A NEW MONOPARAMETRIC CFAR DETECTOR ROBUST AGAINST SPIKY CLUTTER

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RÉSUMÉ

Dans cet article on considère la détection à Taux de Fausse Alarm Constant (TFAC) d'un cible noyé dans un bruit impulsionel non Gaussien. Un nouveau simple détecteur TFAC avec une intégration incohérente est presenté. Ce détecteur, malgré l'estimation d'un seul paramètre, présente une probabilité de fausse alarme peu variable lorsque le fouillis est très impulsionel. L'algorithme est analysé avec un modèle du bruit de Weibull. Les pertes TFAC sont proches a celles des détecteurs avec un seul paramètre, et elle sont moins de celles des systèmes qui estiment deux paramètres. Ainsi le détecteur robuste représente une amélioration des detecteurs classiques qui dégradent intolérablement dans un fouillis non Gaussien, et une alternative aux systèmes TFAC avec deux paramètres ou non paramètriques, qui sont complexes et qui ont beaucoup de pertes.

ABSTRACT

This paper deals with Constant False Alarm Rate (CFAR) detection in non Gaussian "spiky" clutter. A new simple monoparametric CFAR detector with postdetection integration is proposed. In spite of the single-parameter estimation, it exhibits a false alarm probability which is resistant to changes of the degree of clutter spikiness. The algorithm is derived and analysed assuming the Weibull clutter model. The CFAR loss is near to that of conventional monoparametric procedures, and less than that of biparametric CFAR systems. Thus the robust algorithm represents an upgrade of the classical monoparametric CFAR processors, that degrade intolerably in spiky clutter, and an alternative to biparametric or non-parametric CFAR systems, that are complex and lossy.

1. INTRODUCTION

The radar clutter amplitude probability density function (pdf) may largely deviate from Rayleigh in high-resolution and low grazing angle situations (long-tailed or "spiky" clutter). The Weibull family of pdfs, characterized by two parameters (scale and shape), is very often assumed to encompass such situations [1]. Conventional monoparametric CFAR procedures (CA-, OS-, GO-CFAR) degrade intolerably as the Rayleigh distribution assumption is violated [2], [3]. It can be seen (Fig.1) that also conventional CFAR detectors with postdetection integration [4] suffer essentially the same degradation.

We propose and analyse a new and simple monoparametric CFAR algorithm with postdetection integration that guarantees *robustness* to deviations from the Rayleigh distribution. The robustness of the false alarm probability (P_{FA}) is obtained by a *joint* action: the effect of the specific choice of the clutter parameter to be estimated, namely the *root mean square* (RMS) clutter value, in conjunction with the effect of the *integration* itself. This proposal is based on the observation of an interesting behaviour of conventional monoparametric CFAR detectors (Fig.1): for very high nominal P_{FA}, about 10^{-1}, the P_{FA} increases slightly in spiky clutter, or it can even decrease [1]. This happens because at constant clutter level, when the clutter becomes spiky, the pdf exhibits a lengthening of the tail but it accumulates close to the ordinate axis; so nearby a particular threshold value, the false alarm rate (FAR) is slightly sensitive to the shape parameter.

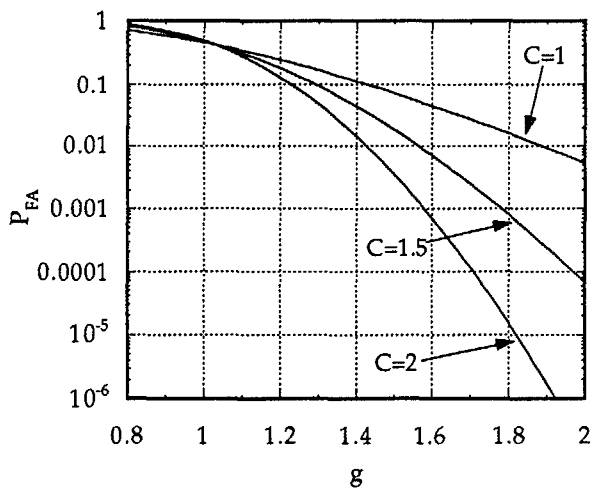

Fig.1 - P_{FA} vs. threshold coefficient g for the CA-CFAR ($N = 10$ pulses integrated, $K \to \infty$ reference cells)

