SPACE–TIME PROCESSING FOR DETECTION OF RANGE–SPREAD TARGETS IN NON-GAUSSIAN ENVIRONMENT

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RÉSUMÉ

Dans cet article nous abordons le probleme de la synthèse de système TFAC pour la détection de cibles noyés dans un fouillis non-gaussienne. Nous présenton un système génerale de seuillage adaptatif, dans l'hypothèse que la loi de répartition du fouillis est du type "location-scale". La strategie utilisée est l'etablissement d'une carte de fouillis après le filtrage spatial de un certain nombre de cellules de résolution. Le calcule des performances démontre que le recepteur est tres robuste aussi pour cibles étendu.

ABSTRACT

This paper faces the problem of CFAR detection of possibly range–spread targets in non–Gaussian disturbance. The proposed system relies on a combination of space and time processing so as to ensure CFAR against clutter depending on two parameters without incurring masking effect from slow, extended and multiple targets. A performance analysis is presented for the case of Weibull clutter so as to give guidelines for system design.

I. Introduction

An increasing attention has been devoted in the last decade to the development of radar system achieving improved performance in non Gaussian environment. This, in turn, poses two problems: the modelization of the stochastic return from the additive disturbance and the development of advanced processing schemes for detection in the presence of a given disturbance model. Moreover, systems often operate in changeable environments, where the cumulative distribution function of the noise is modeled as a member of a multiparametric family with a-priori unknown parameters.

Several procedures have been proposed recently to maintain a Constant False Alarm Rate (CFAR) in the presence of non–Gaussian, time-varying environments. The Sliding–Window (SW) based procedures achieve estimates of the clutter parameters by processing a set of samples from adjacent range cells; in this class we find either monoparametric [1,2] or biparametric [3,4] procedures, suited for adaptive detection in non-Gaussian environment. The main drawback of these procedures is that the system performance impairs in the presence of abrupt variations in the statistical properties of the returns across the reference set.

On a different concept rely the Clutter–Map (CM) based CFAR procedures, wherein the estimate of the relevant distributional parameters is performed by suitably processing the returns from several range cells, as observed in all the scans up to the current one. Intuitively, one expects that CM–CFARs, suffer from range–spread targets or from point targets persisting on the same map cell for several scans [5]. This situation tipically produces an overestimation of the detection threshold and, consequently, a decreased detection probability (masking effect) [6].

We propose here a new biparametric CFAR procedure based on a hybrid space-time processing; CFAR is achieved whenever the amplitude probability density function (apdf) of the clutter returns, possibly upon a parameter-independent transformation, is of the Location–Scale (LS) type. The procedure is applicable to a large class of input distributions and proper setting of the system parameters may confer to the detector robustness against non-homogeneities in the estimation sample.

II. Achieving CFAR against LS-distributions

When the clutter is assumed to depend on two unknown parameters, α and β say, the detection rule can be expressed in the form

$$Y \underset{H_0}{\overset{H_1}{\gtrless}} \widehat{T}(\widehat{\alpha},\widehat{\beta}) \tag{1}$$

where Y is the envelope of the return from the range cell being tested and $\widehat{T}(\widehat{\alpha},\widehat{\beta})$, is a suitable adaptation law, fulfilling the condition

$$\Pr\left\{Y > \widehat{T}(\widehat{\alpha},\widehat{\beta})|H_0\right\} = P_{FA} \tag{2}$$

regardless the values of the parameters α and β.

The problem of finding the estimators of the distributional parameters α and β satisfying the above condition admits a general solution if the clutter apdf is assumed to be of the LS type. The definition of the LS property can be given as follows: A family of random variables Y is said to be of LS type with location parameter $\theta_L \in \mathrm{R}$ and scale parameter $\theta_S > 0$, if any variate in the family can be obtained by linearly transforming a *generating* - variate Y_0, defined as the member with $\theta_L = 0$ and $\theta_S = 1$, namely if $Y = \theta_S Y_0 + \theta_L$. In the following we adopt the shorthand notation $Y \sim \mathrm{LS}(\theta_L, \theta_S)$ to indicate that Y is of LS type with parameters

θ_L and θ_S. In the derivation of the CFAR detector we will exploit some properties of the LS distributions; we report here the most relevant:

P1 Ranking
If Y_i $i = 1, \ldots N$ is a set of N, identically distributed $\mathrm{LS}(\theta_L, \theta_S)$ variates, then the ranked observations $Y_{(1)}, \ldots, Y_{(N)}$, with $Y_{(i-1)} \leq Y_{(i)}$ $\forall i > 1$ are themselves $\mathrm{LS}(\theta_L, \theta_S)$.

