Non-Orthogonal Space-Time Block Coding Design for 3 Transmit Antennas

Berna ÖZBEK[1], Didier LE RUYET[2], Maurice BELLANGER[2],

[1]Izmir Institute of Technology
Electrical and Electronics Eng. Dep., Urla, Izmir, Turkey

[2]CNAM-Electronique
292 rue Saint-Martin 75141 Paris cedex 03, France

bernaozbek@iyte.edu.tr
leruyet@cnam.fr
bellang@cnam.fr

Résumé – Dans cet article, nous proposons un codage spatio-temporel en bloc non orthogonal de rendement symbole 1 pour 3 antennes à l'émission. Ce code permet d'améliorer les performances d'un système de communications sans fil. Le code proposé peut être décodé soit simplement par forçage à zéro soit en utilisant un décodeur à maximum de vraisemblance à complexité réduite. Finalement, nous avons montré que la capacité du code est très proche de la capacité du canal en boucle ouverte.

Abstract – In this paper, we present a non-orthogonal space-time block code for 3 transmit antennas with symbol rate 1. This code improves the performance of the wireless link. The proposed code can be decoded using a simple zero forcing receiver or with a low complexity maximum likelihood decoding. Moreover, we have shown that the achievable capacity of the scheme is almost equal to the open loop channel capacity.

1 Introduction

In the context of wireless personal communications, the objective is to improve the performance of the link by achieving space diversity using M transmit and N receiver antennas. For $M = 2$ transmit antennas, Alamouti [1] proposed the complex orthogonal space-time block code (STBC) that achieves maximum diversity. This is the only symbol rate 1 code which allows to reach the full capacity.

For the downlink of UMTS and the future communication systems, it will be interesting to use more than 2 transmit antennas per sector at the base station. For more than 2 transmit antennas, it has been shown from the Hurwitz-Radon theorem that complex orthogonal STBC designs cannot achieve both maximum diversity and symbol rate 1 [2]. For 3 and 4 transmit antennas, generalized complex orthogonal schemes that give maximum diversity with rates 3/4 and 1/2 were proposed in [3, 4]. Because of the symbol rate decrease, these schemes are not suitable for high data rate applications. However, it is possible to achieve rate 1 for complex constellations for more than 2 transmit antennas by sacrificing orthogonality. These non-orthogonal schemes have been proposed for 3 transmit antennas in [5] at different symbol rates and for 4 transmit antennas with symbol rate 1 in terms of capacity in [6] and diversity gain in [7] at the expense of performance loss.

In this paper, we will propose a non-orthogonal complex space-time block code for 3 transmit antennas with symbol rate 1. After the presentation of the coding scheme and the associate receiver structure, performance criteria aspects and capacity calculations will be explained. The simulation results and comparison with existing schemes will be given.

2 The Coding Scheme

Let define two different channel transfer matrices H_1 and H_2 as,

$$H_1 = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \qquad H_2 = \begin{bmatrix} -h_3 & 0 \\ 0 & -h_3^* \end{bmatrix} \tag{1}$$

The two channel matrices are combined in a system channel transfer matrix H_c by Hadamard transform,

$$H_c = \begin{bmatrix} H_1 & H_2 \\ -H_2 & H_1 \end{bmatrix} = \begin{bmatrix} h_1 & h_2 & -h_3 & 0 \\ h_2^* & -h_1^* & 0 & -h_3^* \\ h_3 & 0 & h_1 & h_2 \\ 0 & h_3^* & h_2^* & -h_1^* \end{bmatrix} \tag{2}$$

where h_i be the complex channel coefficient from the i^{th} transmit antenna to the receiver antenna which can be represented as $h_i = \alpha_i e^{j\theta_i}$. The channel coefficients for each antenna are assumed to be i.i.d zero mean complex Gaussian variables with variance 0.5 per real dimension and fixed during T time periods (quasi static flat fading channel).

The received signals of the scheme are given as

$$R = H_c \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix} + N_c \tag{3}$$

where $R = \begin{bmatrix} r_1 & r_2^* & r_3 & r_4^* \end{bmatrix}^T$ is the received vector and $N_c = \begin{bmatrix} n_1 & n_2^* & n_3 & n_4^* \end{bmatrix}^T$ is the noise vector, elements of which are assumed to be independent samples of zero mean complex Gaussian random variables with variance σ^2 per dimension. Since the average energy of the symbols transmitted from each antenna is normalized to one, at each receiver antenna, the average power is equal to M.

