REDUCED COMPLEXITY FREQUENCY ESTIMATOR APPLIED TO BURST TRANSMISSION

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Résumé

En transmission par paquet, la récupération de porteuse est un point crucial en matière de synchronisation à cause des sauts de cycles qu'une estimation de fréquence imprécise peut favoriser. Un estimateur de fréquence pour signaux modulés en MPSK, sans biais, non bouclé et fonctionnant en mode NDA (Non Data Aided) est dérivé du principe du maximum de vraisemblance. La variance de l'estimateur est minimisée par un compromis filtrage du bruit/saut de l'estimation et comparée à celle de l'estimateur du maximum de vraisemblance. L'algorithme est mis en oeuvre dans une structure entièrement non bouclée avec une modulation QPSK pour laquelle les performances sont fournies.

Abstract

In burst transmission, carrier recovery is a crucial point for synchronization systems, because of cycle slips an imprecise frequency estimation can provoke. An unbiased feedforward frequency estimator, for M-PSK modulated signals, in the non data aided mode (NDA) is derived from the Maximum Likelihood (ML) principle. A compromise is realized between noise filtering and estimation slip probability by minimizing the estimator variance at low signal to noise ratio. Its performance is compared to the ML estimator one. an structure). Global performance of an all feedforward synchronization unit and a QPSK modulation is supplied.

Introduction

In satellite communications, TDMA mode is very attractive because a high throughput can be maintained with an important number of users. Terminals, transmitting at different instants, need to be synchronized with the received signal, to correctly demodulate it.
Burst transmission lays down important constraints as:

- frequency offset varying between successive bursts.
- variable burst length (demodulation has to be realized even for minimum length bursts)
- minimum preamble length to maintain a high efficiency.
- use of channel coding to operate at low Eb/No and then minimize the dimension and the cost of stations.

Therefore, it is very useful to study demodulators working out with synchronization, operating at low Eb/No, able to sell out acquisition with only a few symbols.

Among different synchronization aspects, frequency estimation is a crucial point which has to be performed as precisely as possible in order to avoid the multiplication of cycle slips which involve data loss in the packet.
This paper deals with a frequency estimator, derived from the Maximum Likelihood principle, in non-data aided (NDA) mode for burst transmission and examines the optimization of the different parameters of this one. It is applied to a practical case with a bounded offset frequency to modelize jitters between oscillator.
In the first section, the likelihood principle is applied to exhibit a reduced complexity frequency estimator (RCFE). We derive in the second section the variance expression of this estimator in the unmodulated case when a frequency offset goes along with received signal. Moreover, we examine the optimization of the parameters of this estimator in the QPSK modulated case. The third section compares its performance to the ML estimator one. The fourth section analyses the global performance of a non data aided (NDA) synchronization structure (timing and carrier recovery). Conclusions are given in section five.

1. Approximation of the likelihood function

Assuming that the clock recovery has already been performed, we consider symbols at the output of the timing recovery system:

$$r(kT_s) = c_k e^{j(\theta_0 + 2\pi k \Delta f T_s)} + n(kT_s) \quad (1)$$

where $\{c_k\}$ is a sequence of statistically independent MPSK symbols with $|c_k|=1$, $n(t)$ is a complex additive white gaussian noise, with statistically independent real and imaginary parts, each one having a variance $\sigma^2 = \frac{1}{2E_s / No}$

T_s is the sampling time and the symbol duration, θ_0 is the unknown initial phase shift and Δf is the frequency offset going along with received symbols.

We have to exhibit an estimator of the random variables couple $(\Delta f, \theta_0)$ from a sequence of N received symbols $\{r_k\}$. In the case where known symbols $\{\hat{c}_k\}$ of a preamble are used to estimate $(\Delta f, \theta)$, the maximization of the likelihood function [1], leads to the expression (2), corresponding to the one given in [2] with only one sample per symbol and then no inter symbol interference, with $L = (N-1)/2$

$$(\hat{\Delta f}, \hat{\theta})_{ML} = Arg\left\{ Max\{\mathrm{Re}(\sum_{k=-L}^{L} \hat{c}_k^* e^{-j(\theta + 2\pi k \Delta f T_s)} r_k)\}\right\} \quad (2)$$