Our goal is to extend this robust behaviour to P_{FA} values of practical interest, making use of the above mentioned effect to derive a robust monoparametric CFAR algorithm. We can use a threshold that depends on the clutter RMS value, and a N-pulse linear postdetection integrator. Then when the clutter becomes spiky (decreasing Weibull shape parameter), at equal clutter power and consequently constant threshold, the pdf at the output of the integrator (approximately Gaussian) increases in variance but decreases in mean value. Thus, the pdf widens but shifts leftward with respect to the threshold, so a *compensation* effect arises which results in P_{FA} values of practical interest almost *insensitive* to changes of the shape parameter. This effect can also be enhanced employing a log integrator, since the log non-linearity tends to smooth the tail of the clutter distribution and to decrease its mean value. In this paper we focus on the robust CFAR detector with logarithmic integrator.

We give the mathematical derivation of the robust algorithm, and we perform an approximate robustness analysis by calculation; a thorough assessment of the robustness performance is carried out via Monte Carlo simulation. Design criteria to achieve a specified robustness degree are given, too. Finally we determine detection performance and CFAR loss; a comparison is carried out with the classical CA- and Log/CA-CFAR systems [4], and with a biparametric CFAR detector for Weibull clutter, based on the joint maximum likelihood (ML) estimation of both the shape and scale parameters [5] and optimized binary integration [6].

2. THE ROBUST THRESHOLDING TECHNIQUE

The output $w \geq 0$ of the envelope detector driving the CFAR device is assumed to be Weibull distributed. The Weibull pdf is [1]:

$$p_W(w) = \frac{C}{B}\left(\frac{w}{B}\right)^{C-1} \exp\left[-\left(\frac{w}{B}\right)^{C}\right] \tag{1}$$

where $B > 0$ and $C > 0$ are the scale and shape parameters; it includes the Rayleigh pdf as the special case $C = 2$. Lower values of C indicate "spiky" clutter. The mean clutter power P, that equals the squared RMS value, is [5]:

$$P = B^2 \Gamma\left(\frac{2}{C}+1\right) = RMS^2 \tag{2}$$

where $\Gamma(.)$ is the Gamma function. We assume that the thermal noise level is sufficiently low, relative to the clutter power, to be neglected.

We take into account N postdetection integrated "pulses" in azimuth and K range reference cells, resulting in $K * N$ reference observations. The term "pulses" refers here to true pulses or to samples resulting from batch coherently processed bursts of pulses. We assume for the sake of simplicity independent and identically distributed (iid) Weibull samples.

Consider the limiting situation with infinite K and high N, and assume that the estimator for the clutter level is consistent (zero bias and variance). After the log amplifier the Weibull variate is transformed in a Gumbel variate [2]:

$$f_G(u) = \frac{1}{b}\exp\left(\frac{u-a}{b}\right)\exp\left[-\exp\left(\frac{u-a}{b}\right)\right] \tag{3}$$

where $-\infty < u < \infty$, and

$$a = \ln B,\ b = 1/C \tag{4}$$

are called location and scale parameters. The mean value and variance of the Gumbel variate are:

$$\eta_G = a - \gamma b \quad \text{and} \quad \sigma_G^2 = \frac{\pi^2}{6} b^2 \tag{5}$$

where $\gamma \cong 0.577$ is the Euler's constant. Substituting (4) into (5) and eliminating B by means of eq. (2) we get:

$$\eta_G = \ln(RMS) - \frac{1}{2}\ln\left[\Gamma\left(\frac{2}{C}+1\right)\right] - \gamma\frac{1}{C},\ \sigma_G = \frac{\pi}{\sqrt{6}}\frac{1}{C} \tag{6}$$

Assuming an integration gain $1/N$, and approximating the pdf at the log integrator output with a Gaussian [4] (central limit theorem), we obtain for the false alarm probability:

$$P_{FA} = Q\left(\frac{T-\eta_G}{\sigma_G/\sqrt{N}}\right) = Q(\alpha) \tag{7a}$$

$$\alpha = \left\{T - \ln(RMS) + \frac{1}{2}\ln\left[\Gamma\left(\frac{2}{C}+1\right)\right] + \frac{\gamma}{C}\right\}\frac{C\sqrt{6N}}{\pi} \tag{7b}$$

where $Q(x) \triangleq \frac{1}{\sqrt{2\pi}}\int_x^\infty \exp(-y^2/2)dy$, T is the detection threshold at the integrator output, and RMS is the root mean square clutter value. Then, by setting the adaptive threshold according to:

$$T = g + \ln(RMS) \tag{8}$$

where g is a threshold coefficient and RMS is the perfectly estimated RMS clutter value, the argument of the Q function does not depend on RMS any more:

$$\alpha = \left\{g + \frac{1}{2}\ln\left[\Gamma\left(\frac{2}{C}+1\right)\right] + \frac{\gamma}{C}\right\}\frac{C\sqrt{6N}}{\pi} \tag{9}$$

so P_{FA} is independent of the mean clutter level, and the algorithm is CFAR for a fixed shape parameter C (the threshold coefficient g determines the P_{FA}).

