Synchronization in OFDM Systems
Sensitivity to the Choice of Pulse Shape

Julia Martinez Arenas[1] **Daniel Landström**[2] **Jan-Jaap van de Beek**[2]
Per Ola Börjesson[2] **Marie-Laure Boucheret**[1] **Per Ödling**[2]

Ecole National Supérieure de Telecommunications, Site de Toulouse, BP 4004 CEDEX, Toulouse, France
E-mail: martinez@tlse.enst.fr
Luleå University of Technology, Div. of Signal Processing, SE-971 87 Luleå

RÉSUMÉ

Problème traité : dans cette publication, nous nous intéressons au problème de la synchronisation dans les systèmes à porteuses multiples : estimation du début du symbole et de l'écart de fréquence. Nous utilisons la redondance introduite par le temps de garde ainsi qu'une fonction de mise en forme des symboles.
Originalité de la contribution : Les performances d'un système utilisant uniquement la corrélation introduite par le temps de garde ont été évaluées dans un article précédent dans le cas d'un canal gaussien. Dans cette contribution, nous introduisons une fonction de mise en forme sur les symboles OFDM et nous évaluons les performances du système ainsi obtenu.
Résultat nouveaux : Nous avons déterminé les performances du système proposé pour différentes fonctions de mise en forme.

ABSTRACT

Solved Problem: Symbol arrival time and carrier frequency offset estimation is addressed in an OFDM system using symbol extension with a cyclic prefix and employing pulse shaping.
Originality of the work: Previous work has presented a method that without the use of pilots exploits the inherent correlation in the cyclically extended OFDM symbol in Gaussian channels. This contribution investigates what accuracy is achievable with this method in systems using pulse shaping.
New results: The performance of the joint Maximum Likelihood estimator for a time and frequency offset is evaluated for different pulse shapes.

1 Introduction

OFDM for mobile wireless communications has been a subject of research for some time. One of the main challenges in such systems is the synchronization, both in time and in frequency [1]–[3]. Various time and frequency estimation methods based on pilot symbols have been presented. As an alternative, or a complement, to pilot aided synchronization, redundant information in the cyclic prefix is exploited in [4]–[8] for the synchronization.

Meanwhile, the use of pulse shaping in OFDM systems has been proposed to improve side lobe suppression [9],[10]. A joint Maximum Likelihood estimator of time and frequency offsets for Gaussian channels without pulse shaping was presented in [8], and with pulse shaping in [11]. Here, we investigate the effects of the choice of pulse shape on the performance of the estimator in [11], highlighting two counteracting phenomena introduced by the pulse shaping: the loss of signal power in the cyclic prefix and the introduction of a time-varying power profile of the received signal.

In Section 2 the signal model and the estimator from [11] are briefly outlined. In Section 3 we present the different pulse shapes investigated here and the performance of the estimator in systems employing these pulse shapes. Finally, Section 4 discusses the results of the simulations.

2 Signal Model and Estimator Structure

For synchronization purposes we are interested in the estimation of arrival time and frequency offset of a received OFDM symbol. The optimal joint estimator for time and frequency offsets based on the model described below, has been developed in [11].

An OFDM signal composed of a large number of carriers has statistical properties similar to a discrete-time Gaussian process. In our model, the OFDM signal comprises N carriers and is extended with a cyclic prefix of L samples. Furthermore, a pulse shaping window is employed at the transmitter. We model the received signal, $r(k)$, as

$$r(k) = g(k-\theta)s(k-\theta)e^{j2\pi\varepsilon k/N} + n(k). \tag{1}$$

The parameters θ and ε represent the symbol arrival time and the carrier frequency offset, respectively. The signal $s(k)$ is the transmitted signal (assumed to be Gaussian), $g(k)$ is the square root of the received signal's expected power profile (it represents the attenuation introduced by the pulse shape), and $n(k)$ is additive complex white Gaussian noise at the receiver. We consider an infinite observation interval. The choice of $g(k)$ may be based on a number of assumptions about the system and is clearly an important part of the estimator design. For instance, the transmitted OFDM symbol of one user may be surrounded by OFDM symbols from other users. We have chosen $g(k)$ as the defined pulse shape, $p(k)$, surrounded on both sides by half a pulse shape and then extended at a constant level. Depending on the *system load*, which we define as the probability that a neighbouring time slot is being used, the extended half pulse shapes are attenuated to model an average power level. An example is shown in Figure 1.