P2 Closure under linear transformations
Linearly filtering a sequence $y(n) \sim \mathrm{LS}(\theta_L, \theta_S)$, with one and the same marginal pdf, yields a sequence $v(n) \sim \mathrm{LS}(H(0)\theta_L, \theta_S)$ of identically distributed variates, with $H(0)$ the dc gain of the filter.

P3 Quantiles
Let $Y \sim \mathrm{LS}(\theta_L, \theta_S)$, then its α-quantile Y_α, namely the solution to the equation $\Pr\{Y \leq Y_\alpha\} = \alpha$ can be represented as $Y_\alpha = \theta_S Y_{0\alpha} + \theta_L$ where $Y_{0\alpha}$ denotes the α-quantile of the standard distribution.

The expression for Y_α suggests adopting the structure

$$\widehat{T}(n) = \widehat{\theta}_S[z(n)]\gamma + \widehat{\theta}_L[z(n)] \tag{3}$$

for the adaptive threshold. Consequently, the actual FAR can be written as

$$P_{FA} = \Pr\left\{\frac{Y - \widehat{\theta}_L[z(n)]}{\widehat{\theta}_S[z(n)]} > \gamma | H_0\right\} \tag{4}$$

which implies that CFAR is ensured if the test statistic

$$\frac{Y - \widehat{\theta}_L[z(n)]}{\widehat{\theta}_S[z(n)]} \tag{5}$$

is *ancillary* under H_0, namely if its distribution is independent of the values of the parameters. Such a property is achieved if the estimates of the location and scale parameters are *equivariant*, namely if for any $c \in [0, \infty[$ and $d \in]-\infty, \infty[$, the equations

$$\begin{aligned} \widehat{\theta}_L[cz(n) + d] &= c\widehat{\theta}_L[z(n)] + d \\ \widehat{\theta}_S[cz(n) + d] &= c\widehat{\theta}_S[z(n)] \end{aligned} \tag{6}$$

hold. Based on (6), we have

$$\frac{Y - \widehat{\theta}_L[z(n)]}{\widehat{\theta}_S[z(n)]} = \frac{Y_0 - \widehat{\theta}_L[z_0(n)]}{\widehat{\theta}_S[z_0(n)]} \tag{7}$$

indicating that the statistic (5) is one and the same, whether it is computed based on the original data $(Y, z(n))$ or on the normalized data $(Y_0, z_0(n))$.
As outlined in the previous section, the resilience to masking effect from slow targets can be achieved by properly processing the samples in the map-cell. It is understood that range-spread targets affect the homogeneity of the estimation sample; on the other hand, for high signal-to-clutter ratios (as it is the case for coastal radars where the target radar cross section is typically very large) the samples containing echoes from the target exceed the samples from clear cells with high probability. Thus, some resilience against self-masking from range-spread targets is expected

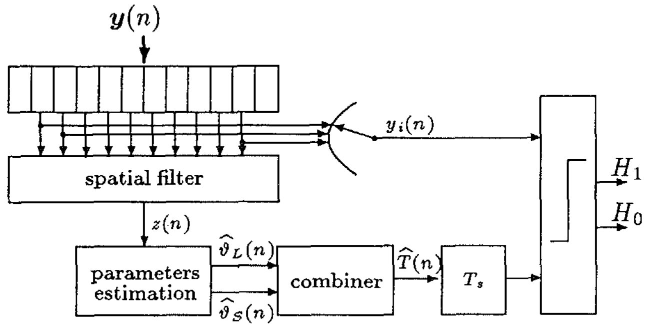

Figure 1. General architecture of a CM-CFAR.

