A Geometrical Algorithm for Blind Separation of Sources

C. Puntonet
Departamento de Electronica y Tecnologia de Computadores
Universidad de Granada. 18071 Granada (Spain)
Email: carlos@casip.ugr.es

A. Mansour, C. Jutten
INPG-TIRF, 46 avenue Félix Viallet, 38031 Grenoble (France) Cedex
and Groupement De Recherche GDR 134 du CNRS
Email: mansour and chris@tirf.inpg.fr

Résumé

Dans cet article, nous proposons une méthode géométrique simple pour la séparation aveugle de sources. Cette méthode s'applique pour des sources de densité de probabilité bornée. Elle est fondée sur l'identification des pentes des arètes d'un parallélépipède. Nous proposons un algorithme dans le cas de deux mélanges de deux sources, dont nous discutons les performances. Actuellement, nous abordons l'extension de l'algorithme au cas de plus de deux mélanges et deux sources.

Abstract

In this paper, we present a geometrical method for solving the problem of blind separation of sources. The method assumes that sources have bounded probability density functions pdf. It is based on estimation of edges of a parallelepiped. We propose an algorithm for two mixtures of two sources, performance of which are discussed. Currently, we address the generalization of the method for more than two mixtures and two sources.

1 Introduction

The problem of blind separation of sources is generally solved by using statistical criteria, minimization of contrast function [2], [8], cancelation or minimization of a cost functions [6], [5], [4], [3]. However, using prior knowledge on the sources, new algorithms, basically simpler and more efficient, may be derived [9], [1], [7].

In this paper, assuming that sources have bounded probability density functions pdf, we propose a method based on geometrical properties of the mixtures.

2 Geometrical representation

Let us consider p observations, say $e_j(t)$ $(1 \leq j \leq p)$, assumed to be unknown linear instantaneous mixtures of n unknown sources, say $s_i(t)$ $(1 \leq i \leq n)$:

$$e_j(t) = \sum_{i=1}^{n} m_{ij} s_j(t). \tag{1}$$

Source separation consists in estimating the unknown sources, only using the mixtures. It is well known that estimated sources are defined up to any permutation and up to any scalar. Because of the last indeterminacy, we may assume, without loss of generality, that issues of principal diagonal of the mixing matrix $\mathbf{M} = (m_{ii})$ equal 1: $m_{ij} = 1$.

For sake of simplicity, we restrict the following analysis to the case $n = p = 2$, but generalization to any n and p is immediate:

$$\begin{cases} e_1(t) = s_1(t) + a s_2(t) \\ e_2(t) = b s_1(t) + s_2(t). \end{cases} \tag{2}$$

Let us suppose that the sources are statistically independent, and have bounded probability density functions. Then, in the plane (e_1, e_2), the observation, at any time t, is a point $(e_1(t), e_2(t))$ which belongs to a parallelogram (see Fig. 1). Using (2), it is clear that the parallelogram edges have slopes equal to to b and $1/a$ in the plane (e_1, e_2). Then, estimation of the slopes gives directly estimation $\hat{\mathbf{M}}$.

For sources with semi-bounded pdf (for instance $s_i(t) \in [0, \infty[)$, observations belong to an angular sector, edge slopes of which still correspond to b and $1/a$ (see Fig. 2). For sources with various pdf, we plotted

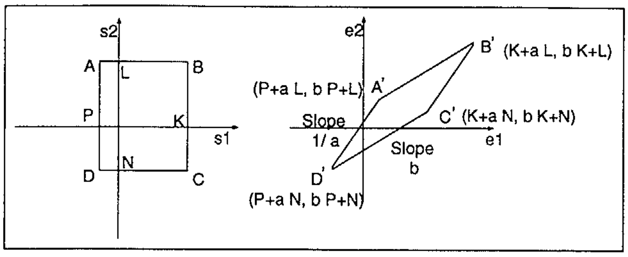

Figure 1: Source and mixture spaces for bounded signals.

