RADAR DETECTION OF TARGETS WITH UNKNOWN PARAMETERS IN COMPOUND-GAUSSIAN CLUTTER

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RÉSUMÉ

Dans cet article nous présentons un recepteur pour la détection de cibles avec paramètres inconnues noyés dans un fouillis de Weibull. Pour surmonter l'indetermination a priori on utilise la strategie du rapport de vraisemblance generalisé. Nous étudions aussi la sensibilité du récepteur a la variation des paramètres de la loi de repartition du fouillis et a la loi même. L'analyse démontre que le recepteur est robuste et on peu l'utilisé pour surmonter l'incertitude sur l'environnement.

ABSTRACT

In this paper we introduce and assess a GLRT-based receiver designed to detect coherent pulse trains with unknown parameters embedded in Weibull-distributed clutter. In particular we address a through sensitivity analysis of such receiver aimed at determining to what extent the performance are influenced by mismatch between the design and the actual clutter statistics. The analysis shows that the proposed detector is relatively insensitive to possible mismatch thus stating its suitability as a means for circumventing the a-priori environmental uncertainty.

1. INTRODUCTION

The design of detectors for targets embedded in additive disturbance is a problem of primary concern among radar engineers. The theory of optimum detection is in principle always applicable for design purposes, regardless the probability density functions (pdf's) of the impinging noise and of the received target echo: in practice, however, general solutions to the problem are hardly found in a closed form even in the simple case of completely known target signal, except for the case of Gaussian noise. Moreover, although the Gaussian distribution relies on the central-limit theorem, it is not the limit pdf for those situations where the received clutter echo is modelled as the sum of a fluctuating number of contributions [1]. High resolution clutter at low grazing angles, for instance, may significantly differ from the Rayleigh law, especially in the upper tail of the amplitude pdf (apdf) and, oftentimes, it is well fitted by the Weibull and K- family of distributions with a shape parameter in addition to the scale one [2, 3].

Moreover, the experimental evidence, supported by theoretical considerations, indicate that non-Gaussian clutter arises as an effect of a doubly stochastic scattering mechanism wherein a spiky component, $s(t)$ say, which accounts for the overall non-Gaussian nature of the clutter, modulates a more rapidly fluctuating Gaussian component (commonly referred to as speckle), $g(t)$ say, i.e.

$$c(t) = s(t)g(t). \tag{1}$$

Thus, insofar as the radar processing interval is sufficiently short, the underlying spiky component can be thought of as a random constant and, in turn, the overall process can be described in terms of a Spherically Invariant Random Process (SIRP), namely

$$c(t) = sg(t). \tag{2}$$

Based on the SIRP model for clutter returns, *conventional* detectors, namely those designed for detecting radar targets embedded in Gaussian clutter, have been assessed in the presence of non-Gaussian disturbance [4]. The analysis highlights that conventional processors suffer marked performance degradation. Moreover, setting the threshold to achieve the desired Probability of false alarm (P_{fa}) requires knowledge of the clutter apdf up to its distributional parameters; more precisely:

- The threshold level is strongly dependent, within a given family of distributions, on the actual clutter shape parameter and mismatch between design and actual clutter statistics may result in unacceptable false alarm rate inflation. For example, if the threshold of an envelope detector, coherently integrating $N = 4$ pulses, is set to ensure $P_{fa} = 10^{-4}$ in presence of Weibull clutter with a shape parameter b equal to 0.5, the actual P_{fa} in presence of Weibull distributed clutter with (the same power and) shape parameter equal to 1 is $\approx 2 \times 10^{-11}$.
- An accurate threshold setting may even require knowledge of the actual family being in force. For example, if the threshold of the above envelope detector is set to ensure $P_{fa} = 10^{-4}$ in presence of Weibull clutter with

a shape parameter b equal to 0.53, the actual P_{fa} in presence of K-distributed clutter with shape parameter $\nu = 0.1$[1] is $\approx 4 \times 10^{-6}$.

Optimized detectors allow to considerably increase the maximum radar range (for a given visibility factor) with respect to conventional receivers [5, 6]. In particular, the Generalized Likelihood Ratio Test (GLRT) was proved to be advantageous in that it allows to trade a small additional loss for reduced complexity as well as robustness with respect to target phase [6]. However, a definite validation of a GLRT-based strategy, as a suitable means to detect coherent target echoes in presence of non-Gaussian noise, requires a through, still lacking, investigation of the effects of the incomplete knowledge of the clutter statistics.