The expression (2) can be rewritten as:

$$(\hat{\Delta f}, \hat{\theta})_{ML} = Arg\left\{ Max\{|H(\Delta f T_s)| \mathrm{Re}(e^{j(-\theta + Arg[H(\Delta f T_s)])})\}\right\} \quad (3)$$

with $$H(\Delta f T_s) = \sum_{k=-L}^{L} \hat{c}_k^* r_k e^{-j2\pi k \Delta f T_s}$$

The joint maximum $(\hat{\Delta f}, \hat{\theta})_{ML}$ is obtained for Δf maximizing $|H(\Delta f)|$ which is independent on θ. The second factor is maximized for $\hat{\theta} = Arg(H(\hat{\Delta f} T_s))$. Then, for frequency estimation, we only need to maximize $|H(\Delta f)|$. Deriving $|H(\Delta f)|^2 = H(\Delta f) H^*(\Delta f)$ with respect to Δf, the ML value $\hat{\Delta f}$ verifies:

$$\frac{\partial}{\partial \Delta f}(|H(\Delta f)|^2)\Big|_{\hat{\Delta f}} = 4\pi |H(\hat{\Delta f})|.$$

$$\mathrm{Re}\left\{ -jT_s \sum_{k=-L}^{L} k\ \hat{c}_k^* r_k e^{-j2\pi k \hat{\Delta f} T_s} e^{-j\hat{\theta}} \right\} = 0 \quad (4)$$

Since $|H(\hat{\Delta f})|$ is not equal to zero, we only have to consider the real part described in (4).
Neglecting the noise influence, what is correct at high signal to noise ratio, but coarse when the noise becomes more important, we can write:

$$\mathrm{Im}\left\{ \sum_{k=-L}^{L-D} h_k \frac{\hat{c}_{k+D}^*}{\hat{c}_k^*} r_{k+D} r_k^* e^{-j2\pi D \hat{\Delta f} T_s} \right\} = 0 \quad (5)$$

with $h_L = h_{L-1} = \cdots = h_{L-D+1} = 0$ and $\sum_{l=1}^{D} h_{k-l} = \frac{1}{2}\left(\frac{N^2-1}{4} - k(k-1)\right)$ (6)

A frequency estimator can immediately be derived:

$$2\pi \hat{\Delta f} T_s = \frac{1}{D} Arg\left\{ \sum_{k=-L}^{L-D} h_k \frac{\hat{c}_{k+D}^*}{\hat{c}_k^*} r_{k+D} r_k^* \right\} \quad (7)$$

This expression exhibits the use of timing spaced samples and may be interpreted as an estimation obtained from decided symbols $\{\hat{c}_k\}$ instead of preamble symbols in the case of differentially encoded transmission schemes.
The same maximization can be executed for the non data aided (NDA) case, by averaging over all different data sequences. For an MPSK modulated signal, the estimator can be generalized with a non linear function F_f:

$$2\pi \hat{\Delta f} T_s = \frac{1}{MD} Arg\left\{ \sum_{k=-L+D}^{L} F_f(|r_k r_{k-D}^*|) e^{jM(Arg(r_k) - Arg(r_{k-D}))} \right\} \quad (8)$$

It is shown in [3] that such an estimator may be simplified using equal weights instead of $\{h_k\}$ without any variance loss at low SNR (our interesting case). $\{h_k\}$ coefficients improve estimator performance only at high SNR. The influence of F_f has been studied in [3] and will not be more detailed in this paper. As long as Eb/No < 8 dB, $F_f(r) = r$ gives better results than $F_f(r) = 1$.

Qualitatively, the influence of D can easily be understood taking into account the two following phenomena:
We consider the simplified expression:

$$r_k = c_k e^{j(\theta_0 + 2\pi k \Delta f T_s)} + n_{Ik} + j n_{Qk} \quad (9)$$

where $\Delta f T_s$ has a fixed value and θ_0 is chosen equal to zero to simplify calculations.