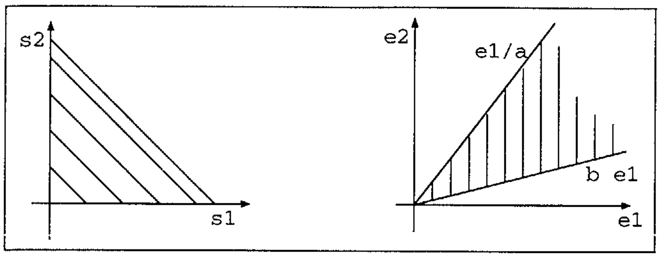

Figure 2: Source and mixture spaces for positive sources.

5000 points $(e_1(t), e_2(t))$ in the plane (e_1, e_2). In Fig. 3, sources have both uniform distributions. In Fig. 4, one has Gaussian distribution, the other has uniform distribution. In Fig. 5, sources are both Gaussian. Finally, in Fig. 6, we plotted mixtures for two deterministic sine sources: $sin(2\pi f_0 t)$, and $sin(2\pi f_1 t + \phi)$. The parallelogram of mixture distribution clearly appears for sources with bounded distributions, and the method can only be applied in these cases.

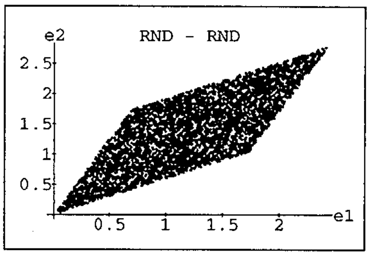

Figure 3: Mixture of two uniform signals.

3 Analytical study

First assume sources have distributions on positive bounded intervals $[0, M_i]$. One of the vertices of the mixture parallelogram is then located in $(0,0)$, and the slope estimation may be very simple.

In fact, let us consider $r(t) = \frac{e_2(t)}{e_1(t)}$. We may compute the maximum and minimum value, say r_{max} and r_{min} respectively. Clearly, if the number of samples is large enough, r_{max} and r_{min} tend toward the slopes

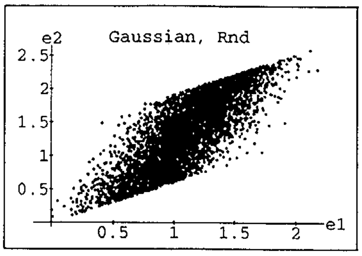

Figure 4: Mixture of Gaussian and uniform signals.

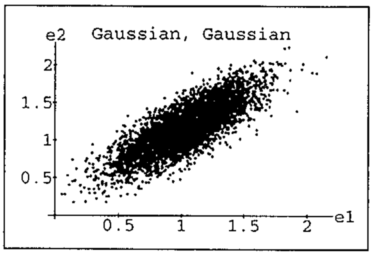

Figure 5: Mixture of two Gaussian signals.

of the parallelogram, that is toward parameters of the unknown mixing matrix.

3.1 Indeterminacies

According to the unknown values, a and b, of the mixing matrix, maximum and minimum values of the ratio r can satisfy: $r_{min} = \hat{b}$ and $r_{max} = 1/\hat{a}$, or $r_{min} = 1/\hat{a}$ and $r_{max} = \hat{b}$. The two solutions imply two different estimated matrices:

$$\hat{\mathbf{M}}_1 = \begin{pmatrix} 1 & \hat{a} \\ \hat{b} & 1 \end{pmatrix} \text{ or } \hat{\mathbf{M}}_2 = \begin{pmatrix} 1 & 1/\hat{b} \\ 1/\hat{a} & 1 \end{pmatrix}. \quad (3)$$

