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Reducing the Peak to Average Power Ratio in OFDM Systems

Mario BONACCORSO[1], Karim MHIRSI[1], Karim MAOUCHE[1], Véronique BUZENAC[2]

[1]Institut Eurécom
2229 Route des Crêtes, BP 193, 06904 Sophia-Antipolis

[2]Centre de Recherche Motorola Paris
Espace Technologique Saint-Aubin, 91193 Gif-Sur-Yvette

bonaccor@eurecom.fr, mhirsi@eurecom.fr, maouche@eurecom.fr, buzenac@crm.mot.com

Résumé – Nous donnons un aperçu de l'état de l'art en réduction du rapport puissance maximale à puissance moyenne (PAPR) pour les systèmes OFDM, puis, nous présentons une nouvelle méthode permettant une bonne réduction du PAPR tout en ayant une complexité de calculs et une redondance relativement faibles. Cette méthode introduit de l'interférence entre porteuses (ICI). Nous montrons analytiquement comment réduire l'ICI et donnons quelques résultats de simulations.

Abstract – We review some techniques for reducing the Peak to Average Power Ratio (PAPR) in OFDM systems and present a new method called Discrete Clipping. This method achieves a good reduction of the PAPR with a low computational complexity and a small redundancy at the expense of some Inter Carrier Interference (ICI). Through analysis, we show how to reduce the amount of ICI. Finally, some simulations are given.

1 Introduction

Although ignored during several years, multi-carrier modulation has become a strong competitor to classical single carrier modulations, mainly because of their very attractive properties. Indeed, besides flexibility and bandwidth efficiency which are inherent qualities of OFDM (Orthogonal Frequency Division Multiplexing) systems, they also allow communications in severe channels without heavy equalization. That is the reason why OFDM is now used in a wide range of standards such as DAB (Digital Audio Broadcasting), ADSL (Asymmetric Digital Subscriber Line), HIPERLAN/2 (HIgh PErformance Radio Local Area Network).

However, one of the main drawbacks of OFDM systems is that they can present a high Peak to Average Power Ratio (PAPR) when the phases of the different carriers add constructively to form large peaks. When such peaks occur, they may be cut-off by the amplifier nonlinearities, which leads to out of bands radiations and Inter Carrier Interference (ICI) at the receiver, which may degrade considerably the performances. To avoid this degradation, one has either to use amplifiers with large back-offs, which are both expensive and less efficient, or to reduce the PAPR before the transmission. This paper focuses on the second solution. Namely, we first provide a review of the already existing PAPR reduction techniques for OFDM systems. Afterwards, we propose a new PAPR reduction method that we have called discrete clipping, whose performances approach those of the best known methods while being much more simple. This new method introduces ICI that we have analyzed by deriving an upperbound on the Signal-to-Interference Ratio (SIR).

2 OFDM systems and PAPR

In this section, useful definitions on OFDM systems and their PAPR are provided. The complex envelope of an OFDM system may be written as:

$$x(t) = \sum_{\mu=-\infty}^{+\infty} \sum_{n=-K/2}^{K/2} A_{\mu,n} g(t-\mu(T-T_g)) e^{2j\pi nt/T} \ , \quad (1)$$

where the $A_{\mu,n}$ are the complex informative symbols (chosen in an alphabet of M symbols) to transmit (for example QPSK symbols), T is the inverse of the carrier spacing and T_g is the guard interval duration. Let us note $T_s = T+T_g$ the whole OFDM symbol duration. Remember that a guard interval is needed to ensure the classical OFDM easy equalization scheme.

It can be easily shown that sampling $x(t)$ with a period $T/N, N \geq K$, leads to the digital baseband implementation of the OFDM modulator represented on figure 1. N is the number of carriers, and K the number of useful carriers. The $N-K$ carriers containing no information are called virtual carriers.