These scheme is implemented by using the code matrix given below,

$$S_c = \begin{bmatrix} s_1 & s_2 & -s_3 \\ -s_2^* & s_1^* & -s_4^* \\ s_3 & s_4 & s_1 \\ -s_4^* & s_3^* & s_2^* \end{bmatrix} \tag{4}$$

In equation (4), since $Q = 4$ symbols are sent over 3 transmit antennas during $T = 4$ time interval, the symbol rate $R = Q/T$ is equal to 1.

It should be noted that this code can be derived from the non-orthogonal code for 4 transmit antennas proposed in [6] by setting $h_4 = 0$ in the channel transfer matrix H.

3 Performance Criteria

The design criteria for space-time codes in Rayleigh fading channels are formulated in terms of the codeword difference matrix $D_{ce} = S_c - S_e$. Here S_c and S_e are the code matrices corresponding respectively to the encoded and possibly erroneously detected sets of information bits. Minimizing the pairwise error probability of deciding in favor of S_e when transmitting S_c leads to the rank and determinant criterion, which determine respectively the diversity order and the coding gain [8]. Furthermore, the trace criterion is an important parameter for designing non-orthogonal STBCs [9]. The distance matrix which defines the Hermitian square of the difference matrix is given by

$$D_{ce}^H D_{ce} = \sum_{k=1}^{4} |s_k^c - s_k^e|^2 \, I_M + \mathrm{N} \tag{5}$$

where $\mathrm{N} = \sum\limits_{m<k}^{4} (D_{ce})_{mk}^H (D_{ce})_{mk}$ is a non-orthogonal matrix.

The Rank Criterion: The minimum rank of $D_{ce}^H D_{ce}$ for any possible pair of codewords determines the maximum diversity order. The minimum rank of the proposed code is 2 with a mean close to 3. For QPSK symbol constellation, there exist $4^4(4^4 - 1) = 65280$ error events, $S_c \rightarrow S_e$, where the pairs S_c, S_e are counted with $S_c \neq S_e$. It is found that only 2080 error events have rank 2 instead of rank 3. The degradation is due to the self-symbol interference, resulting from non-orthogonality of the code matrix. It should be remembered that the rank of existing codes for 3 and 4 transmit antenna [5, 6] is also equal to 2.

The Determinant Criterion: The maximum value of $\min\limits_{c \to e} \det(D_{ce}^H D_{ce})$ determines the coding advantage, which measures the effective product distance of the code. The criterion is satisfied if the eigenvalues of $D_{ce}^H D_{ce}$ are close to each other and the off-diagonal elements which result self-interference are minimized.

The Trace Criterion: To maximize the minimum Euclidean distance between all possible codeword pairs, the minimum value of $Tr(D_{ce}^H D_{ce})$ should be maximized. If $Tr(N) \neq 0$, the Euclidean distance between a symbol and its nearest neighbors differs from the distance between an equivalent rotated symbol and its nearest neighbors. In order to avoid this, the non-orthogonality matrix should be traceless $Tr(N) = 0$ and Euclidean distance squared should be proportional to a sum of $|s_k^c - s_k^e|^2$ value. Since the proposed code has a traceless nonorthogonality matrix, the criterion is fulfilled.

4 The Receiver Structure

The reconstructed signals are obtained by applying the matched filter H_c^H to the received vector,

$$Y = H_c^H H_c S + H_c^H N_c \tag{6}$$

$$Y = \begin{bmatrix} G & 0 & \varepsilon & 0 \\ 0 & G & 0 & \varepsilon \\ -\varepsilon & 0 & G & 0 \\ 0 & -\varepsilon & 0 & G \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix} + H_c^H N_c \tag{7}$$

where $\varepsilon = h_3^* h_1 - h_3 h_1^* = j2\mathrm{Im}\{h_3^* h_1\}$ and $G = |h_1|^2 + |h_2|^2 + |h_3|^2$.