In this paper we introduce and assess a GLRT-based receiver designed to detect coherent pulse trains embedded in correlated Weibull-distributed clutter. In particular, we assess its performance in presence of mismatch between the design and the actual statistics of both the useful target and the disturbance: in other words, we investigate to what extent a possible mismatch affects the P_{fa} and the Probability of detection (P_d). The paper is organised as follows: next section reviews the clutter model and introduces the structure of the GLRT-based receiver; section 3 contains the performance assessment and, finally, section 4 some concluding remarks.

2. OPTIMISED DETECTION IN SIRP

The problem of detecting signals embedded in additive disturbance can be stated in terms of the following hypotheses test

$$\begin{cases} H_0: & \mathbf{r} = \mathbf{c} \\ H_1: & \mathbf{r} = \alpha\mathbf{p} + \mathbf{c} \end{cases} \tag{3}$$

where $\mathbf{r} = \mathbf{r}_I + j\mathbf{r}_Q$, $\mathbf{p} = \mathbf{p}_I + j\mathbf{p}_Q$ and $\mathbf{c} = \mathbf{c}_I + j\mathbf{c}_Q$ are $N-$dimensional, complex vectors whose components are samples from the baseband equivalent of the received signal, of the transmitted signal and of the clutter, respectively, while $\alpha = Ae^{j\theta}$ is a complex gain accounting for the channel effects and the target Radar Cross Section (RCS).

As already mentioned, the experimental evidence suggests that the complex envelope $c(t)$ of the clutter returns is to be described in terms of a SIRP. Equivalently, the row vector $\mathbf{c}$ is a Spherically Invariant Random Vector (SIRV) and can be written as

$$\mathbf{c} = s\mathbf{g}, \tag{4}$$

where the Gaussian vector $\mathbf{g}$ is assumed to have the circular property associated with the inphase and the quadrature components of a Wide Sense Stationary bandpass process, and, without loss of generality, the so-called *modulating* variate s, is assumed to have unitary second moment.

Remarkably, the SIRP model allows a complete specification of the clutter process even in the case of correlated observations. Precisely, the $N-$dimensional pdf of a zero-mean complex SIRV $\mathbf{c}$ can be cast as

$$f_{\mathbf{c}}(\mathbf{x}) = \mathrm{B}\, h_N\left(\| \mathbf{x} \|_{\mathbf{M}}\right) \tag{5}$$

[1]That value of ν and a proper choice of the corresponding scale factors yields a K-apdf with the same first two moments of a Weibull apdf with $b = 0.53$.

where B is a suitable normalization factor, $h_N(\cdot)$ is related to the marginal apdf of the process and $\| \mathbf{x} \|_{\mathbf{M}}$ is the norm of $\mathbf{x}$ with respect to the positive-definite matrix $\mathbf{M}^{-1}$, where $\mathbf{M}$, in turn, is the autocovariance matrix of $\mathbf{c}$.

When the clutter covariance matrix $\mathbf{M}$ is known, the closure property of SIRV's [7] allows one to apply the whitening approach to detect signals in correlated disturbance. Therefore, at the design stage, we limit ourselves to the case of uncorrelated noise (i.e., the vector $\mathbf{c}$ possesses identity covariance matrix) with the understanding that, if the clutter is correlated, $\mathbf{p}$ represents the useful signal at the output of the whitening filter. It is easy to see that the GLRT for the hypotheses testing problem (3) is given by [5]

$$\frac{h_N\left(\sqrt{\| \mathbf{r} \|^2 - \frac{|\mathbf{r}\cdot\mathbf{p}|^2}{\|\mathbf{p}\|^2}}\right)}{h_N(\| \mathbf{r} \|)} \underset{H_0}{\overset{H_1}{\gtrless}} T. \tag{6}$$

The corresponding receiving structure, referred to as ML detector in the following, can be interpreted as a generalization of the conventional minimum-distance receiver, see [5, 6] for more details.