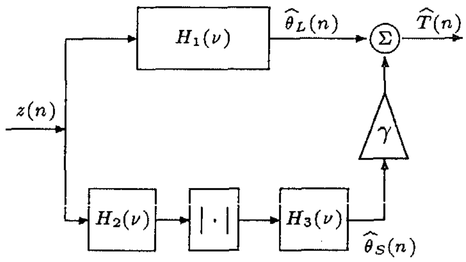

Figure 2. Equivariant estimation of the location and scale parameters

if preliminary observation censoring is performed, namely if the top-rank statistics are discarded prior to estimating the adaptive threshold.
Summing up, the general architecture of a CM-CFAR system against clutter with LS-distribution is depicted in Figure 1 where $\boldsymbol{y}(n)$ is the N-dimensional set of returns forming the map cell and $z(n) = y_{(k)}(n)$ is the k-th order statistic at the nth scan. The estimation block and the combiner are described with more details in Figure 2. With reference to such a figure, it is possible to demonstrate that the upper branch provides an equivariant estimate of the location parameter if $H_1(0) = 1$ while the lower branch provides an equivariant estimate of the scale parameter if $H_2(0) = 0$. In our design $H_1(\nu)$ is a single-pole, single-zero lowpass IIR filter, $H_2(\nu)$ is a backward difference and $H_3(\nu)$ is a single-pole lowpass AR filter. If the poles of $H_1(\nu)$ and $H_3(\nu)$ are located at the same point, a say, it is possible to demonstrate that the variance of the estimators vanishes as $a \to 1$, i.e. that the system achieves a perfect estimate of the noise parameters under this limiting situation.

III. Example: CM-CFAR against Weibull clutter

A widely accepted model for the clutter apdf relies to the Weibull family of distributions, whose cdf is

$$F_{X|H_0}(x) = 1 - \exp\left[-\left(\frac{x}{\alpha}\right)^\beta\right] \quad x \geq 0,\ \alpha > 0,\ \beta > 0 \tag{8}$$

which is not of a LS type. However, if the received variate undergoes a logarithmic transformation, under H_0, the Weibull distribution is converted into a Gumbel variate, whose CDF is

$$F_{Y|H_0}(y) = 1 - \exp\{-\exp[\beta(y - \ln\alpha)]\} \tag{9}$$

which is $\mathrm{LS}(\ln\alpha, 1/\beta)$.

3.1 FAR regulation

The threshold multiplier γ ensuring a preassigned P_{FA} is implicitely defined by the equation

$$\Pr\left\{\frac{y_i(n) - \widehat{\theta}_L(n-1)}{\widehat{\theta}_S(n-1)} > \gamma | H_0\right\} = P_{FA} \qquad (10)$$

whose solution requires the first-order characterization of the normalized test statistic

$$Y(n) = \frac{y_i(n) - \widehat{\theta}_L(n-1)}{\widehat{\theta}_S(n-1)} \ . \qquad (11)$$

Since $Y(n)$ is ancillary under H_0, γ is one and the same independent of the values of the location and scale parameters of $y_i(n)$ and, eventually, of the scale and the shape parameters of the clutter apdf. Consequently, with no loss of generality, we can assume $\theta_L = 0$ and $\theta_S = 1$, implying $\alpha = \beta = 1$. The analytical evaluation of the threshold multiplier turns out to be mathematically unwieldy; thus we present here a technique for approximating the high-amplitude tail of the pdf of the test statistic by a suitable function depending on some parameters. Assume, at first, that perfect estimates of the location and scale parameters are available; in this case the estimated parameters coincides with their expectations $E[\widehat{\theta}_L] = \mu_{0k}$ and $E[\widehat{\theta}_S] = \eta_{0k}$ and we have

$$1 - F_Y(x) = \exp(-e^{\eta_{0k}x + \mu_{0k}}) \ . \qquad (12)$$

However, equation (12) applies only in the limiting case of $a \to 1$, for intermediate situations we can exploit the limiting form of (12)

$$\exp(-e^{\eta_{0k}x+\mu_{0k}}) = \lim_{N_e \to \infty}\left[1 + \frac{1}{N_e}\left(1 + \frac{\eta_{0k}x + \mu_{0k}}{N_e}\right)^{N_e}\right]^{-N_e} \qquad (13)$$

where N_e is defined as $N_e = (1+a)/(1-a)$, as a hint for envisaging the approximating family. Consequently, we assume

$$\mathcal{F}(x) = \frac{1}{\left[1 + \frac{1}{\nu N_e}\left(1 + \frac{\eta_{0k}x+\delta}{\nu N_e}\right)^{\nu N_e}\right]^{\nu N_e}} \qquad x >> 0 \ . \qquad (14)$$