Equation (7) can be rewritten in terms of $\{s_1, s_3\}$ and $\{s_2, s_4\}$, since the pairs are decoupled from each other completely.

$$\begin{bmatrix} y_1 \\ y_3 \end{bmatrix} = \Delta \begin{bmatrix} s_1 \\ s_3 \end{bmatrix} + \begin{bmatrix} n_1' \\ n_3' \end{bmatrix} \tag{8}$$

$$\begin{bmatrix} y_2 \\ y_4 \end{bmatrix} = \Delta \begin{bmatrix} s_2 \\ s_4 \end{bmatrix} + \begin{bmatrix} n_2' \\ n_4' \end{bmatrix} \tag{9}$$

where $\Delta = \begin{bmatrix} G & \varepsilon \\ -\varepsilon & G \end{bmatrix}$.

In order to reconstruct the symbols at the receiver side, a maximum likelihood (ML), a zero forcing (ZF) and a minimum mean squared error (MMSE) processing can be applied to each decoupled pair of matched filter outputs.

The generalization of the proposed code to N=2 receiver antennas is derived from the structure given in [1]. The combined signals from the two receiver antennas are a simple addition of the combined signals from each antenna, where the diversity gain and interference term in Equation (7) are equal to $G_2 = |h_{11}|^2 + |h_{21}|^2 + |h_{31}|^2 + |h_{12}|^2 + |h_{22}|^2 + |h_{32}|^2$ and $\varepsilon_2 = j2\mathrm{Im}\{h_{31}h_{11}^* + h_{32}h_{12}^*\}$ respectively, where h_{ij} is the channel coefficient from i^{th} transmit antenna to j^{th} receiver antenna.

4.1 Zero Forcing Processing

The ZF receiver is applied to the matched filter outputs in (8) and (9) as,

$$\begin{bmatrix} y_{1,ZF} \\ y_{3,ZF} \end{bmatrix} = \Delta^{-1} \begin{bmatrix} y_1 \\ y_3 \end{bmatrix} \tag{10}$$

$$\begin{bmatrix} y_{2,ZF} \\ y_{4,ZF} \end{bmatrix} = \Delta^{-1} \begin{bmatrix} y_2 \\ y_4 \end{bmatrix} \tag{11}$$

where $\Delta^{-1} = \frac{1}{G^2+\varepsilon^2} \begin{bmatrix} G & -\varepsilon \\ \varepsilon & G \end{bmatrix}$.

Since only the inversion of 2x2 matrix (instead of 4x4 matrix) is required ZF processing is a computationally simple receiver, however it comes at the cost of noise enhancement.

4.2 Maximum Likelihood Processing

The ML receiver is implemented to recover the symbols by taking into account the effect of self symbol interference for each decoupled symbol pairs.

$$\min_{\{s_1,s_3\}\in \mathrm{A}\times\mathrm{A}} \left\| \begin{bmatrix} y_1 \\ y_3 \end{bmatrix} - \Delta \begin{bmatrix} s_1 \\ s_3 \end{bmatrix} \right\|^2 \qquad (12)$$

$$\min_{\{s_2,s_4\}\in \mathrm{A}\times\mathrm{A}} \left\| \begin{bmatrix} y_2 \\ y_4 \end{bmatrix} - \Delta \begin{bmatrix} s_2 \\ s_4 \end{bmatrix} \right\|^2 \qquad (13)$$

where A is the alphabet shared by all the substreams.

The use of decoupling structure leads to less computational complexity compared to the full ML processing.

5 Capacitive Derivations

The comparison between the open-loop (M,N) channel capacity and the maximum mutual information that a (M,N) STBC code can achieve is important to evaluate the efficiency of the code. The channel capacity of a (M,N) antenna system under the condition that the channel is known at the receiver and no feedback is sent back to the transmitter, is calculated in [10] as,

$$C(\rho, M, N) = E\log_2 \det\left(\mathrm{I}_N + \frac{\rho}{M} HH^H\right) \qquad (14)$$

where ρ is the signal-to-noise ratio at each receive antenna and H is the channel coefficient matrix.

In [10], the achievable maximum mutual information by a (T,M,N) space-time code is given as,

$$C(\rho, T, M, N) = \frac{1}{T} E\log_2 \det\left(\mathrm{I}_{NT} + \frac{\rho}{M} H_c H_c^H\right) \qquad (15)$$

It can be shown that for the (4,3,1) proposed code using ML receiver, the achievable capacity is equal to

$$C_{ML}(\rho) = \frac{1}{4} E\left[\log_2 \det\left(\mathrm{I}_4 + \frac{\rho}{3}\Delta\right)\right] \qquad (16)$$

When a zero forcing detector is used, the capacity is reduced to the following expression:

$$C_{ZF}(\rho) = E\left[\log_2\left(1 + \frac{\rho}{3}\left(\frac{G^2 + \varepsilon^2}{G}\right)\right)\right] \qquad (17)$$

The open loop channel capacity and maximum mutual information that the proposed and existing codes can be obtained by using equations (14) and (15).

