Throughput evaluation of HARQ Schemes with Packet and Code Combining over multipath fading channels for DS-SSS

M. Wissem El Bahri, Hatem Boujemaa, Mohamed Siala

Ecole Supérieure des Communications de Tunis, MEDIATRON Laboratory
Route de Raoued Km 3.5, 2083 El Ghazela, Ariana, Tunisia
`wissem_bahri@yahoo.fr, boujemaa.hatem@supcom.rnu.tn , mohamed.siala@supcom.rnu.tn`

Résumé – Dans cet article, nous évaluons les performances des schémas ARQ et HARQ I, utilisant la stratégie Packet Combining, et HARQ II, utilisant la stratégie Code Combining, en présence d'un canal multi-trajets à évanouissements de Rayleigh. L'analyse est effectuée dans deux scénorios extrèmes où le canal est constant et indépendent durant les différentes retransmissions. Les résultats théoriques obtenus sont ensuite validés par des simulations.

Abstract – In this paper, the performance of both ARQ and HARQ I, with Packet Combining (PC), and HARQ II, with Code Combining (CC), are evaluated in terms of throughput efficiency over multipath fading channels. The analysis is done in two extreme scenarios where the channel is assumed to be constant and independent during the different transmissions. The theoretical results are then compared to those obtained by computer simulations and confirm the great advantage of using a PC strategy.

1 Introduction

The High Speed Downlink Packet Access (HSDPA) is a new mechanism proposed by 3GPP [1] to support higher data transmission rate for mobile users, and to provide streaming, interactive and background services with a good quality of service. This is accomplished by using different techniques such as Hybrid Automatic Repeat reQuest (HARQ) scheme. HARQ consists in combining ARQ and FEC schemes [2]. Two types of Hybrid ARQ schemes [2] have been identified in which error correction followed by error detection are applied at every received packet. In HARQ I schemes, the correction of errors is first attempted at each received packet. If this fails, the entire packet is discarded and its retransmission is requested. Each subsequent retransmission is decoded independently of the prior retransmissions. The main disadvantage of ARQ and HARQ I schemes is their low throughput at low signal to noise ratios. In fact, when a packet is detected in error, it must be retransmitted and, therefore, for low or mean signal to noise ratios, the number of transmissions needed before receiving a packet correctly is high and then the transmission rate is reduced to an unacceptable level. In HARQ II schemes, the receiver stores the erroneous packets in a buffer so that they can be reused after subsequent retransmissions. Storing the previously received packets allows the technique of Code Combining [3] to be exploited which consists in alternately sending one of the outputs of a convolutional encoder. In this strategy, the first transmission occurs without coding and successive repetition of the convolutional encoder outputs yields a family of repetition codes of decreasing rates equal to $1/j$ where j is the number of transmission tentative.

Significant improvement of the performance of ARQ and HARQ I schemes can be obtained by using a Packet Combining strategy. In this technique, all received packets are averaged in a soft manner at the input of the channel decoder. This strategy is considered in the HSDPA mechanism since it is less complex than HARQ II. Packet combining was initially introduced by Sindhu [5]. It uses hard decisions and tries to correct errors based on the differences between received packets. Benelli [6], Metzner and Chang [7] generalised Sindhu's idea by using soft decisions. The performance of ARQ and HARQ I with packet combining have been analyzed in [5] for a gaussian and binary symmetric channel. The analysis carried in [4] assumes a single block fading path where the channel is assumed to be independent between different transmissions. In this paper, we evaluate the performance of HARQ schemes with Packet Combining and Code Combining strategy over multipath fading channels for Direct Sequence Spread Spectrum System (DS-SSS). The analysis is done into extreme scenarios in which fading is assumed to be constant and independent during the different transmissions.

The paper is organized as follows. Section II gives the system model. Sections III, IV and V give respectively the performance of ARQ with PC, HARQ I with PC and HARQ II with CC, over multipath fading channels. Section VI compares simulation and theoretical results of the different schemes in terms of throughput efficiency (Thr). Finally, section VII draws some conclusions.