A relevant feature of the proposed detection scheme is that the pdf of the test statistic and, hence, the detection performances depend upon the signal-to-noise ratio (snr)

$$\mathrm{snr} = \frac{\| \alpha\mathbf{p} \|^2}{\mathrm{E}\left[\| \mathbf{c} \|^2\right]} = \frac{A^2 \| \mathbf{p} \|^2}{N}, \tag{7}$$

but they are otherwise independent of the signal pattern $\mathbf{p}$ and of the train phase θ, whether θ is an unknown deterministic parameter or a random variate.

The actual implementation of the above receiver requires knowledge of the clutter apdf up to its distributional parameters. With regard to, the most credited non-Rayleigh apdf's are the Weibull [3]

$$f_R(u) = abu^{b-1}\exp\left(-au^b\right) \qquad u \geq 0, \quad a,\, b > 0 \tag{8}$$

and the K-distribution [2]

$$f_R(u) = \frac{a^{\nu+1}u^{\nu}}{2^{\nu-1}\Gamma(\nu)}K_{\nu-1}(au) \qquad u \geq 0, \quad a,\, \nu > 0 \tag{9}$$

where $\Gamma(\cdot)$ is the Eulerian function, $K_\nu(\cdot)$ is the modified second-kind Bessel function of order ν, b and ν are *shape* parameters ruling the tail decay, and, in both cases, a is a *scale* parameter related to the common variance of the clutter quadrature components. Both the Weibull and the K-distribution are compatible with the compound-Gaussian model in the range of interest of the respective parameters [7]. In particular, for a SIRV with a Weibull apdf $h_N(x)$ is given by [7]

$$h_N(x) = \sum_{k=1}^{N} A_k x^{kb-2N} e^{-ax^b} \tag{10}$$

where

$$A_k = \sum_{m=1}^{k} (-1)^{m+N} \frac{a^k 2^N}{k!} \begin{pmatrix} k \\ m \end{pmatrix} \frac{\Gamma\left(m\frac{b}{2}+1\right)}{\Gamma\left(m\frac{b}{2}+1-N\right)}, \tag{11}$$

with $a = (\Gamma(1 + 2/b)/2)^{b/2}$ in eqns. (10) and (11).

3. PERFORMANCE ASSESSMENT

In the previous section we have derived the structure of the ML detector when the clutter is modelled as a SIRP process with a Weibull-apdf and given distributional parameters. As in practice both the target and the clutter statistics are not perfectly known, the need arises to evaluate the effects of a possible mismatch between the actual and the design assumptions. Since closed-form expressions for the pdf of the test statistic under the two hypotheses are not available, the performances were obtained by computer simulations assuming, regard to the transmitted signal, a rectangular pulse-train with zero-doppler shift.

3.1 Sensitivity to the target model

The analysis of the detector (6) can be carried on with reference to the case of uncorrelated clutter only: in fact, the clutter correlation can be easily accounted for through a detection gain, depending upon the clutter and the signal spectral properties [5]. While at the design stage we modelled A and θ as unknown parameters, now we also consider the case that they fluctuate according to a given pdf. Obviously, the ML detector is *canonical* with respect to their law in its nature; in addition, its performance is independent of the actual pdf of the target phase. In order to analyse the effects on the performance of the distribution of A, we assume that the RCS of the target fluctuates according to the exponential law.

The Receiver Operating Characteristics (ROC's) of the ML detector are reported in figure 1 for both steady and fluctuating targets, several values of the shape parameter b of the clutter, $N = 4$ integrated pulses[2], and $P_{fa} = 10^{-6}$. The shape parameter of the clutter is seen to critically affect the performance in the case of non-fluctuating targets. On the contrary, in case of exponentially fluctuating RCS, the performances are significantly influenced by the shape parameter only in the range of low snr's - where the curves corresponding to the two fluctuation models are very close to each other - while for high snr values they are primarily ruled by the Rayleigh fluctuation law. The performance of the conventional detector under the same instances of clutter distribution and target amplitude fluctuation are reported as well: the comparison of the operating characteristics of the two receivers shows that the ML detector largely outperforms the conventional one in presence of fluctuating targets for any value of snr.