The PAPR of the signal $x(t)$ on an interval $[\mu T_s, (\mu+1)T_s]$, that is on one single OFDM symbol, is:

$$\chi_{cont,\mu} = \frac{\max\limits_{t\in[\mu T_s,(\mu+1)T_s]} |x(t)|^2}{\int_{\mu T_s}^{(\mu+1)T_s} |x^2(t)|dt} \ . \quad (2)$$

A PAPR can also be defined in the discrete-time domain:

$$\chi_{disc,\mu} = \frac{\max\limits_{\rho} |a_{\mu,\rho}|^2}{E\{|a_{\mu,\rho}|^2\}} \ , \quad (3)$$

where $\mathbf{a}_\mu = [a_{\mu,0} \cdots a_{\mu,N-1}]$ is obtained by pre and post-concatenating the vector $\mathbf{A}_\mu = \left[A_{\mu,-\frac{K}{2}} \cdots A_{\mu,\frac{K}{2}}\right]$ with zeros to obtain a N length vector and by taking its Inverse Discrete Fourier Transform (IDFT). In [1], it is proved that the difference between the PAPR of the continuous and discrete signals is marginal. In the sequel, we will focus on the discrete-time definition of the PAPR. The PAPR of one single symbol can be as high as N, that is the maximum PAPR increases with the number of carriers.

3 State of the art

Several techniques have been proposed for reducing the PAPR in OFDM systems. The common idea of many of these methods is the introduction of redundancy. Intuitively, if R_{ac} bits of the OFDM symbols do not contain information but redundancy in some way, it is obvious that the same information is represented by $2^{R_{ac}}$ different OFDM symbols. Then the OFDM symbol having the smallest PAPR can be chosen for the transmission. This consideration is used in many PAPR reduction methods.

The different techniques can be divided in three families, as follows.

3.1 Scaling techniques

Scaling techniques such as Block Scaling and Peak Windowing [2] are used to reduce the continuous-time PAPR. In such methods, the maximum value of the continuous-time signal corresponding to an OFDM symbol is computed and compared to one or several thresholds. If this value exceeds one of the thresholds, the whole signal is scaled by a predefined coefficient so that the amplifiers work in linearity. As an alternative, it can also be multiplied by a window. These methods are quite simple and do not introduce redundancy, but in both cases, the main drawback is that not only the peaks but the whole signal is attenuated, resulting in an increase of the bit error rate

3.2 Coding techniques

The basic idea of these techniques is to avoid the transmission of symbols that present a high PAPR, which in fact, is equivalent to use redundant codes. Many coding techniques have been proposed [3], [4], [5], [6], [7]. In some of them the introduced redundancy is also interestingly used for error detection or correction. Among all the used codes, one may mention Rudin-Shapiro codes [4], Golay sequences [5], M-sequences [7] and Reed-Muller codes [6]. The main drawback of all these coding schemes is that the computational complexity which can be very high when the number of carriers is relatively large.

3.3 Miscellaneous techniques

Some of the methods do not really belong to a specific family, and are described in this section. All of them introduce redundancy. In the Selective Mapping method (SLM) [1], U different OFDM symbols representing the same information are produced using U fixed rotation vectors supposed to be known at the receiver. Each OFDM symbol PAPR is computed and the symbol having the lowest PAPR is selected for transmission. A variant called Partial Transmit Sequence (PTS) [1] consists to split the OFDM symbol in V disjoint subsets of carriers and to choose V rotational factors to apply to the subsets. An exhaustive search of the rotational factors that minimize the PAPR is done. In these two methods, the introduced redundancy is in fact the information on the chosen rotation vectors that has to be transmitted to the receiver as side information. Both methods present really good performances and work for high number of carriers. Their main drawback is their complexity, since several IDFTs have to be computed for each transmitted OFDM symbol. One may also cite the selective scrambling technique where the coefficients of the OFDM symbol $\mathbf{A}_\mu$ are scrambled (using scrambling sequences) and the sequence with the lowest PAPR is sent with the corresponding scrambling sequence. This technique behaves nearly like the the SLM method but due to the fact that scrambling is easier to perform than computing DFTs, it is much easier to implement.

In what follows, we present a new technique for PAPR reduction which is relatively simple and which introduce a small amount of redundancy.

4 Discrete Clipping

Our method is based upon the following idea: rather than clipping or scaling the continuous time signal $x(t)$ like is the case for some systems, we propose to clip or scale the discrete signal $x[k]$, which is equivalent to clip or scale the coefficients $a_{\mu,\rho}$. However, in order to reduce the distortion of the OFDM signal, we constrain the number of the modified coefficient to be relatively small.

From eq.(3), it appears that if some coefficients $a_{\mu,\rho}$ that are beyond a certain threshold are scaled then the numerator can be significantly reduced. Meanwhile, the denominator remains nearly the same if the number of modified coefficient is small. Hence, clipping a small part of the most important coefficients $a_{\mu,\rho}$ can reduce significantly the PAPR.