2 System model

In this section, we describe the channel decoder input for DS-SSS using a Rake receiver. The Rake receiver is assumed to have a perfect knowledge of complex path gain modules $\{\alpha_l\}_{l=1}^{L}$, where L is the number of paths. If path delays are well separated and in the absence of Inter Symbol Interference and Multi-User Interference, the Rake receiver output for symbol s_n can be written as [9]

$$\hat{s}_n = \sum_{l=1}^{L} \alpha_l^2 s_n + w_n \tag{1}$$

where s_n is the n-th transmitted symbols which is dropped from a BPSK constellation and w_n is a complex Gaussian noise with variance

$$Var(w_n) = \sum_{l=1}^{L} \alpha_l^2 N_0 \tag{2}$$

where N_0 is the Power Spectral Density of the complex channel noise.

The wireless link implements the Selective Repeat (SR) protocol for retransmission of erroneous packets with perfect code detection and suitably large buffers at the transmitter and the receiver. Furthermore, we assume an error free feedback channel over which positive or negative acknowledgements can be sent.

3 ARQ with Packet Combining

In the conventional ARQ scheme, the transmitter sends a packet consisting of k information bits and n_p parity bits for error detection. At the receiver, packets declared in error are discarded and replaced by another copy. The performance of conventional ARQ schemes can be improved if the erroneous packets are not discarded but averaged with the new packet to reduce the effect of the channel noise and therefore the average number of transmissions. Let Tr be the average number of transmission attempts that must be made before a packet is accepted by the receiver. The average number of transmission Tr is given by

$$Tr = 1 + \sum_{j=1}^{+\infty} P(R_d^j) \tag{3}$$

where R_d^j is the event "received sequence obtained by combining the first j packets contains detected errors".

Assuming that the probability of undetected errors is negligible, the probability of detected errors at the j-th retransmission can be evaluated numerically as follows

$$P(R_d^j) = 1 - \int [1 - p(\boldsymbol{\beta}^j)]^{k+n_p} f(\boldsymbol{\beta}^j) d\boldsymbol{\beta}^j \tag{4}$$

where $\boldsymbol{\beta}^j = (\boldsymbol{\alpha}^1, ..., \boldsymbol{\alpha}^j)$, $\boldsymbol{\alpha}^j = (\alpha_1^j, ..., \alpha_L^j)$, α_l^j is the l -th path gain during the j-th transmission, $p(\boldsymbol{\beta}^j)$ is the Conditioned Bit Error Rate (CBER) at the channel decoder input and $f(\boldsymbol{\beta}^j)$ is the joint probability density function of path gains.

If the channel is independent during the different transmissions, the CBER is given by

$$p(\boldsymbol{\beta}^j) = \frac{1}{2} \mathrm{erf_c} \left(\sqrt{\frac{E_s}{N_0} \sum_{i=1}^{j} \sum_{l=1}^{L} (\alpha_l^i)^2} \right) \tag{5}$$

where E_s is the average transmitted energy per symbol.

When the channel is constant during the different transmissions ($\boldsymbol{\alpha}^1 = ... = \boldsymbol{\alpha}^j = \boldsymbol{\alpha} = (\alpha_1, ..., \alpha_L)$), the CBER is given by

$$p(\boldsymbol{\beta}^j) = \frac{1}{2} \mathrm{erf_c} \left(\sqrt{\frac{E_s}{N_0} j \sum_{l=1}^{L} (\alpha_l)^2} \right). \tag{6}$$

The throughput efficiency Thr is given by

$$Thr = \frac{1}{Tr} \frac{k}{(k + n_p)} \tag{7}$$

where the factor $k/(k + n_p)$ is the loss in throughput due to the added parity check bits for error detection.

4 HARQ I with Packet Combining

In HARQ I with packet combining, erroneous received packets are averaged with the new packet before decoding. In this paper, we assume that a convolutional encoder of rate $R_c = u/v$ and memory m is used for error correction. At the receiver, a soft input Viterbi decoder is used.

If the channel is independent during the different transmissions, the packet error probability at the $j - th$ transmission $P(D_d^j)$ can be upper-bounded as

$$P(D_d^j) \leq 1 - \int [1 - P_E(\boldsymbol{\beta}^j)]^{\frac{k+n_p}{u}} f(\boldsymbol{\beta}^j) d\boldsymbol{\beta}^j \tag{8}$$

where D_d^j is the event "decoded sequence obtained by combining the first j packets contains detected errors", $P_E(\boldsymbol{\beta}^j)$ is the Error Event Probability (EEP) upper-bounded by

$$P_E(\boldsymbol{\beta}^j) \leq \sum_{d=d_f}^{+\infty} a_d P_d(\boldsymbol{\beta}^j) \tag{9}$$

where d_f is the free distance, a_d is the number of incorrect paths at distance d and

$$P_d(\boldsymbol{\beta}^j) = \frac{1}{2} \mathrm{erf_c} \left(\sqrt{\frac{dE_s}{N_0} \sum_{i=1}^{j} \sum_{l=1}^{L} (\alpha_l^i)^2} \right) \tag{10}$$

is the conditioned probability that a wrong path at distance d is selected.