Given the received signal $r(k)$ in (1), the log-likelihood function of θ and ε can be written as [11]

$$\Lambda(\theta, \varepsilon) = |\gamma(\theta)| \cos(2\pi\varepsilon + \gamma(\theta)) + \beta(\theta), \qquad (2)$$

where

$$\gamma(\theta) = \sum_{k=\theta}^{\theta+L-1} h_1(k-\theta) r^*(k) r(k+N), \qquad (3)$$

$$\beta(\theta) = \sum_{k=-\infty}^{\infty} h_2(k-\theta) |r(k)|^2, \qquad (4)$$

and where

$$h_1(k) = \frac{2g(k)g^*(k+N)}{|g(k)|^2 + |g(k+N)|^2 + \frac{1}{\text{SNR}}}, \qquad (5)$$

$$h_2(k) = \begin{cases} \left(\frac{1}{|g(k)|^2+|g(k+N)|^2+\frac{1}{\text{SNR}}}\right)|g(k)|^2, \\ \qquad k \in [\theta, \theta+L-1] \\ \left(\frac{1}{|g(k-N)|^2+|g(k)|^2+\frac{1}{\text{SNR}}}\right)|g(k)|^2, \\ \qquad k \in [\theta+N, \theta+N+L-1] \\ \left(\frac{1}{|g(k)|^2+\frac{1}{\text{SNR}}}\right)|g(k)|^2, & \text{otherwise.} \end{cases} \qquad (6)$$

The joint ML-estimate of the OFDM symbol arrival time θ and the frequency offset ε is obtained as the arguments maximizing the log-likelihood function (2). In [11] it is shown that

$$\widehat{\theta}_{\text{ML}} = \arg\max \{|\gamma(\theta)| + \beta(\theta)\},$$

$$\widehat{\varepsilon}_{\text{ML}} = -\frac{1}{2\pi}\measuredangle\gamma(\widehat{\theta}_{\text{ML}}).$$

This estimator can be realized using two filters, $\{h_1(k)\}$ and $\{h_2(k)\}$. The filter coefficients depend on the choice of $g(k)$ and on the Signal-to-Noise Ratio (SNR). The filter $\{h_1(k)\}$ extracts information contained in the correlation introduced by the cyclic prefix and the filter $\{h_2(k)\}$ can be viewed as a filter matched to the expected power profile.

The offsets are estimated individually for each symbol. We make no use of the potential correlation between the time and frequency offsets from one OFDM symbol to the next. In a real system averaging of the likelihood function or the estimates would typically be used if such correlation exists.

3 Performance Simulations

A number of pulse shapes have been proposed to reduce the out-of-band power of OFDM signals. By simulations, the estimator's performance is evaluated for the following four different pulse shapes:

1. We include the regular rectangular pulse shape, for instance given in [12], [10], as our reference pulse shape. The rectangular pulse shape is defined as

$$p(k) = \begin{cases} 1 & 0 \leqslant k < N+L \\ 0 & \text{otherwise} \end{cases}.$$

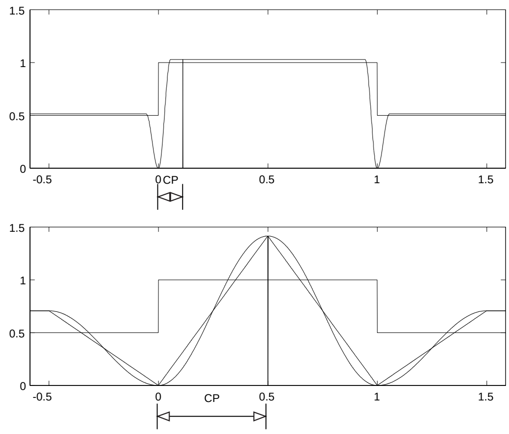

Figure 1 — The choices of the expected power profile $g(k)$ used in this contribution. Based on the rectangular and Tukey pulse (upper figure) and on the rectangular, triangular, and Hanning pulse (lower figure).

2. An alternative pulse shape is the Tukey pulse shape [10]:

$$p(k) = \begin{cases} 1 & \left|k - \frac{N+L}{2}\right| \leqslant \alpha\frac{N+L}{2} \\ \frac{1}{2} + \frac{1}{2}\cos\left(\frac{k-(1+\alpha)(N+L)/2}{(1-\alpha)(N+L)/2}\pi\right) \\ \qquad \alpha\frac{N+L}{2} \leqslant \left|k - \frac{N+L}{2}\right| \leqslant \frac{N+L}{2} \\ 0 & \text{otherwise} \end{cases},$$

where α is the ratio between the number of shaped samples and the total amount of samples. Compared to the rectangular pulse it has the edges shaped with a raised cosine. In the simulations we use the values $\alpha = \frac{1}{36}$ and $\alpha = \frac{1}{9}$ and denote these choices by Tukey-16 and Tukey-64, respectively.