This is the principle of the method. The complete process is described as follows:

At the Transmitter

- Consider one symbol $\mathbf{A}_\mu$ of length K corresponding to one time interval T_s, add the virtual carriers to obtain a N length vector whose IDFT is $\mathbf{a}_\mu$.
- Compare $\mathbf{a}_\mu$ to a fixed threshold ψ and apply a fixed scaling function f to all the coefficients exceeding the threshold. The new vector is denoted $\tilde{\mathbf{a}}_\mu$: $\forall \mu, \quad \forall 0 \leq \rho < N \quad , \quad$ if $|a_{\mu,\rho}| \leq \psi \Rightarrow \tilde{a}_{\mu,\rho} = a_{\mu,\rho}$ else $\tilde{a}_{\mu,\rho} = f(a_{\mu,\rho})$.
- Modulate the continuous signal $x(t)$ with $\tilde{\mathbf{a}}_\mu$ and transmit it.
- Transmit as side information the indices of the scaled coefficients : $\mathcal{R}_\mu = \{\rho / |a_{\mu,\rho}| > \psi\}$.

At the Receiver

- From the received signal $y(t) = x(t) * c(t)$ ($c(t)$ is the channel impulse response), sample at $t = \mu + \rho T$ and compute the vector $\tilde{b}_\mu$ corresponding to the transmitted vector $\tilde{a}_\mu$: $\tilde{b}_{\mu,\rho} = y(\mu + \rho T)$.
- Compute the vector $\mathbf{b}_\mu$ corresponding to $\mathbf{a}_\mu$ at the transmitter using the received side information : $\forall \rho \in \mathcal{R}_\mu$, $b_{\mu,\rho} = f^{-1}(\tilde{b}_{\mu,\rho})$.
- Compute B_μ which corresponds to the vector A_μ at the transmitter. A simple DFT is required : $B_\mu = \mathrm{DFT}(b_\mu)$.

This technique offers a significant PAPR reduction for all values of N. Unlike coding techniques, it is very effective for large values of N and M. Performances are comparable to those of SLM and PTS if enough scaling is introduced. Unlike SLM and PTS which are computationally heavy, this technique is very simple to implement. At the transmitter only one IDFT, N comparisons and very few divisions (case of the scaling function being a simple division) need to be computed. At the receiver, demodulation is even simpler. It requires one DFT, and few multiplications (applying f^{-1} on the scaled coefficients).
In the discrete clipping technique, redundancy is due to the coding of the indices of the scaled coefficients. The number of redundant bits per symbol is $R_{ac} = \log_2 \binom{\gamma N}{N}$, where γ is the average percentage of scaled coefficients. Henceforth, the ratio of redundant bits is $R_{ac}/2N \log_2 M$ which is low even Redundancy is low (compared to coding techniques) even for small alphabet sizes.
The PAPR is bounded if f is bounded. Indeed, since $\forall \mu$, ρ $|\tilde{a}_{\mu,\rho}| \le f_{\max}$ where $f_{\max} = \max_a |f(a)|$, the PAPR is ensured to be within a fixed limit:

$$\chi_{\max} \simeq \frac{f_{\max}^2}{\int_0^{f_{\max}} u^2 p_{|a|}(u) du} . \tag{4}$$

This property is not present in both SLM and PTS which only reduce the probability of exceeding a certain threshold but never ensure that this PAPR will be under a certain value. However, and this is the strongest drawback of this technique, scaling introduces ISI between symbols of different carriers. One of the most important advantages of OFDM transmission is lost. However, as only a few number of coefficients are modified, the ISI may be small. In fact, there is a trade off between the permitted amount of ISI and the PAPR reduction as is shown in the next section. Finally, note that unlike usual clipping (where this phenomenon is also present), this method does not cause out of band radiation.

5 ICI Analysis

The important predictable drawback of this technique is the introduction of ISI between symbols of different carriers. This is a direct consequence of the modifications applied to vectors $\mathbf{a}_\mu$. In this section we propose to measure this ISI. Our aim is to compute the relation between the end vectors $\mathbf{A}$ and $\mathbf{B}$. We assume that the side information $\mathcal{R}$ is safely transmitted. Using the result obtained in classic OFDM transmission we have:

$$\tilde{\mathbf{b}} = \tilde{\mathbf{a}} \otimes \mathbf{c} . \tag{5}$$

Then, assuming that f is a linear contracting function ($f(x) = \eta x$ where $\eta \in]0, 1[$), the scaling operation can be written analytically as :

$$\tilde{\mathbf{a}} = \mathbf{a} \odot (\mathbf{1} - (1 - \eta)\mathbf{1}_\mathcal{R}) , \tag{6}$$

where $\mathbf{1}$ denotes the all one vector, and $\mathbf{1}_\mathcal{R}$ denotes the vector containing ones on the indices where scaling is performed, and zeros elsewhere. The rescaling operation can similarly be written as:

$$\mathbf{b} = \tilde{\mathbf{b}} \odot (\mathbf{1} - (1/\eta - 1)\mathbf{1}_\mathcal{R}) . \tag{7}$$