In the equation above $\delta = \delta(N_e, k)$ and $\nu = \nu(N_e, k)$ are suitable parameters to be inferred so as to minimize a given measure of discrepancy between the true and the approximating distribution. An example of optimization procedure relies on the following steps:

- Numerical inversion of (14) and choice of an interval P_{FA1}, P_{FA2} such that the estimation, via Monte-Carlo counting, of the corresponding threshold multiplier requires an acceptable number of trials;
- Determination, via computer simulations, of the true threshold multipliers, namely of the $(1 - P_{FA})$–quantiles of the empirical distributions, for $P_{FA1} \le P_{FA} \le P_{FA2}$;
- Selection of ν and δ so as to minimize the mean square error between the quantiles of the empirical and the approximating distributions in the above interval of P_{FA}.

The applicability of this procedure for extrapolation purposes, is demonstrated in Figure 3 wherein the empirical

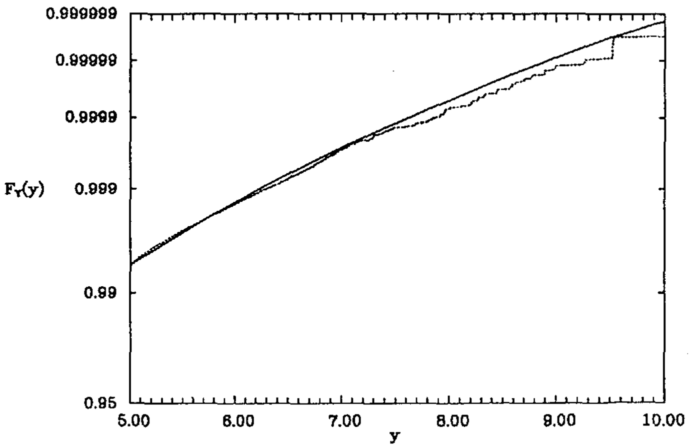

Figure 3. High-amplitude tail of the empirical and of the approximating CDF for Weibull background; $N = 12$, $k = 7$, $a = 0.9$.

CDF of the test statistic, evaluated for $N = 12$ and $k = 7$ is contrasted with the corresponding approximator: results show that the threshold multipliers may be computed based on (14) at a good confidence level.

3.2 Detection performance for point-like targets

Assume that all of the samples in the map cell, up to the $(n-1)$th scan, have one and the same distribution; at the nth scan, a useful target whose amplitude A is assumed to fluctuate according to a Rayleigh pdf, i.e.:

$$f_A(x) = \frac{x}{\sigma_A^2}\exp\left(-\frac{x^2}{2\sigma_A^2}\right), \quad x \ge 0 \qquad (15)$$

enters the map cell, occupying one of the range cells therein. The detection probability is defined as

$$P_D = \Pr\left\{y_i(n) > \widehat{T}(n-1) | H_1\right\} \qquad (16)$$

with $y_i(n)$ the clutter-plus-signal envelope, as observed at the output of the logarithmic amplifier. Exact evaluation of the above probability requires the knowledge of the first-order pdfs of both $y_i(n)$ and $\widehat{T}(n-1)$ whose closed-form expression is not available. Thus, we resorted to computer simulation for performance evaluation. Detection performance under Weibull distributed clutter are shown in Figure 4 versus the signal-to-noise ratio

$$\rho = \frac{2\sigma_A^2}{\alpha^2\Gamma(1+2/\beta)} \qquad (17)$$

assuming that the 7th Order Statistic ($k = 7$) is singled out and used for estimation purposes. We accounted for the two cases of $\beta = 1$ and $\beta = 2$ in order to elicit the effect of the clutter spikyness: it can be easily noticed that higher tails of the noise distribution cause worse performance. The curves are indexed through the parameter K_s – the number of persistence scans of the target –; the performance of the fixed-threshold detector is also reported for comparison purposes. Figure show that the CFAR loss is not very sensitive to K_s, an evidence that the system is practically immune to masking effect from slow targets.