- The estimator expression (8) imposes to satisfy the following relation:

$$|2\pi M D \Delta f T_s| < \pi \quad (10)$$

For QPSK modulated signal, when D takes large values, $8\pi D \Delta f T_s$ is directed towards the limit $\pm\pi$. The *argument* function bounded by $[-\pi;\pi]$ may create an "estimation slip" because of the noise influence. ***Figure 1*** shows an "estimation slip" when D takes a large value (D_2 for instance), at low signal to noise ratio. D has then to be chosen small enough (as D_1) to avoid the occurrence of such "estimation slip".

- As it can be seen on ***figure 3***, when D takes values close to 1, the estimated slope with $M[Arg(r_k) - Arg(r_{k-D})]$ takes an imprecise values because of the important degradation, due to the noise, on the phase of the samples r_k even if a summation allows to reduce this influence. On the contrary, with an important value of D, the slope, estimated with more spaced samples one from each other, is more precise. The summation then allows to minimize the noise influence. Therefore, taking an important value of D allows to minimize the noise influence.

When $\Delta f T_s$ and Eb/No are fixed, an optimal value D_{opt} can be found to minimize the joint influence of noise and "estimation slip".

2. Analytical study of the estimator statistic

In this section, we investigate mean and variance of the estimator given in (8) with equal weights, $F_f(r) = r$ and an unmodulated signal:

$$r_k = e^{j2\pi k \Delta f T_s} + n_{Ik} + j n_{Qk} \quad (11)$$

Mean
The mean of the estimator can be written as:

$$E(2\pi \hat{\Delta f} T_s) = \frac{1}{D} E\left\{ Arg\left(\sum_{k=-L+D}^{L} r_k r_{k-D}^* \right) \right\} \quad (12)$$

Taking the following notation, $x'_i + jy'_i = r_{k-i} r_{k-i-D}^*$ (13)
the *Argument* operation can be linearized without severe restriction (the interesting case is at low variance), and (12) can be rewritten as:

$$E(2\pi \hat{\Delta f} T_s) = \frac{1}{D} Arg\left(\sum_i E(x'_i + jy'_i) \right) \quad (14)$$

$$\text{and} \quad E(2\pi \hat{\Delta f} T_s) = 2\pi \Delta f T_s \quad (15)$$

The estimator is then unbiased.

Variance
Taking the same notations as in (13), we evaluate the estimator variance for any value $\Delta f T_s$ verifying $|2\pi D \Delta f T_s| < \pi$. It can be expressed as:

$$Var(2\pi \hat{\Delta f} T_s) = E\left\{ \left[\frac{1}{D} Arg\left(\frac{1}{N} \sum_i x'_i + jy'_i \right) \right]^2 \right\} - \left[E(2\pi \hat{\Delta f} T_s) \right]^2 \quad (16)$$

When $\sum y'_i << \sum x'_i$ is verified (ie when $|2\pi D \Delta f T_s| << 1$), ie when the imaginary part has zero mean and variances of both real and imaginary parts are neglectible compared to the squared real mean, (16) can be written as:

$$Var(2\pi \hat{\Delta f} T_s) \approx E\left\{ \left[\left(\frac{1}{ND} \sum_i y'_i \right) \right]^2 \right\} - (2\pi \Delta f T_s)^2 \quad (17)$$

and finally,

$$Var(2\pi \hat{\Delta f} T_s) \approx \frac{1}{D^2} \left\{ \left(1 + 4\frac{\sigma^2}{N}\right) \sin^2(2\pi D \Delta f T_s) + \frac{2}{N}\sigma^4 + 2\frac{D\sigma^2}{N^2} \cos(4\pi D \Delta f T_s) \right\} - (2\pi \Delta f T_s)^2 \quad (18)$$

with $\sigma^2 = E\left[n^2{}_{Ik}\right] = E\left[n^2{}_{Qk}\right]$.