Combining equations (5), (6) and (7), we obtain:

$$\begin{aligned} \mathbf{b} \;=\; & \mathbf{a} \otimes \mathbf{c} \\ - & (1-\eta)\,(\mathbf{a} \odot \mathbf{1}_\mathcal{R}) \otimes \mathbf{c} \\ + & (1/\eta - 1)\,\{(\mathbf{a} \otimes \mathbf{c}) \odot \mathbf{1}_\mathcal{R}\} \\ - & (1/\eta - 1)\,(1-\eta)\,\{[(\mathbf{a} \odot \mathbf{1}_\mathcal{R}) \otimes \mathbf{c}] \odot \mathbf{1}_\mathcal{R}\} . \end{aligned} \tag{8}$$

Finally by taking the DFT, we obtain:

$$\mathbf{B} = \mathbf{A} \odot \mathbf{C} + \mathbf{I_1} + \mathbf{I_2} + \mathbf{I_3} . \tag{9}$$

The first term in equation (9) corresponds to the zero-ISI received signal. The three other terms , denoted by $\mathbf{I_1}$, $\mathbf{I_2}$ and $\mathbf{I_3}$, which are the DFT of the second, third and forth terms in (8), correspond to ISI. Using Parseval's identity and Cauchy-Schwarz inequality we are able to bound the energies of these signals:

$$\begin{array}{lll} \|\mathbf{I_1}\|^2 & \le & (1-\eta)^2 \, \|\mathbf{C}\|^2 \, \|\mathbf{a} \odot \mathbf{1}_\mathcal{R}\|^2 \\ \|\mathbf{I_2}\|^2 & \le & \frac{|\mathcal{R}|}{N'} \, (1/\eta - 1)^2 \, \|\mathbf{A} \odot \mathbf{C}\|^2 \\ \|\mathbf{I_3}\|^2 & \le & \frac{|\mathcal{R}|}{N'} \, (1/\eta - 1)^2 (1-\eta)^2 \, \|\mathbf{a} \odot \mathbf{1}_\mathcal{R}\|^2 \, \|\mathbf{C}\|^2 \end{array} . \tag{10}$$

Note that the signal energy is given by $\|\mathbf{S}\|^2 = \|\mathbf{A} \odot \mathbf{C}\|^2$. Considering ISI as noise, we define the ISI-SNR, where only noise due to ISI is considered, by :

$$\mathrm{SNR}_{\mathrm{ISI}}^{-1} = \frac{\|\mathbf{I_1}\|^2 + \|\mathbf{I_2}\|^2 + \|\mathbf{I_3}\|^2}{\|\mathbf{S}\|^2} \tag{11}$$

This noise-to-signal ratio can be bounded by :

$$\begin{aligned} \mathrm{SNR}_{\mathrm{ISI}}^{-1} \;\le\; & (1-\eta)^2 \, \frac{\|\mathbf{C}\|^2 \, \|\mathbf{a} \odot \mathbf{1}_\mathcal{R}\|^2}{\|\mathbf{A} \odot \mathbf{C}\|^2} + \frac{|\mathcal{R}|}{N} \, (1/\eta - 1)^2 \\ & + \frac{|\mathcal{R}|}{N} \, (1/\eta - 1)^2 \, \frac{\|\mathbf{C}\|^2 \, \|\mathbf{a} \odot \mathbf{1}_\mathcal{R}\|^2}{\|\mathbf{A} \odot \mathbf{C}\|^2} . \end{aligned} \tag{12}$$

Note that $\gamma = \frac{|\mathcal{R}|}{N}$ is the percentage of scaled coefficients and is of the order of 1%, both second and third terms of equation (12) are kept within reasonable limits. However the first term is in general much larger due to the fraction $\frac{\|\mathbf{C}\|^2 \, \|\mathbf{a} \odot \mathbf{1}_\mathcal{R}\|^2}{\|\mathbf{A} \odot \mathbf{C}\|^2}$. However, by keeping η close to one, the total ISI can also be kept within reasonable bounds.