6 Simulation Results

In this section, we provide simulation results which are obtained by using QPSK over Rayleigh fading channels for the proposed code and existing codes.

In Figure 1, we show the bit-error-rate (BER) performance of several schemes. According to results, the proposed code with ML detection gives better performance than the ZF processing at the cost of some computational expense. For BER= 10^{-4}, the (3,1) proposed code with ML provides 2dB diversity gain compared to the (2,1) scheme in [1]. It has almost the same BER performance as the non-orthogonal code for 4 transmit antennas given in [7]. The (3,2) proposed scheme provides an additional diversity gain compared to the (2,2) code in [1].

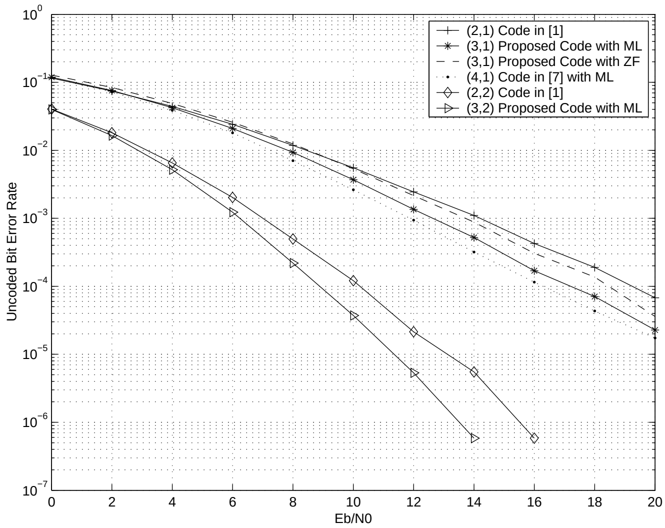

FIG. 1: Bit error rate performances in Rayleigh fading channel for proposed and existing code schemes

Figure 2 shows the open loop channel capacity compared to maximum mutual information obtained by the proposed code and the codes given in [5], [7] and [11]. According to these results, the proposed code which has low complexity encoding and decoding scheme, achieves a higher mutual information than the R=1 code in [5] and gives the same mutual information as the LD code given in [11] which was obtained by using information-theoretic optimization criterion. Furthermore, compare to the LD code, all the elements of the diagonal $H_c^H H_c$ of the proposed code are identical. Also, the proposed code has almost the same capacity with the (4,1) code in [7].

For the (3,2) code, the achievable mutual information is significantly lower than the open loop channel capacity with M=3 and N=2 as shown in Figure 3. For example, at $SNR = 20dB$ the capacity of (3,2) proposed scheme is only 61.36% of the open loop channel capacity while the capacity of the (3,1) proposed code is 97.48%.

7 Conclusion

In this paper, we introduced a non-orthogonal space-time block code scheme for complex symbols with symbol rate 1 employing 3 transmit antennas by constructing low complexity receiver structure. Simulation results demonstrate that the scheme achieves diversity gains without bandwidth loss. The achieved capacity of this scheme is almost equal to the capacity of the open loop channel.

8 Acknowledgement

The authors would like to thank for their support the Institut Aeronautique et Spatial (IAS).

References

[1] S. M. Alamouti, “A Simple Transmitter Diversity Scheme for Wireless Communications," IEEE J. Select. Areas