Note that this approximation is only valid for small values of $2\pi D\Delta fT_s$. In the contrary case, the approximation $Arctg(y/x) \approx y/x$ is unsatisfactory and the preceding result is not valid.
When the frequency offset Δf is null, the variance estimator is, as specified in [4]:

$$Var(2\pi\hat{\Delta f}T_s) \approx \frac{1}{D^2}\left(2\frac{D\sigma^2}{N^2} + 2\frac{\sigma^4}{N}\right) \qquad (19)$$

For ΔfT_s varying in $[-(\Delta fT_s)_{\max};(\Delta fT_s)_{\max}]$, with $(\Delta fT_s)_{\max}$ close to zero (as here 4.10^{-3}) and a low D value, expression (19) gives a precise approximation of (18). But it may be erroneous for higher ΔfT_s and D. These expressions validity will be analyzed in the following chapter.

3. Simulation of the estimator performance

Performance evaluation has been realized with COSSAP, a powerful tool for communication systems simulations.

- ***Unmodulated signal***

We examine the ML and the RCFE estimators variance for different values of N.
The signal to noise ratio is fixed at Eb/No=2dB. The frequency offset Δf is bounded as $|\Delta fT_s| < (\Delta fT_s)_{\max} = 4.10^{-3}$, value corresponding to a symbol frequency F_s=512 kHz and a frequency offset $\Delta f = 2kHz$. When the estimate verifies $|\hat{\Delta fT_s}| > (\Delta fT_s)_{\max}$, $\hat{\Delta fT_s}$ is automatically truncated and takes the value $\hat{\Delta fT_s} = (\Delta fT_s)_{\max} Sgn(\hat{\Delta fT_s})$.
Compare theoretical and simulated results. ***Figure 3*** shows the validity domain for expression (18) and (19). While $|D(\Delta fT_s)_{\max}| < 4e-2$, the RCFE estimator variance is accurately approximated by the middle variance expression calculated by averaging (18) over $[-(\Delta fT_s)_{\max};(\Delta fT_s)_{\max}]$. But for higher values, (18) becomes erroneous whereas (19) is still valid. (19) becomes imprecise when estimation slips occur ie for higher values of $(D\Delta fT_s)_{\max}$ (typically when $2\pi(D\Delta fT_s)_{\max}$ is not neglectible compared to π).
The ML estimator is determined by computing the frequency value which maximizes $|H(\Delta fT_s)|^2$ given in (3). This calculation is realized by using a granularity to examine completely the domain in which ΔfT_s can take its value ($|\Delta fT_s| < (\Delta fT)_{\max}$). When Eb/No=2 dB and $(\Delta fT_s)_{\max}$=4.10^{-3},

$$Var((\hat{\Delta fT_s})_{ML}) = 1.35\ 10^{-8}\ \ if\ \ N = 200\cdot$$

$$Var((\hat{\Delta fT_s})_{ML}) = 3.79\ 10^{-9}\ \ if\ \ N = 300\cdot$$

When ΔfT_s and Eb/No take fixed values, the optimal value allowing to minimize the RCFE estimator variance is noted D_{opt}. When Eb/No=2 dB and $(\Delta fT_s)_{\max}$=4.10^{-3}, D_{opt} #110, and $Var((\hat{\Delta fT_s})_{RCFE}) \approx$ 6.24 10^{-9} when N=300 (only twice more than the ML one).
We can say that, in the unmodulated case and even at low Eb/No, performance of the RCFE estimator is close to the ML one, for each burst length examined.

- ***QPSK modulated signal***

In this simulations series, the estimators variance is analyzed with QPSK modulated signals. Simulations conditions are the same as before (Eb/No=2dB, $|\Delta fT_s| < (\Delta fT_s)_{\max} = 4..10^{-3}$ and truncation in case of $(\Delta fT_s)_{\max}$ overstepping).
Figure 4 shows the variance of the RCFE estimator versus D in the QPSK modulated case. The optimal value D_{opt} is obtained when the noise influence is well averaged and $2\pi\Delta fT_s$ is still low enough to avoid "estimation slip".
When Eb/No=2 dB, for $\Delta fT_s = 4.10^{-3}$, D_{opt} #23 and $Var((\hat{\Delta fT_s})_{RCFE})$ is about 1,5.10^{-6} with N=300.
In ***figure 4***, we can see that D can belong to an interval (for instance 16< D <24 with $(\Delta fT_s)_{\max} = 4.10^{-3}$) almost without loss for the variance estimator. For a modem using the RCFE estimator, the value D will be chosen according to the modem operating point (minimum Eb/No). Moreover, in a practical point of view, it is preferable to adopt a low value D to decrease the logic gates number used to realize the D symbol duration delay. Finally, we choose D=16 or 18 for future simulations.
The ML estimator, with the same granularity and the same parameters (N and ΔfT_s) than in the unmodulated case, performs as: $Var((\hat{\Delta fT_s})_{ML})$ = 1,74 10^{-8} $if\ N = 200$.