Discrete clipping resorts a trade-off between PAPR reduction and level of ICI. The level of ICI can be adjusted by manipulating two parameters that are : η (the scaling factor), and γ (the percentage of scaled factors). ICI increases when both η and γ are fixed, and the number of carriers increases (in fact the amount of ICI depends on the number of scaled coefficients), so in order to make fair comparisons at constant ICI, we take γ inversely proportional to N: $1-\gamma = \text{const}/N$. Figures 2 and 3 compare the performances of discrete clipping and selective mapping techniques. In the first figure, SIR $= 20dB$ and one sees that the performances of discrete clipping are not far from those of SLM. In the second figure, we see that the performances are similar to those of SLM at the expense of an increase in ICI (SIR $= 6dB$).

6 Conclusions

Various techniques for reducing the PAPR in OFDM signals were reviewed. Basic scaling techniques reduce PAPR but decrease the SNR. Coding techniques present good reduction performances but generally introduce a lot of redundancy. Other miscellaneous techniques such as SLM and PTS methods provide both good reduction performances and introduce low redundancies. However, they are still too computationally expensive to be implemented. We have proposed a method called discrete clipping which provides a significant reduction of the PAPR at low computational complexity and with a little a mount of redundancy. In conclusion, the great advantage of discrete clipping remains its simplicity. This new method allows to get the same PAPR reduction as the best other methods, at the expense of an ICI introduction, whose amount may be chosen by tuning two parameters.

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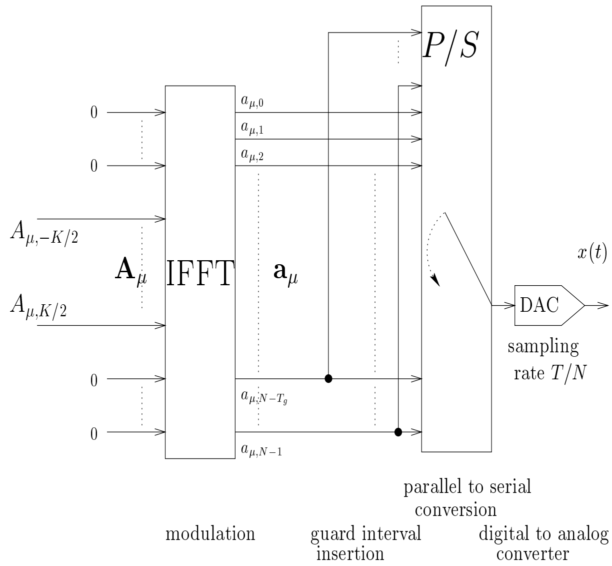

FIG. 1: *OFDM modulator digital baseband implementation.*

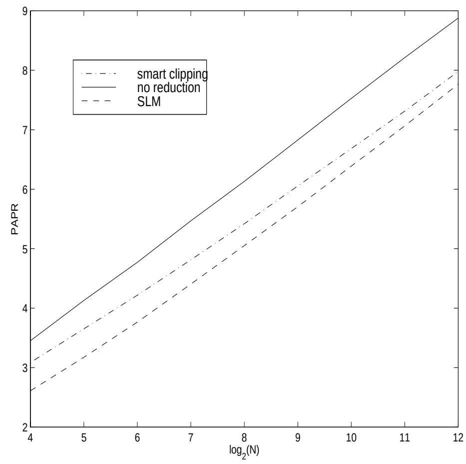

FIG. 2: *Mean PAPR in discrete clipping for SIR $= 20dB$.*

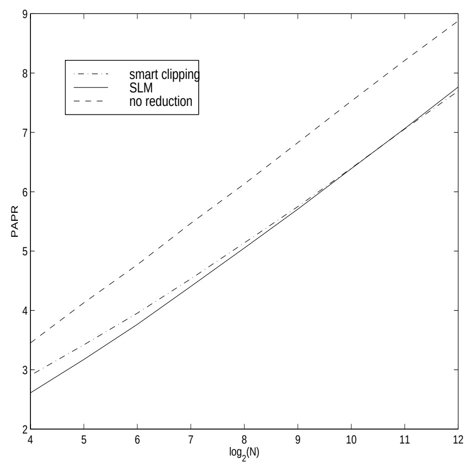

FIG. 3: *Mean PAPR in discrete clipping for SIR $= 6dB$.*