3. Other approaches to choosing the shape of the OFDM symbol have been presented. A doubling of the symbol, *i.e.*, a cyclic prefix of a 100%, and a subsequent shaping of the entire symbol with a triangular weighting window preserves the orthogonality between every second sub-carrier. The resulting triangular window is defined as (to be applied with $N = L$)

$$p(k) = \begin{cases} 4k/(N+L) \\ \qquad 0 \leqslant k < (N+L)/2 \\ 4 - 4k/(N+L) \\ \qquad (N+L)/2 \leqslant k < N+L \\ 0 & \text{otherwise} \end{cases}.$$

4. Similarly, the Hanning window [13], [10] can be used, where

$$p(k) = \begin{cases} \frac{1}{2}\left(1 - \cos\left(\frac{2\pi k}{N+L-1}\right)\right) \\ \qquad 0 \leqslant k < N+L \\ 0 & \text{otherwise} \end{cases}.$$

From these pulse shapes we can calculate the expected power profile.$g(k)$. In Figure 1 the expected power profile

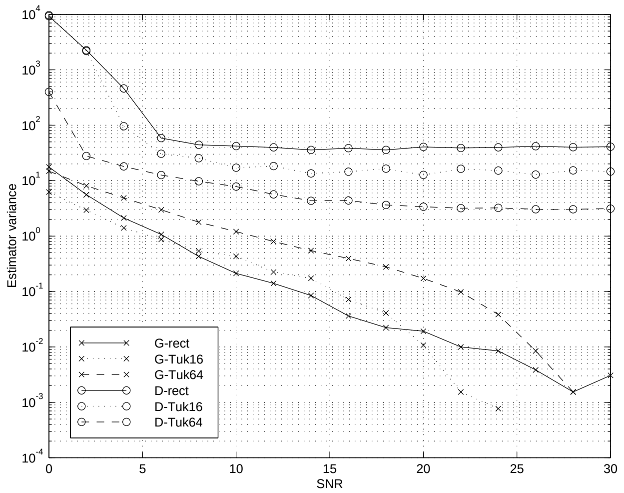

Figure 2 — The performance of the timing estimator for the AWGN channel (G) and the dispersive channel (D). Curves are shown for the rectangular pulse (solid), the Tukey-16 pulse (dotted), and the Tukey-64 pulse (dashed).

for the different pulse shapes and a system load of 50% is depicted.

An OFDM system with 1024 carriers and a system load of 1 has been considered for the performance simulations. For the simulation using the rectangular and Tukey shape, a cyclic prefix of 128 samples has been used. In the case of the rectangular, the triangular and the Hanning shape the cyclic prefix was 1024 samples, *e.g.,* a repetition of the symbol.

Figures 2 - 5 show the variance of the joint time and frequency offset estimator for the different pulse shapes. Both an additive white Gaussian noise (AWGN) channel and a dispersive channel (with exponentially decaying power profile and a delay spread of 40 samples) have been considered. The impulse response of the dispersive channel used in the simulations is

$$h(n) = \begin{cases} e^{-n/20} & 1 \leqslant n < 40 \\ 0 & \text{otherwise} \end{cases} .$$

Note that the estimators are designed for the AWGN channel, while we evaluate them for both the AWGN and a dispersive case.

4 Discussion

We have investigated two different types of OFDM systems: one with a small cyclic prefix where we have compared the standard rectangular pulse shape with the Tukey pulse shape, and one where the symbol has been doubled and where we have compared a triangular and a Hanning pulse shape. Comparing the rectangular with the Tukey pulse shape; the curves indicate that, for arrival time estimation in the AWGN channel, the loss of signal power in the cyclic prefix due to the shaping is only partly compensated for by the information in the time varying pulse shape. For the estimation of frequency