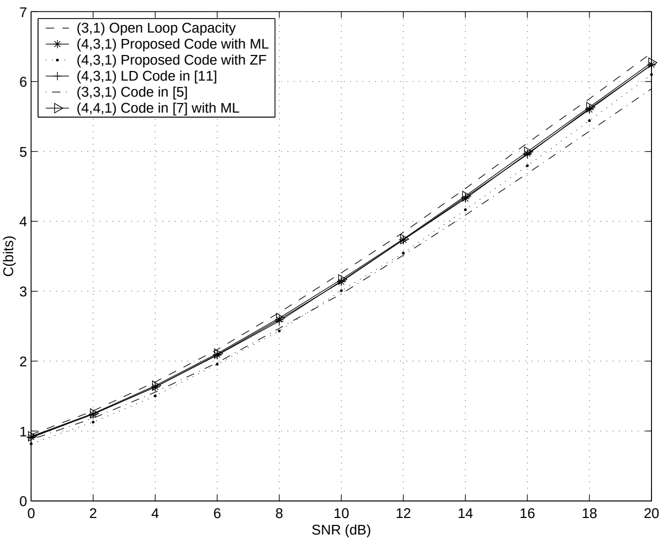

FIG. 2: Maximum mutual information achieved by the (4,3,1) proposed code, (4,3,1) LD code in [11], (3,3,1) code with R=1 in [5] and the (4,4,1) code in [7] compared to open loop channel capacity

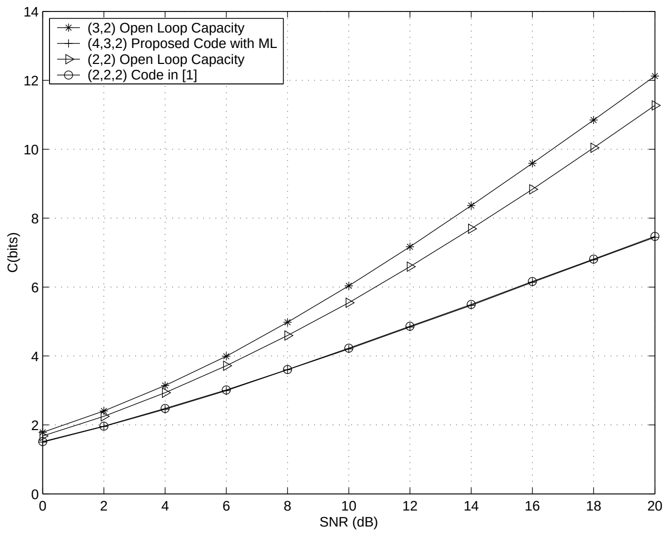

FIG. 3: Maximum mutual information achieved by the (4,3,2) proposed code and (2,2,2) code in [1] compared to open loop channel capacity

Communication, vol. 16, no. 8, pp. 1451-58, October 1998.

[2] V. Tarokh, H. Jafarkhani, A.R. Calderbank, “Space-Time Block Codes from Orthogonal Designs", IEEE Trans. on Information Theory, vol. 45, no. 5, pp.1456-67, July 1999.

[3] O. Tirkkonen, A. Hottinen, “Square-Matrix Embeddable Space-Time Block Codes for Complex Signal Constellations", IEEE Trans. on Information Theory, vol.48, no.2, pp.384-395, February 2002.

[4] B.M.Hochwald, T.L. Marzetta, C.B. Papadias, “A Transmitter diversity scheme for wideband CDMA systems based on space-time spreading", IEEE J. Select Areas Communication, vol.19, no.1, pp.48-60, January 2001.

[5] M. Uysal, C.N. Georghiades, “Non-Orthogonal Space-Time Block Codes for 3TX Antennas", IEE Electronics Letters, vol. 38, no.25, December 2002.

[6] C. B. Papadias, G. J. Foschini, “A Space-Time Coding Approach for Systems Employing Four Transmit Antenna", In Proc. of IEEE ICASSP’01, vol.4, pp.2481-85, 2001.

[7] H. Jafarkhani, “A Quasi-Orthogonal Space-Time Block Code", IEEE Trans. Commun., vol.49, no.1, pp.1-4, January 2001.

[8] V. Tarokh, N. Seshadri, and A. R. Calderbank, “Space-time codes for high data rate wireless communication: Performance criteria and code construction," IEEE Trans. on Information Theory, vol.44, no.2, pp.744-765, March 1998.

[9] O. Tirkkonen, A. Hottinen, “Improved MIMO Performance with Non-Orthogonal Space-Time Block Codes", Globecom 2001, Texas, USA, November 2001.

[10] G.F. Foschini and M. J. Gans, “On limits of wireless communications in a fading environment when using multiple antennas," Wireless Personal Communications, vol. 6, pp. 311-335, 1998.

[11] B. Hassibi, B. M. Hochwald, “High-Rate Codes That are Linear in Space and Time", IEEE Trans. on Information Theory, vol.48, no.7, pp.1804-1824, July 2002.