If the channel is constant during the different transmissions, equation (8) remains valid and equation (10) reduces to

$$P_d(\boldsymbol{\beta}^j) = \frac{1}{2} \mathrm{erf_c} \left(\sqrt{\frac{dE_s}{N_0} j \sum_{l=1}^{L} (\alpha_l)^2} \right). \tag{11}$$

The average number of transmissions is given by

$$Tr = 1 + \sum_{j=1}^{+\infty} P(D_d^j) \tag{12}$$

The throughput efficiency of HARQ I with PC is given by

$$Thr = \frac{R_c}{Tr} \frac{k}{(k + n_p + um)}, \tag{13}$$

where the factor $k/(k + n_p + um)$ is the loss in throughput due to the added parity bits for error detection and to the tail of um known bits appended to each transmitted sequence.

5 HARQ II with Code Combining

HARQ II with Code Combining [3] belongs to the class of incremental redundancy schemes in which incremental bits are progressively transmitted in order to optimize the throughput. In HARQ II with Code Combining, the transmitter alternatively sends one of the outputs of a convolutional encoder. At the reception, the receiver has the possibility to extract the information from the last uncoded received packet or from the combined version of all previously received packets. By using this strategy, the first transmission occurs only with code detection and successive transmissions yield a family of repetition codes of decreasing rates equal at $1/j$ where $j \geq 2$. The average number of transmissions is upper bounded by [3]

$$Tr \leq 1 + P(R_d) + \sum_{j=1}^{+\infty} P(D_d^j). \tag{14}$$

where R_d is the event "last received sequence contains detected error". Assuming that the probability of undetected errors is negligible, the probability that a received sequence is detected in error is given by

$$P(R_d) = 1 - \int (1 - p(\boldsymbol{\alpha}))^{k+n_p} f(\boldsymbol{\alpha}) d\boldsymbol{\alpha} \tag{15}$$

where $p(\boldsymbol{\alpha})$ is the CBER given by

$$p(\boldsymbol{\alpha}) = \frac{1}{2}\mathrm{erf_c}\left(\sqrt{\frac{E_s}{N_0}\sum_{l=1}^{L}(\alpha_l)^2}\right). \tag{16}$$

The packet error probability $P(D_d^j)$ is also given by (8). If the channel is constant during the different transmissions, $\boldsymbol{\beta^j}$ has to be replaced by $\boldsymbol{\alpha} = (\alpha_1, ..., \alpha_L)$ and the EEP is upper-bounded by

$$P_E(\boldsymbol{\alpha}) \leq \sum_{d=d_f^{(j)}}^{+\infty} a^{(j)}(d) P_d(\boldsymbol{\alpha}) \tag{17}$$

where

$$P_d(\boldsymbol{\alpha}) = \frac{1}{2}\mathrm{erf_c}\left(\sqrt{\frac{dE_s}{N_0}\sum_{l=1}^{L}(\alpha_l)^2}\right), \tag{18}$$

$a^{(j)}(d)$ and $d_f^{(j)}$ are respectively the distance spectra and the free distance of rate $1/(1+j)$ code.

If the channel is independent during the different transmissions, the total distance between two paths has to be divided into component distances, $d = d_1 + ... + d_j$, where d_l gives the distance between the error path and the zero path during the $l - th$ transmission. The EEP is upper bounded by

$$P_E(\boldsymbol{\beta^j}) \leq \sum_{d_1=d_{1,f}} ... \sum_{d_j=d_{j,f}} a(d_1, ..., d_j) P_d(\boldsymbol{\beta^j}) \tag{19}$$

where

$$P_d(\boldsymbol{\beta^j}) = \frac{1}{2}\mathrm{erf_c}\left(\sqrt{\frac{E_s}{N_0}\sum_{p=1}^{j} d_p \sum_{l=1}^{L}(\alpha_l^p)^2}\right), \tag{20}$$

$a(d_1, ..., d_j)$ is the number of error events with distance vector $\boldsymbol{d} = (d_1, ..., d_j)$. The lower summation limits, $\boldsymbol{d_f} = (d_{1f}, ..., d_{jf})$, are the free component distances associated with each subcode. To compute this distance spectra, we can use the generalized transfer function [10] of the code, or the modified version of the fast Cederval algorithm searching the code tree [8]. The throughput efficiency is given by

$$Thr = \frac{1}{Tr}\frac{k}{k + n_p + um} \tag{21}$$

where the factor $k/(k + n_p + um)$ is the loss in throughput due to the added parity bits for error detection and to the tail of um known bits appended to each transmitted sequence.