3.2 Sensitivity to the clutter model

3.2.1 Compound-Gaussian clutter vs SIRP: At the design stage the slowly fluctuating spiky component was approximated with a random constant in order to keep the analytical complexity at a reasonable level. However, strictly speaking, $s(t)$ is a non-constant random process. It is not really difficult to quantify a-posteriori the effects of our approximation, evaluating the performances of the receiver (6) in presence of uncorrelated, compound-Gaussian clutter: to this end realizations of the clutter with an exponential apdf

[2]Hereafter we will implicitly assume $N = 4$ integrated pulses.

ρ_S	ρ_G	$P_{fa}/10^{-4}$
0.95	0.93	0.4
0.90	0.87	0.3

Table 1: Estimated P_{fa} in presence of compound-Gaussian clutter.

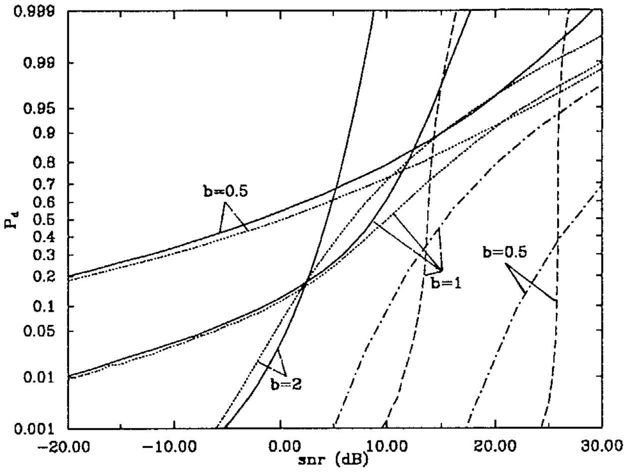

Figure 1: ROC's of the ML (steady: ————, fluctuating targets: ·········) and of conventional detector (steady :– – – – –, fluctuating targets: – · – · – ·) .

($b = 1$) are generated resorting to the procedure illustrated in [7]. More precisely, the modulating sequence, generated according to the Wiener model, is the output of a plain full wave rectifier fed by a correlated Gaussian process: the Zero-Memory Non-Linearity controls the pdf of $s(\cdot)$ while a single-pole Auto-Regressive (AR) filter controls the value of $\rho_S = \frac{r_S(1)}{r_S(0)}$, the one-lag normalised correlation of the modulating sequence. To be more definite, the output of the AR filter is a complex, exponentially correlated, Gaussian process whose one-lag correlation coefficient, ρ_G say, is fixed, based on a "trial and error" procedure, in order to ensure the desired value of ρ_S.

The effects of a non-constant spiky component on the P_{fa} are summarized in table 1: although we have considered values of ρ_S well beyond those of practical interest [8], the estimated P_{fa}'s are close to and less than the nominal value. In addition, it can be seen that the mismatch results in a negligible improvement for the values of P_d's of interest (> 0.5) provided that the threshold is set in order to ensure the nominal FAR.

3.2.2 Unknown clutter apdf: So far, we have assumed complete knowledge of the clutter marginal apdf: precisely, the clutter apdf was a Weibull distribution with known shape and scale parameters. In practice, the noise statistics are not known up to the distributional parameters, moreover not even the membership of the received clutter echo to either the Weibull or the K- family (of univariate distributions) can be definitely claimed, since both of them achieve reasonable fit to experimental data. Thus, it is of primary concern to investigate the sensitivity of the previously introduced detector to possible mismatching between the design and the actual marginal distribution of the noise.

As a case study, we evaluated the actual performance in

b_D	b	$P_{fa}/10^{-4}$
0.5	1.0	1.2
1.5	1.0	8.2

Table 2: Estimated P_{fa} in presence of mismatch between the design and the actual value of the clutter shape parameter.