In the practical situation of finite K, we employ a sample root mean square estimator (which is consistent). The threshold becomes:

$$T' = g + \ln\left(R\hat{M}S\right) = g + \ln\sqrt{\frac{1}{KN}\sum_{j=1}^{K}\sum_{i=1}^{N} w_{ij}^2} \tag{10}$$

where $R\hat{M}S$ is the estimated RMS, and w_{ij} are the total $K * N$ samples from the reference cells, with j and i range and azimuth indexes. The false alarm probability is now given by:

$$P_{FA} = \int_{-\infty}^{\infty} p_{T'}(z)\int_z^\infty p_I(v)\,dv\,dz \tag{11}$$

where $p_I(v)$ is the pdf at the log integrator output, and $p_{T'}(z)$ is the pdf of random variable T'. These pdfs can be expressed by means of a characteristic function approach. After some manipulations, it can be proved that the value of P_{FA} (11) does not depend on the mean clutter power, hence the proposed algorithm is still CFAR, for a fixed C, with a finite K value. In the proof the assumption of Gaussian approximation at the integrator output is removed: the result is valid for any N. The proposed monoparametric detection scheme is depicted in Fig.2.

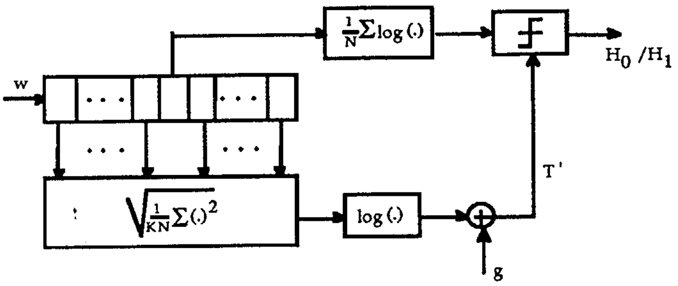

Fig.2 - Robust monoparametric CFAR detector

3. ANALYSIS OF ROBUSTNESS

We now analyse the approximate expression (7a, 9) obtained for P_{FA}, looking for compensation conditions: as an example we impose the same $P_{FA}=10^{-5}$ for $C=2$ and $C=1$, and verify that for $1<C<2$ P_{FA} does not change significantly. It is worth noting that this range of C comprehends most of the situations of practical interest. The previous condition yields, independently of N: $g=(\ln 2)/2\cong 0.347$. For $P_{FA}=10^{-5}$, substituting $g=0.347$ and, say, $C=2$ in the expression for P_{FA}, we get $N\cong 18$. It can also be verified that the maximum P_{FA} within the interval $C\in[1,2]$ is $P_{FA}\cong 2.2\cdot 10^{-5}$. Thus under the Gaussian approximation the system, with $N=18$, $K\to\infty$ and a nominal $P_{FA}=10^{-5}$ in Rayleigh clutter, maintains practically the same P_{FA} when C moves to $C=1$. It can be shown that similar robustness conditions occur at higher or lower values of P_{FA}, respectively with lower and higher N.

We could not find a closed form expression for P_{FA} when K is finite or N low. To analyse exactly the P_{FA} robustness, a data set was built, via Monte Carlo simulation (10^8 trials), as a table of values of P_{FA} versus g, C and N, for $K=10$ and $K\to\infty$. Some results are reported in Fig.3, that shows P_{FA} versus the threshold coefficient g and C, for $N=10$, $K\to\infty$.