$$Var((\hat{\Delta fT_s})_{ML}) = 3{,}74\ 10^{-9}\ \ if\ \ N = 300.$$

These results show that the RCFE estimator variance is more than two decades worse than the ML one but its main interest is its very reduced complexity.

For $(\Delta fT_s)_{\max}$=4.10^{-3} and N=300, as long as Eb/No≥2 dB, the RCFE estimator variance is less than 1,6.10^{-6}. Under this Eb/No value, the variance becomes to important and the correction realized on r_k samples not precise enough for an accurate phase estimation. Then, the estimator performance is satisfying until this signal to noise ratio.

4. Global simulation of a synchronization structure using the rkrk-D estimator

The modelized structure is an all feedforward modem synchronization unit, operating in TDMA mode on QPSK modulated signals. Clock, frequency and phase are entirely recovered with non data aided (NDA) algorithms. In tracking mode, the frequency offset does not exceed $\Delta f = 1kHz$ with a minimum symbol frequency $F_s = 512kHz$, $|\Delta fT_s|_{\max} = 2.10^{-3}$.
The modem performance is analyzed on a channel which adds a white gaussian noise. Global transmitting and receiving filters are modelized with two raised cosine filters, satisfying the first Nyquist criterion.
Timing is recovered with an all feedforward system, constituted with a timing estimator [5] using four samples per symbol and a linear interpolation to regenerate the sample at the optimal eye aperture instant. The RCFE estimator then allows to estimate the frequency offset. The Viterbi and Viterbi phase estimation algorithm [6] is used to estimate the symbol phase and a rotation matrix using these two results corrects the timing recovery system output signal.
The bit error rate has been computed in tracking mode, parametered with the burst length L_b=200, 2000 symbols.
Figure 5, gives the modem performance.
With long bursts, the frequency offset can slowly vary. It can be tracked with the help of an integrating filter. Thus, performance is improved when the burst length increases. With L_b=200 and L_b=2000 symbols, the improvement is about 0.3 dB near Eb/No=2 dB (modem operating point), and 0.1 dB when Eb/No>4 dB. That can easily be explained with cycle slips. At low Eb/No (2 dB), the frequency estimation tracking allows to decrease the cycle slips rate. At high Eb/No, the frequency estimation is more precise and cycle

slips, less likely, give a BER closer to the QPSK modulation one.

The global performance of the complete synchronization structure, in the worse case examined ($\Delta f = \Delta f_{max}$ =2kHz, $F_s = (F_s)_{min}$ =512 kHz, L_b =200 symbols, Eb/No=2 dB, no frequency estimation tracking) is only about 0.65 dB degradation compared to a QPSK modulation BER.

5. Conclusion

Parameters of the RCFE estimator have been optimized to operate at low signal to noise ratio by minimizing the frequency error variance. D has to be chosen taking into account the modem operating point (lowest Eb/No), $(\Delta f T_s)_{max}$, and the realization complexity. The RCFE estimator offers a satisfying complexity / performance compromise. A precise frequency estimation then allows to decrease the cycle slips rate (created during the phase post-processing). The frequency estimation tracking improves performance and makes the cycle slips rate decrease.

Even if the estimator variance is more than one decade worse than the ML one, the all feedforward synchronization structure it here belongs to, allows to recover timing and carrier phase in suitable conditions to take correct decisions on received symbols and then have a BER close to the QPSK modulation one and a high frame efficiency even with minimum length burst.