6 Numerical and simulation results

In this section, we compare theoretical and simulation results in terms of throughput efficiency evolution with respect to E_s/N_0 for a two paths Rayleigh fading channel ($L = 2$). The code used for HARQ I and HARQ II with Code Combining is a half rate convolutional encoder of constraint length $m + 1 = 7$ and generators polynomials (133,171). The free distance d_f and the distance spectra a_{d_f} of repetition codes of HARQ II with Code Combining are listed in table VI of [3]. In order to compare the performance of the different schemes, the same length of the transmitted packets must be maintained. In the simulations, the transmitted packet length was equal to 960 bits. The parameters of the different schemes are listed in Table 1. Figure 1 compares the performance of ARQ and ARQ with PC in the constant and the independent channel scenarios. We can observe that at high E_s/N_0, the PC strategy does not yields a throughput improvement. However, as the channel degrades, the throughput without PC drops rapidly to zero, whereas with PC a high throughput is still achieved even at very low E_s/N_0. We also notice that performance improvement brought by packet combining is greater when the channel is constant.

Tab. 1: Parameters for different schemes used for numerical and simulation results

Schemes	k (bits)	n_p (bits)
ARQ with and without PC	940	20
HARQ I with and without PC	454	20
HARQ II with CC	940	14

Figure 2 shows the performance of HARQ I and HARQ I with PC in the same context. We show that PC strategy improves also the performance of HARQ I.

In figures 3 and 4, we have compared the performance of packet combining to Code Combining schemes respectively in the constant and the independent channel scenarios. We see that at low E_s/N_0 the performance of ARQ and HARQ I with PC are close to those of HARQ II with Code Combining. We also notice that ARQ with PC offers close performance to HARQ II with Code Combining at high E_s/N_0. Finally, we verify that simulation results are in a good agreement with the theoretical ones.

7 Conclusion

In this paper, we have derived the performance of ARQ and HARQ I with PC in multipath Rayleigh fading channels. The analysis is done in two extreme scenarios where the channel is assumed to be constant and independent during the different transmissions. We have shown that the performance improvement brought by PC is greater when the channel is constant during the different transmissions.

References

[1] 3GPP TR 25.848 V4.0.0, Physical layer aspects of UTRA high speed downlink packet access (release 4), 2001.

[2] M. Wissem El Bahri, Hatem Boujemâa and Mohamed Siala "Performance Comparison of Type I, II and III Hybrid ARQ Schemes over AWGN channels," *IEEE ICIT*, Hammamet, Tunisia, Dec. 2004.

[3] S. Kallel, "Analyis of a type II hybrid ARQ schemes with code combining", *IEEE Trans. Commun.*, Vol. 38, pp. 1133-1137, Aug. 1990.

[4] K Akhavan, S. Farahvash and M. Kavehrad , "Performance of Hybrid ARQ Schemes with ML Code Combining over a Block-Fading Rayleigh Channel", *Inter. Jour. of Wirel. Infor. Netw.*, Vol. 7, No. 1, 2000.

[5] Y. M. Wang and S. Lin "A modified selective type II hybrid ARQ systems and its performance analysis", *IEEE Trans. Commun.*, Vol. COM-31 pp. 593-608, May 1983.

[6] G. Benelli, "An ARQ scheme with memory for mobile communications", *Euro. Trans. on Telecom.* 3(5) pp. 11-15, 1992.

[7] J. J. Metzner, and D. Chang "Efficiency selective repeat ARQ strategies for very noisy and fluctuating channels", IEEE Trans. Commun., Vol. COM-33 pp. 409-416, 1985.

[8] M. Cedervall and R. Johannesson "A fast algorithm for computing distance spectrum of convolutional codes ", *IEEE Trans. on Infor. Theory*, Vol. 35 pp. 1146-1159, Nov. 1989.