If we compute the global matrix $\mathbf{H} = \hat{\mathbf{M}}^{-1}\mathbf{M}$, we get:

$$\mathbf{H}_1 = \frac{1}{1-\hat{a}\hat{b}} \begin{pmatrix} 1-\hat{a}b & a-\hat{a} \\ b-\hat{b} & 1-a\hat{b} \end{pmatrix} \quad (4)$$

or

$$\mathbf{H}_2 = \frac{1}{1-\hat{a}\hat{b}} \begin{pmatrix} \hat{a}(b-\hat{b}) & \hat{a}(1-a\hat{b}) \\ \hat{b}(1-\hat{a}b) & \hat{b}(a-\hat{a}) \end{pmatrix}. \quad (5)$$

Then, if $\hat{a} \to a$ and $\hat{b} \to b$, the two solutions only differ from a scale factor and a permutation.

In the following, we will propose a 2-step algorithm: the first step consists in translating the parallelogram, so that the origin corresponds to any corner, the second step consists in estimating the slopes. First, we

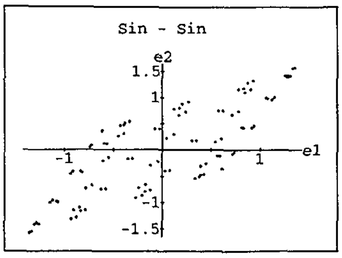

Figure 6: Mixture of two sine signals.

wonder if the solution differs according to the chosen translation. Assume the sources satisfy: $s_1 \in [0, M_1]$ and $s_2 \in [0, M_2]$. If we choose the point D' (see Fig. 1) as new origin, we may obtain two solutions taking into account the slope indeterminacy. Now, assume, the point B' is the new origin. The slope estimation provides the same values and the same indeterminacy, but the new origin implies estimates sources are now $-s_1(t)$ and $-s_2(t)$. Taking the other points (A' or C') as origin, we obtain similar results. Consequently, the new origin may be any corner of the parallelogram.

3.2 Algorithm

We can resume the algorithm in 2 main successive steps (for more information about this algorithm, see [10]).

- We first compute the new origin O' as the point $(e_1(t), e_2(t))$ with the maximum norm (complexity $O(4N)$, where N is the sample number): $O' = (e_1(t_0), e_2(t_0))$, with $t_0 = \arg\max_t(e_1^2(t) + e_2^2(t))$.

- Then, we estimate slopes of the parallelogram. We then deduce the estimated mixing matrix and its inverse. Finally, estimated sources are obtained by multiplying observations by the estimated inverse of the mixing matrix (complexity $0(7N)$):
  Slope estimation
  $r_{min} = \min_t(\frac{e_2(t)-e_2(t_0)}{e_1(t)-e_1(t_0)})$
  and $r_{max} = \max_t(\frac{e_2(t)-e_2(t_0)}{e_1(t)-e_1(t_0)})$.
  Source estimation
  $(\hat{s}_1(t), \hat{s}_2(t))^T = \hat{\mathbf{M}}^{-1}((e_1(t), e_2(t))^T$.

4 Experimentally result

4.1 Accuracy

With this method, necessity of source independence does not appear directly. However, an accurate estimation will only be possible if samples $(e_1(t), e_2(t))$ exist in the neighborhood of parallelogram edges. If such points are scarce, the algorithm will need a lot of samples. In particular, if the sources are not independent, this situation may occur. Conversely, if we know source pdf and assume source independence, we may compute the probability of points in the neighborhood of edges and deduce information on algorithm speed and accuracy.
Consider the estimation of b: $\hat{b} = \min_t(\frac{e_2(t)-e_2(t_0)}{e_1(t)-e_1(t_0)})$, and assume $\hat{b} = b + \epsilon$. Let us denote α the sector bounded by the 2 straight lines with slopes b and $b + \epsilon$ (see fig 7). It is easy to prove that: $\alpha = \arctan(\frac{\epsilon}{1+b^2+b\epsilon})$. Using the inverse of the mixture matrix, we can calculate the straight line, say Δ, in the source space, corresponding in the observation (mixtures) space to D: $e_2 = (b + \epsilon)e_1$:

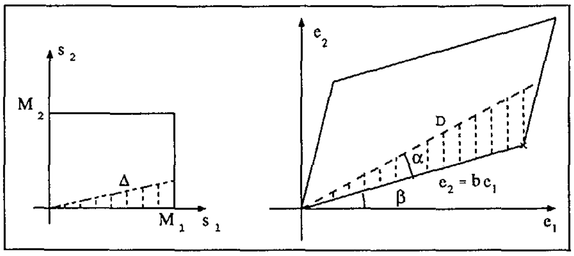

Figure 7: Security sector.

$$\begin{pmatrix} s_2 \\ s_1 \end{pmatrix} = \frac{1}{1-ab}\begin{pmatrix} 1 & -a \\ -b & 1 \end{pmatrix}\begin{pmatrix} e_1 \\ e_2 \end{pmatrix}$$
$$= \frac{1}{1-ab}\begin{pmatrix} 1 & -a \\ -b & 1 \end{pmatrix}\begin{pmatrix} e_1 \\ (b+\epsilon)e_1 \end{pmatrix}$$

The sector in source space is limited by the straight line Δ, equation of which is $s_2 = \frac{\epsilon}{1-ab-a\epsilon}s_1$.
Now, assuming sources have uniform pdf: $s_1 \in [0, M_1]$ and $s_2 \in [0, M_2]$, the probability of samples in the sector S is:

$$P = \int_0^{M_1}\int_0^{\frac{\epsilon s_1}{1-ab-a\epsilon}} \frac{1}{M_1 M_2} ds_2 ds_1 = \frac{\epsilon M_1}{2M_2(1-ab-\epsilon a)}. \qquad (6)$$

Practically, if the total sample number N, and the sample number N_s in sector S, are large enough, the ratio $\frac{N_S}{N}$ tends toward the probability P. Then, we may deduce that the number of samples in sector S must satisfy: $N_S = \frac{\epsilon M_1 N}{2M_2(1-ab-a\epsilon)} >> 1$. The minimum sample number N can then be deduced from this relation. Note that the accuracy on $\hat{a}$ and $\hat{b}$ directly corrupts separation performance. With $\hat{a} = a+\epsilon_a$ and $\hat{b} = b+\epsilon_b$, and assuming sources have the same power, it is easy to compute the residual crosstalk: $C_i = \frac{\epsilon_i}{1-ab-a\epsilon_b-b\epsilon_a}$, with $i \in \{a, b\}$. Then, from (6), we deduce $N = \frac{N_s}{2\sqrt{C}}$. Taking $N_s \geq 10$, we finally obtain $N \geq \frac{5}{\sqrt{C}}$. For $C = 0.01$ (- 20 dB), we will choose $N \geq 50$.

4.2 Algorithm performance

With 1000 samples and for two sources and two sensors, we obtain a crosstalk of about -20 dB to -24 dB. In the case of more sources (three sources), the same algorithm can be applied for particular mixing matrices with similar performance. However, in the general case, the algorithm consisting in estimating slopes from the ratio $r = \frac{e_i}{e_j}$ does not work any more. It is necessary to estimate the planes (or hyperplanes in more general case) which bound the parallelepiped in the mixture space.
Finally, the algorithm is very sensitive to additive noise in the mixtures. In fact, the additive noise implies noise around parallelogram edges, and consequently poor performance in slope estimation.

5 Conclusion

In this paper, we propose a source separation algorithm, based on geometrical properties:

- It is very simple, and does not need computation of any order statistics.
- The convergence is fast, and depends only on the probability of points close to the parallelogram (or parallelepiped) edges.

The algorithm suffers from a few limitations:

- Sources must have bounded pdf.
- It is sensitive to noise.
- It cannot be applied directly for more than two sources, although the geometrical idea still holds.

In further works, we would like to apply image processing techniques for noisy mixtures and more sophisticated geometrical methods for more than two sources.

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