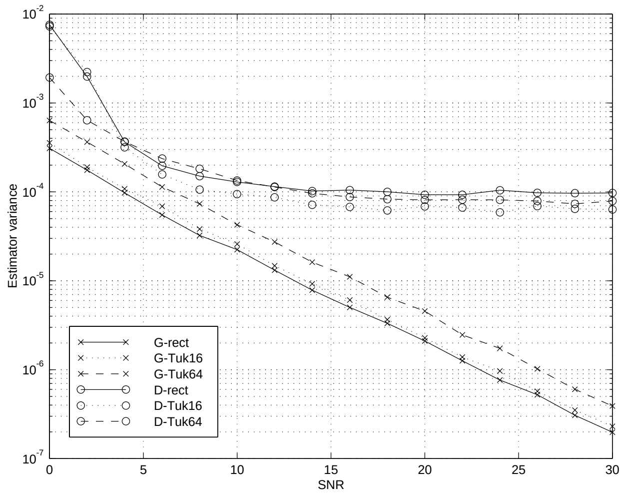

Figure 3 — The performance of the frequency offset estimator in the AWGN channel (G) and the dispersive channel (D). Curves are shown for the rectangular pulse (solid), the Tukey-16 pulse (dotted), and the Tukey-64 pulse (dashed).

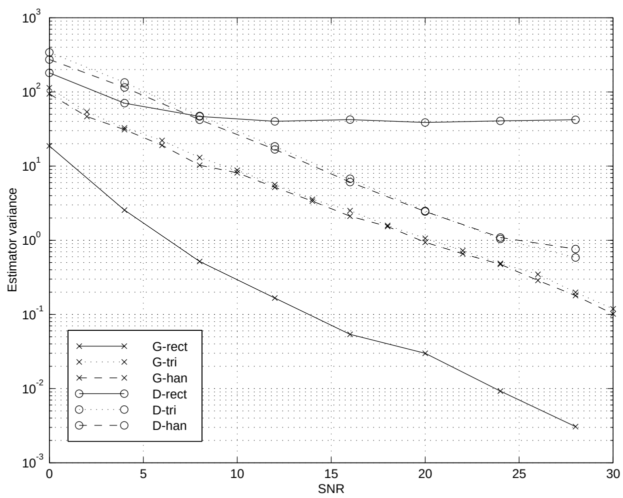

Figure 4 — The performance of the timing estimator for the AWGN channel (G) and the dispersive channel (D). Curves are shown for the rectangular pulse (solid), the triangular pulse (dotted), and the Hanning pulse (dashed).

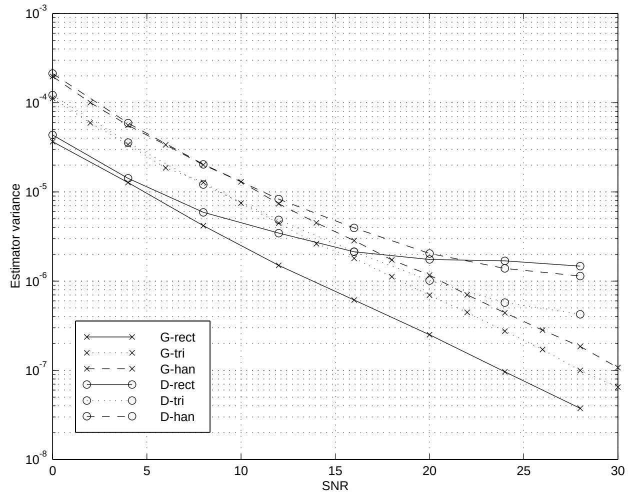

Figure 5 — The performance of the frequency offset estimator for the AWGN channel (G) and the dispersive channel (D). Curves are shown for the rectangular pulse (solid), the triangular pulse (dotted), and the Hanning pulse (dashed).

offset moderate shaping can be done essentially maintaining performance. In the case of dispersive channels, we can see how the absolute performance decreases due to the dispersion introduced by the channel. This is not unexpected as our signal model contains no dispersion. As opposed to the AWGN case, the robustness introduced by the time-varying signal power adds significantly to the estimator's performance.

The Hanning and the triangular pulse shapes have similar performance for both time and frequency estimation. However, the difference between their performance and the performance obtained with the rectangular window is notable. The rectangular window clearly has superior performance for the AWGN channel, but suffers from a high error floor for the dispersive channel. The error floor arises as the peak of the likelihood function is broadened by the dispersion and the data on the sides of the cyclic prefix mix with the repeated parts and act as strong additive noise.

It is straightforward to extend the signal model to include a dispersive channel. However, the optimal estimator seems to become very complex and does not have an obvious filter implementation. As an alternative, introducing pulse shaping may offer some robustness against dispersion. This presumes that the pulse shaping is done in such a way that time-variations in the received power occur, which, for instance, may not be the case using overlapping pulses.

We conclude that the use of pulse shaping may be advantageous unless the limiting factor is the frequency offset estimation. Especially, it seems to introduce some robustness against modelling errors, of which we have investigated unmodelled channel time-dispersion.

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