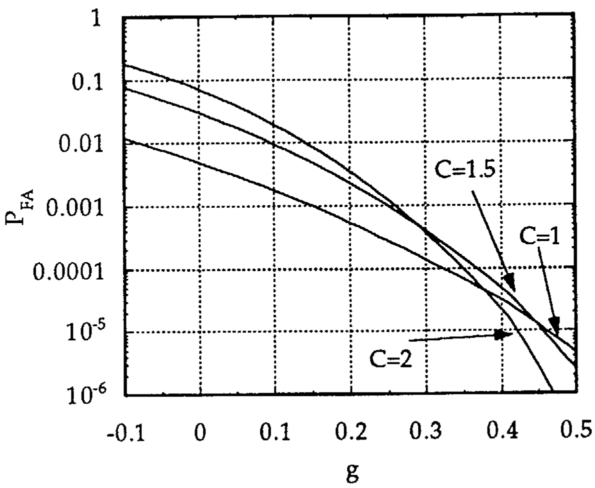

Fig.3 - P_{FA} vs. g for the robust system ($N=10, K\to\infty$)

It results that if the nominal P_{FA} is designed to be, say, 10^{-5} for $C=2$, a very slight maximum FAR increment factor ($\cong 3$) occurs when C moves to $C=1$. The intolerable FAR inflation that the classical multipulse CA-CFAR detector exhibits in the same conditions is evident from Fig.1 ($\cong 1400$). A similar dramatic inflation stands for the multipulse Log/CA-CFAR.

It is useful, also for design purpose, to plot on the "N- nominal P_{FA}" plane the curves corresponding to various robustness degrees, for a given range of C. Following [3] we characterize the robustness by means of the maximum FAR inflation:

$$\gamma=\max_{C\in[C_{min},C_{nom}]}\frac{P_{FA}(C)}{P_{FA}(C_{nom})} \tag{12}$$

where C_{nom} is the nominal shape parameter, say $C_{nom}=2$, C_{min} is the lowest expected C, and of course $P_{FA}(C)$ is evaluated for the same g used to set the nominal false alarm probability $P_{FA}(C_{nom})$. The closer to 1 is γ, the higher the robustness. These curves have been obtained by a data reduction program from the complete data set $P_{FA}(g,C,N)$. Fig.4 shows graphs for $C\in[1,2]$, $K=10$, $\gamma=4,8$, and 16.

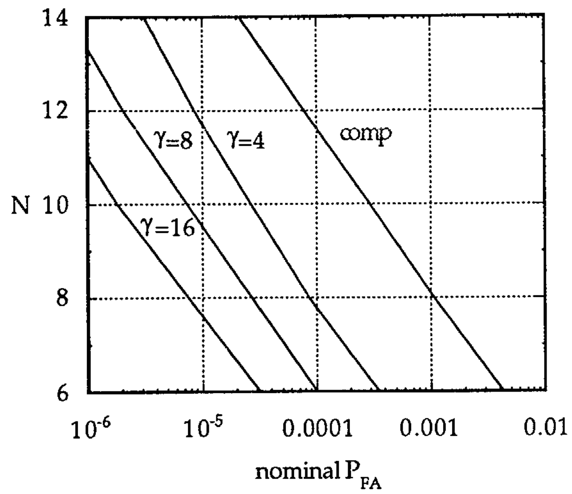

Fig.4 - Curves at constant γ on the "N- nominal P_{FA}" plane ($C\in[1,2]$, $K=10$)

The curve marked "comp" corresponds to a perfect compensation at the extreme points of the range of C; if the point (nominal P_{FA}, N) is above this curve the P_{FA} can be lower than the nominal one, when $C=1$ (see Fig.3). It can be seen that operating with $N=10$, for a nominal P_{FA} from $7\cdot 10^{-6}$ to $3\cdot 10^{-4}$, it is guaranteed that the actual P_{FA} for $C\in[1,2]$ never increases more than 8 times neither decreases with respect to the nominal one. Notice that $\gamma=8$ for $P_{FA}=7\cdot 10^{-6}$ relates to a very good robustness; the classical CA- and Log/CA-CFAR exhibit respectively $\gamma\cong 2100$ and $\gamma\cong 3300$ in the same conditions. Moreover, if we want a nominal $P_{FA}=10^{-5}$ that never decreases, neither increases more than 8 times for $C\in[1,2]$, the system should be designed with N from 10 to 15.