[9] H. Boujemâa, M. Siala, "On the rake receiver performance," *IEEE Veh. Tech. Conf.*, Vol. 3, pp. 1483-1488, Sept. 2000.

[10] Y. S. Leung, S. G. Wilson, J. W. Ketchung, Multifrequency treillis coding with low delay for fading channels", *IEEE Trans. on Commun.*, Vol. 41, no. 10, pp. 1450-1459, Oct. 1993.

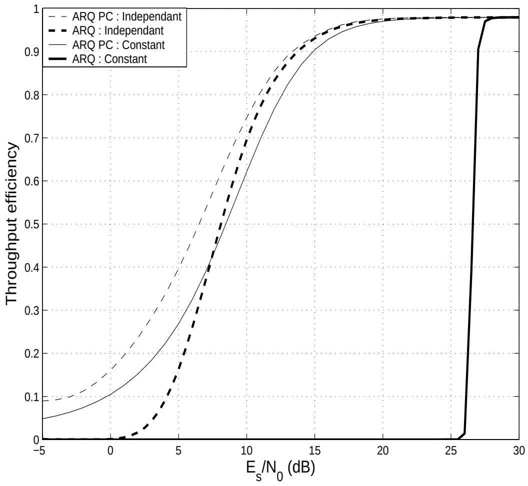

FIG. 1: Performance improvement of ARQ using PC strategy: $k = 940, n_p = 20$ and $L = 2$.

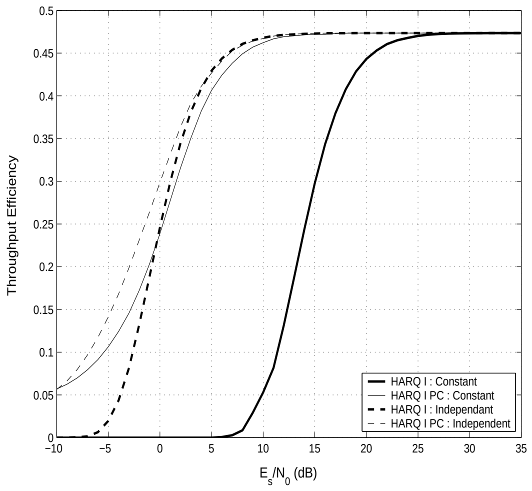

FIG. 2: Performance improvement of HARQ I using PC strategy: $k = 454, n_p = 20$ and $L = 2$.

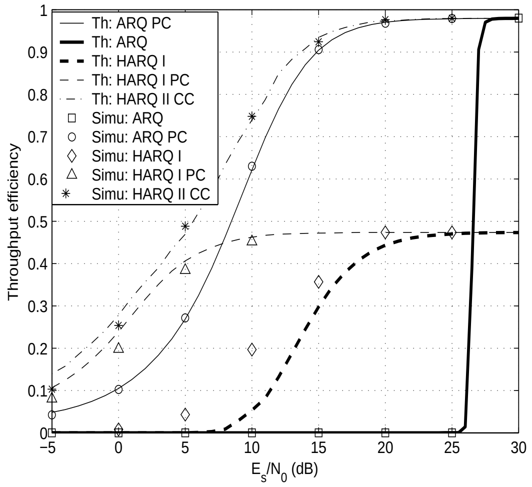

FIG. 3: Performance comparison of ARQ and HARQ I with PC to HARQ II with Code Combining for a constant channel ($L = 2$).

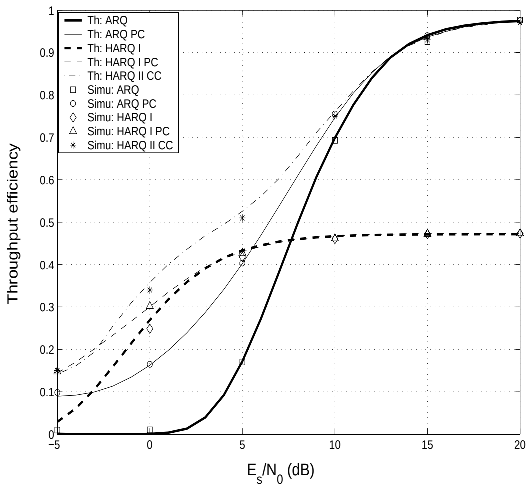

FIG. 4: Performance comparison of ARQ and HARQ I with PC to HARQ II with Code Combining for an independent channel ($L = 2$).