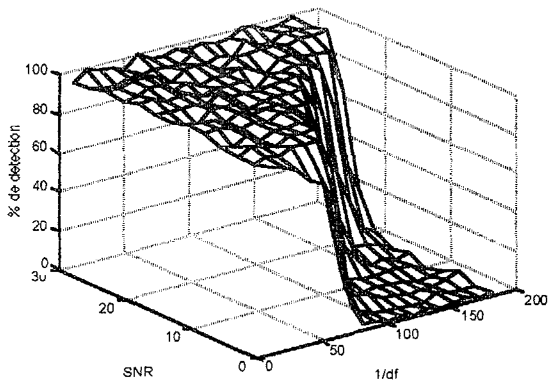

Figure n°2 : MDL

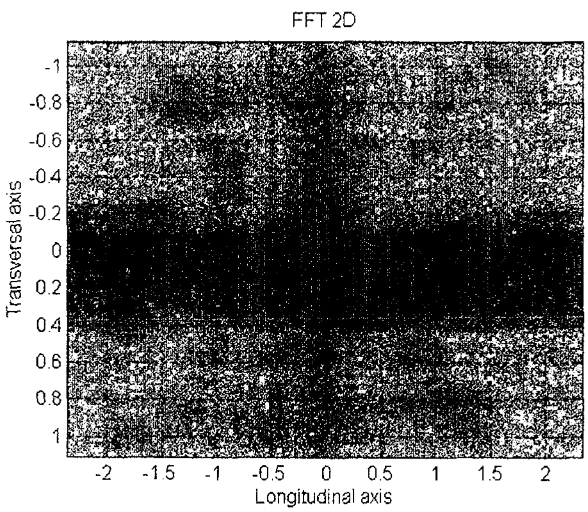

Figure n°3: Radar signature of a metallic cone with the 2D FFT method.

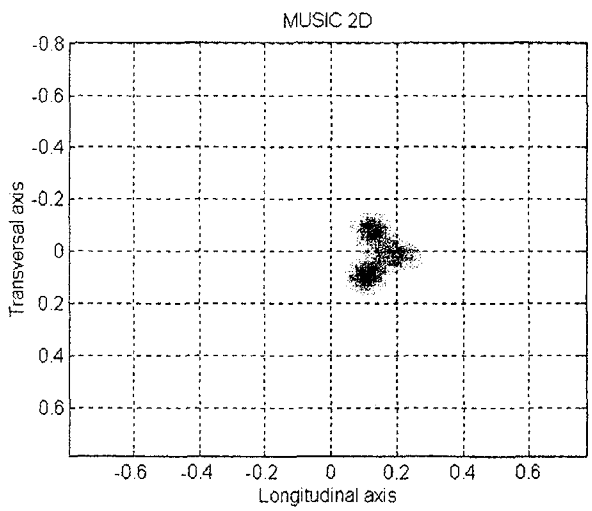

Figure n°4 : Radar signature of a metallic cone with the 2D MUSIC method.