References

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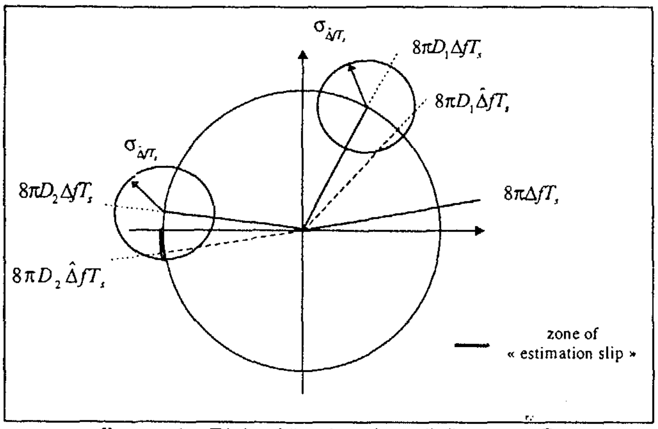
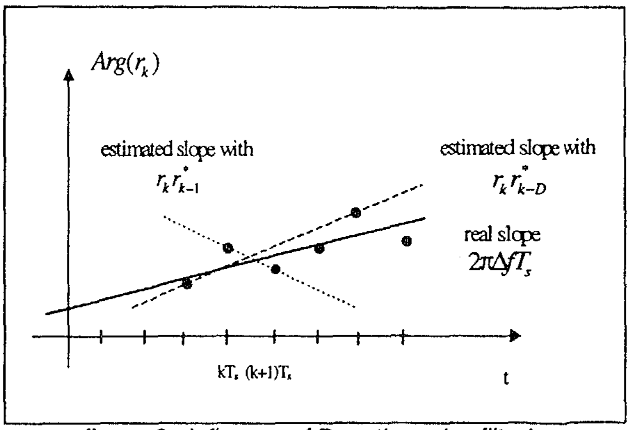
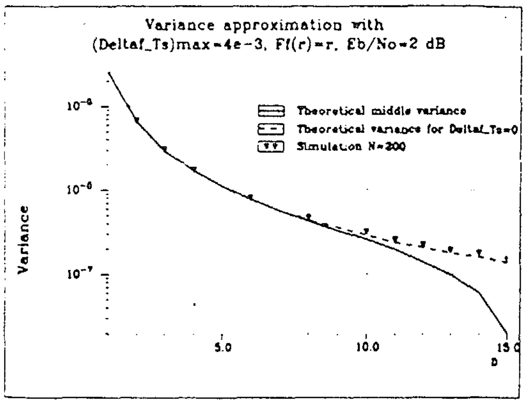
Figures



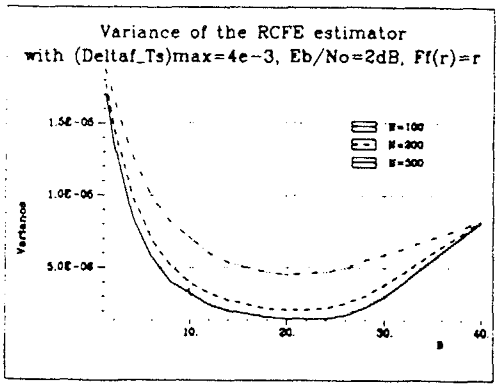
- figure 1 - Risk of "estimation slip" versus D



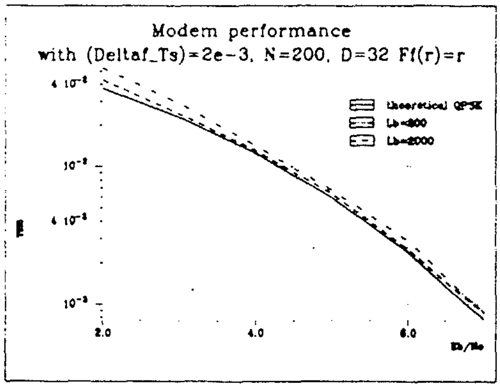
- figure 2 - Influence of D on the noise filtering



- figure 3 - Comparison of theoretical and simulated variance for the unmodulated case.



- figure 4 - RCFE estimator variance versus D with QPSK modulated signal, Eb/No=2dB.



- figure 5 - Modem performance for different burst lengths.