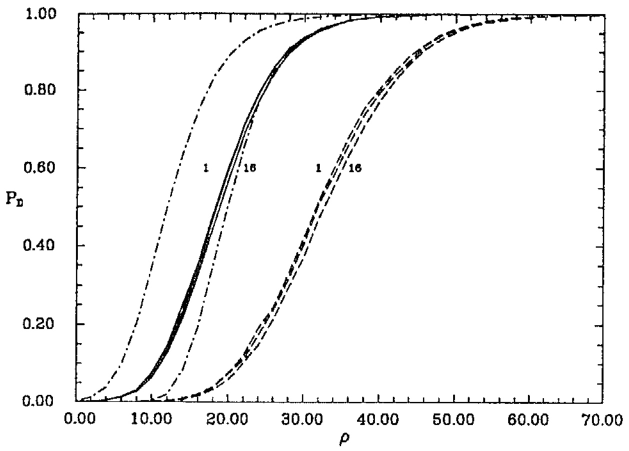

Figure 4. Detection performance of the CM-CFAR for point-like targets in Weibull background:
$N = 12$, $P_{FA} = 10^{-5}$, K_s=1,4,16, $a = 0.9$, $k = 7$;
solid: $\beta = 2$; dashed: $\beta = 1$; dot-dash: fixed-threshold detector

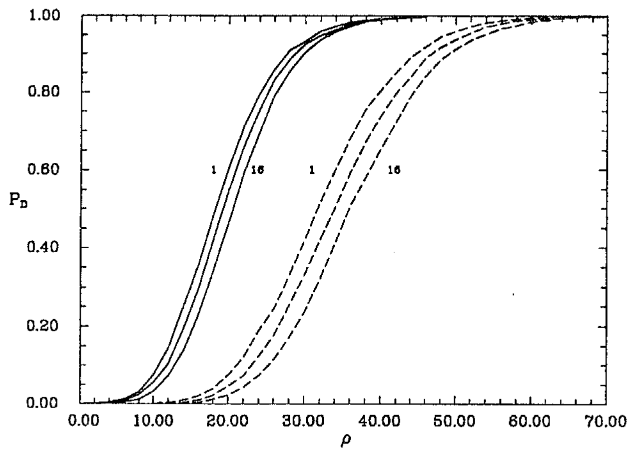

Figure 5. Detection performance for range-spread persisting targets in Weibull background:
$N = 12$, $P_{FA} = 10^{-5}$, $K_c = 3$, K_s=1,4,16, $a = 0.9$, $k = 7$;
solid: $\beta = 2$; dashed: $\beta = 1$.

3.3 Performance in the presence of extended and multiple targets

Now we consider the case of a range-spread target whose extension in the range direction is longer than a single cell; this may be the case as a slow, large ship enters the map cell, yielding strong returns in a significant number of range cells surrounding the cell being tested. The extended nature of the target results in a close-to-one correlation between the amplitudes of the target component of the returns.

The detection performance of the system in the presence of extended targets are shown in Figure 5 where a target extension $K_c = 3$ is simulated. We observe that the system is still effective in combating the self-masking effect, but the CFAR loss is larger than for a point target. The companion case where the primary target, assumed point-like, is in close proximity to two spurious targets, themselves point-like, is also of interest. We consider here the limiting case of completely independent echoes: all the intermediate instances of target echoes correlation should lay between the two aforementioned cases.

After running the simulations, we noticed that, as the signal to interference ratio, equals the signal to noise ratio, the performance reproduces that of a range-spread target whose extension equals the number of interfering targets, an evidence that the target correlation does not have any remarkable effect on the system performance.

V. Concluding remarks

In this paper we have introduced a new clutter-map CFAR procedure, aimed at detecting range-spread targets in non-Gaussian clutter with unknown distributional parameters. The system relies on a combination of time and space filtering, in the sense that, at each scan, it singles out the $k-$th order statistic from an $N-$dimensional sample, representing the clutter returns in the range cells forming the map cell. The clutter parameters are suitably estimated by time-processing these observations on a scan-by-scan basis and the adaptive threshold is eventually delivered. The system admits the size N of the clutter map, the rank k of the order statistic and the location of the pole of the linear filters as "free" parameters to be chosen, according to the environment nature, as a compromise between the conflicting requirements of on-line tracking of clutter variations and small CFAR loss.

The statistical analysis confirms that the system achieves satisfactory performance in the presence of slow targets with the understanding that a masking effect from extended targets is prevented only if the censoring depth from the upper end of the estimation sample at least equals the number of range cells occupied by the target.

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