4. DETECTION PERFORMANCE

The detection performance has been compared to that of the classical Log/CA-CFAR system [4]. As a further comparison, we have evaluated the performance of a system with a ML estimator of both the clutter parameters [5] and binary integration [6]. This system exhibits a perfectly constant P_{FA}, while the robust procedure guarantees that the variations of P_{FA} for changing shape parameter are small. In order to get a meaningful comparison, the threshold coefficient has been adjusted when necessary to keep the P_{FA} fixed. Detection performance has been evaluated for Swerling II target, by means of Monte Carlo simulation. We show results in terms of the mean signal-to-clutter power ratio per pulse (SCR) [5] necessary to get a detection probability $P_D = 0.9$ with $P_{FA} = 10^{-5}$, for various C and N, and $K = 10$ (Fig.5).

We have evaluated also the CFAR loss (Fig.6), that is the ratio between the SCR required to achieve a specified P_D and P_{FA}, and the SCR required in the case $K \to \infty$, i.e. exactly-known clutter level. For $K \to \infty$, the robust detector performance is, of course, the same as every system employing log integration (very good in spiky clutter [1]).

It can be seen that the robust detector ("Rob.Detector") outperforms both the comparison systems ("Log/CA", "ML+Bin"). It exhibits better detection performance than the Log/CA-CFAR detector, that is affected by a well-known high CFAR loss [4]. More interestingly, it outperforms (of $1.1 \div 3.5$ dB) the biparametric CFAR system. For $C = 1$ this is due in part to the different integrators employed (log integration outperforms binary integration of about 2 dB). The rest of the performance difference is due to the low CFAR loss that we have achieved in conjunction with robustness.

In fact, the CFAR loss of the robust detector is near (within 0.2 dB) to that of a CA-CFAR detector, moreover it is smaller than that of the biparametric system. This happens because the adaptive threshold of the robust detector is based on the estimate of only one parameter and hence exhibits a lower variance.

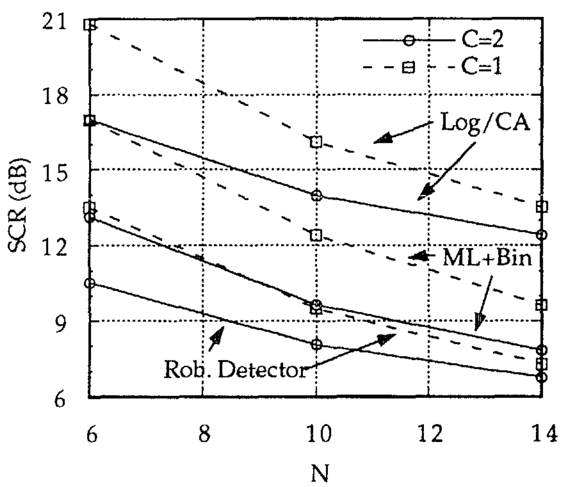

Fig.5 - Detection perf. ($P_D = 0.9$, $P_{FA} = 10^{-5}$, $K = 10$)

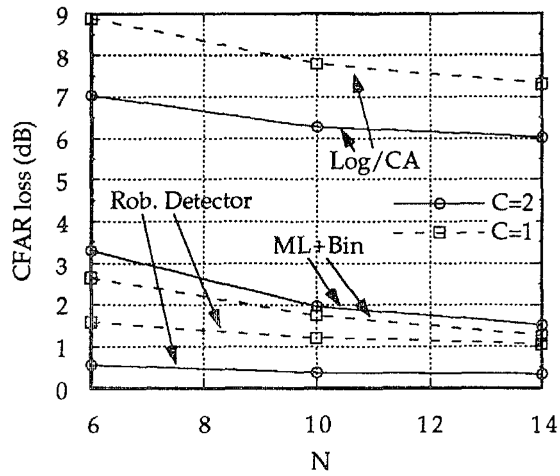

Fig.6 - CFAR loss ($P_D = 0.9$, $P_{FA} = 10^{-5}$, $K = 10$)

5. CONCLUSIONS

We have shown that when postdetection integration is employed a very good CFAR behaviour can be obtained for a two-parameter distribution with variable skewness, by means of a new single-parameter CFAR procedure. A comparison with conventional CFAR systems, failing in spiky clutter [2], has been made. The robust algorithm guarantees that the variations of P_{FA} for changing shape parameter are small and can be bounded by a suitable design procedure; this can satisfactorily meet practical applications requirements. The algorithm is simple and robustness has been achieved in conjunction with low CFAR loss. The proposed algorithm represents an upgrade of the classical CA-CFAR detectors and an alternative to biparametric or non-parametric CFAR systems, that are affected by high complexity [5] and losses [4].

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