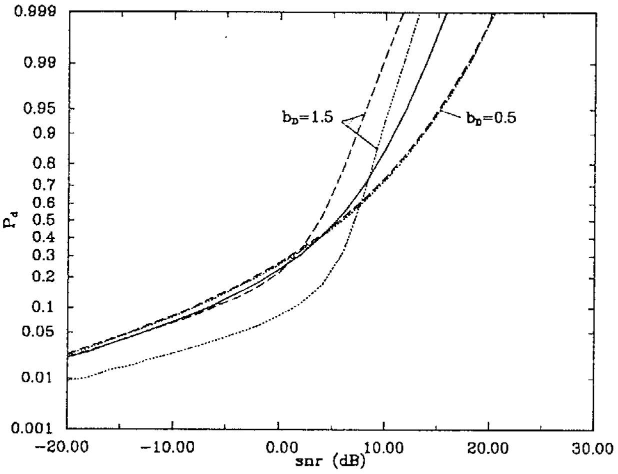

Figure 2: Performance of the ML detector in Weibull clutter with $b = 1$: ——— ML detector with $b_D = 1$, $P_{fa} = 10^{-4}$; ········ ML detector with $b_D = 0.5$ or $b_D = 1.5$, $P_{fa} = 10^{-4}$; - - - - - ML detector with $b_D = 0.5$ or $b_D = 1.5$, and the threshold set to ensure $P_{fa} = 10^{-4}$ under the design assumptions.

presence of Weibull clutter with (known power and) $b = 1$ when the detector is designed for $b_D = 0.5$ or $b_D = 1.5$. In both cases the threshold is set in order to ensure $P_{fa} = 10^{-4}$ under the design assumptions. The actual values of P_{fa} are reported in table 2 while the corresponding ROC's are plotted in figure 2 together with the curve (solid one) referring to a perfect knowledge of the clutter statistics.

For sake of completeness the ROC's (dotted curves) relative to the nominal P_{fa} are plotted too. The results show that the FAR does not substantially vary as compared to the nominal level and that the actual shape parameter is overall influential on the detection performance with the understanding that the *snr*'s variations, for any P_d of interest, are kept within some decibels (dB's).

The robustness of the ML detector to mismatch between the adopted and the actual family of distributions can be studied through table 3 and the curves of figure 3. We considered a ML detector designed under the hypothesis of Weibull-distributed clutter when the input noise is K-distributed: the shape and the scale parameters of the input noise are such that its first two moments are equal to those of the design distribution [7]. Table 3 shows for two values of b_D (ν) that the actual P_{fa}'s are only marginally affected by such a mismatching; figure 3 confirms the overall robustness of the ML detector in the interest region of P_d's.

4. CONCLUSIONS

This paper has handled the detection of coherent pulse trains embedded in compound-Gaussian clutter. In particular we have introduced a GLRT-based detector and analysed its robustness in regard to mismatch between the design and the operating conditions. The analysis has showed that this receiver may be a suitable means to detect both fluctuating and non-fluctuating targets embedded in non-Gaussian clutter for those situations where the clutter statistics can be estimated off-line.

b_D	ν	$P_{fa}/10^{-4}$
0.53	0.1	1.2
1.54	2.0	1.8

Table 3: Estimated P_{fa} in presence of mismatch between the design and the actual family of apdf's.

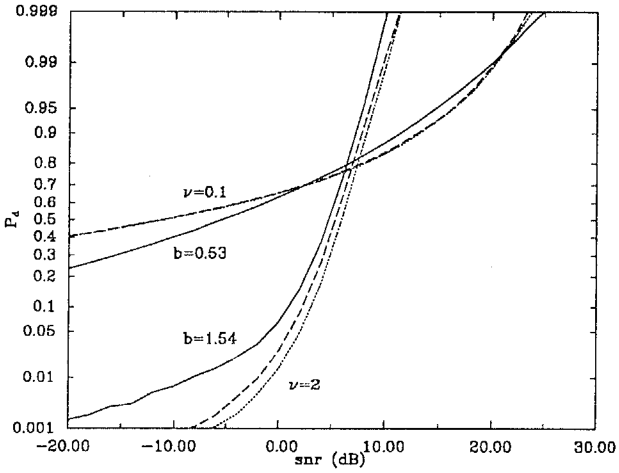

Figure 3: Performance of the ML detector designed under the assumption of Weibull-distributed clutter: ——— Weibull input noise, $P_{fa} = 10^{-4}$; ········ K-distributed input noise, $P_{fa} = 10^{-4}$; - - - - - K-distributed input noise and the threshold set to ensure $P_{fa} = 10^{-4}$ in presence of Weibull-distributed clutter.

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