Empirical Mode Decomposition Analysis of Experimental Homogeneous Turbulence Time Series

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Résumé – Il s'agit d'une mise en application de la méthode d'analyse de séries temporelles non-linéaires EMD (décomposition modale empirique), et de la transformation de Hilbert-Huang, à des données expérimentales de turbulence, possédant des fluctuations invariantes d'échelle dans la zone inertielle de cascade d'énergie. Nous montrons que la méthode EMD permet de décomposer une série temporelle turbulente en une somme de modes intrinsèques appartenant aux échelles inertielles. Nous estimons le spectre de Fourier de chaque mode, et montrons qu'ajouter des modes correspond à remonter en échelles, incluant les basses fréquences dans la zone inertielle. Cette propriété de filtre peut avoir d'intéressantes applications en modélisation de la turbulence. Nous montrons aussi que le spectre de Hilbert-Huang est invariant d'échelle, avec une pente différente de la pente classique turbulente de $-5/3$.

Abstract – In this paper the Empirical Mode Decomposition (EMD) method and Hilbert-Huang transform are used to analyse experimental homogeneous turbulence time series. With this method, one can decompose nonlinear time series into a sum of different modes, each narrow-banded. Here we consider experimental turbulent velocity time series with a large Reynolds number ($Re_\lambda = 720$). The Fourier power spectrum reveals a wide inertial range with a classical $-5/3$ Kolmogorov power-law spectrum. We show that the EMD method applies very nicely to the turbulent velocity time series, with a dyadic filter bank in the inertial range. We estimate the Fourier power spectra of each mode, showing that adding more and more modes corresponds to including lower and lower frequencies. This filtering property can have interesting applications in the field of turbulence modelling. We estimate the Hilbert-Huang power spectrum of the turbulent time series and show its scaling properties, with an exponent different from $-5/3$.

1 Introduction

In this paper the Empirical Mode Decomposition (EMD) method is used to analyze experimental homogeneous turbulence time series in order to decompose nonlinear time series into a sum of different modes, each one having characteristic frequencies [1,2]. Since it was introduced, it has been successfully applied to many topics in the natural and applied sciences. It has also been applied to numerically simulated fractional Gaussian noise (fGn) time series, and shown to act as a dyadic filter bank [3]. In the same paper, it was shown how to use the hierarchy of modes to estimate the fGn scaling exponent H.

However, to our knowledge, it has seldom been applied to fully developed turbulent time series, characterized by a high Reynolds number, a large scaling range for the fluctuations, and strong intermittency [4]. Here we consider experimental turbulent velocity time series with a large Reynolds number. These time series possess multifractal properties, classically characterized in the turbulence literature using structure function scaling exponents [4]. We show in the following that the EMD method applies very nicely to the turbulent velocity time series, with a dyadic filter bank in the inertial range.

2 Material and Method

2.1 Material

We consider here a turbulent channel flow database obtained from an active-grid experiment characterized by the Taylor-based Reynolds number $Re_\lambda = 720$. The sampling frequency is $f_s = 40kHz$, and a low-pass filtered at a frequency of $20kHz$ is applied on the experimental data. The sampling time is $30\,s$, and the total number of data points per channel for each measurement is 1.2×10^6. We used data in the streamwise direction at position $x_1/M = 20$, where M is the grid size (the mean velocity at this location is $12\,m/s$). For details about the experiment and the data see [6] ; the data can be found at http ://www.me.jhu.edu/˜meneveau /datasets.html.

2.2 Empirical Mode Decomposition

Empirical Mode Decomposition is a recently developed method [1, 2] that can be applied to study the nonlinear and non-stationary properties of a time series. This method contains the following two steps : Empirical Mode Decomposition (EMD) and Hilbert Spectra Analysis (HSA). The main idea of EMD is to locally estimate a signal as a sum of a local trend and a local detail : the local trend is a low frequency part, and the local detail a high frequency. When this is done for all the oscillations composing a signal, the high frequency part is called an Intrinsic Mode Function (IMF) and the low frequency part is called the residual. The procedure is then applied again to the residual, considered as a new times series, extracting a new IMF and a new residual. At the end of the decomposition process, the EMD method expresses a time series $x(t)$ as the sum of a finite number of IMFs $C_i(t)$ and a final residual $r_n(t)$ [1, 3]. The procedure is precisely described below.

An IMF is a function that satisfies two conditions : (i) the difference between the number of local extrema and the number of zero-crossings must be zero or one ; (ii) the running mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero. The procedure to decompose a signal into IMFs is the following [1, 2] :

1 The local extrema of the signal $x(t)$ are identified ;
2 The local maxima are connected together forming an upper envelope $e_{\max}(t)$, which is obtained by a cubic spline interpolation. The same is done for local minima, providing a lower envelope $e_{\min}(t)$;
3 The mean is defined as $m_1(t) = (e_{\max}(t)+e_{\min}(t))/2$;
4 The mean is subtracted from the signal, providing the local detail $h_1(t) = x(t) - m_1(t)$;
5 The component $h_1(t)$ is then examined to check if it satisfies the conditions to be an IMF. If yes, it is considered as the first IMF and denoted $C_1(t) = h_1(t)$. It is subtracted from the original signal and the first residual, $r_1(t) = x(t) - C_1(t)$ is taken as the new series in step 1. If $h_1(t)$ is not an IMF, a procedure called "sifting process" is applied as many times as needed to obtain an IMF. In the sifting process, $h_1(t)$ is considered as the new data ; the local extrema are estimated, lower and upper envelopes are formed and their mean is denoted $m_{11}(t)$. This mean is subtracted from $h_1(t)$, providing $h_{11}(t) = h_1(t) - m_{11}(t)$. Then it is checked if $h_{11}(t)$ is an IMF. If not, the sifting process is repeated, until the component $h_{1k}(t)$ satisfies the IMF conditions. Then the first IMF is $C_1(t) = h_{1k}(t)$ and the residual $r_1(t) = x(t) - C_1(t)$ is taken as the new series in step 1.

By construction, the number of extrema decreases when going from one residual to the next ; the above algorithm ends when the residual has only one extrema, or is constant, and in this case no more IMF can be extracted. The complete decomposition is then achieved in a finite number of steps, of the order $n \leq \log_2 N$, for N data points. The signal $x(t)$ is finally written as :

$$x(t) = \sum_{i=1}^{N} C_i(t) + r_n(t) \tag{1}$$

The IMFs are orthogonal, or almost orthogonal functions (mutually uncorrelated). This method does not require stationarity of the data and is especially suitable for nonstationary and nonlinear time series analysis [1, 2]. Each mode is localized in frequency space [7, 8]. This decomposition can be used to express the original time series as the sum of a trend (sum of modes from p to N) and small-scale fluctuations (sum of modes from 1 to $p-1$), where p is an index whose value depends on the trend decomposition which is desired.

EMD is a time-frequency analysis [3]. It can represent the original signal in a energy-frequency-time form at local level, using a complementary method called Hilbert-Huang spectrum [1], which is not detailed here. We only mention the fact that the local energy spectrum (called Hilbert-Huang spectrum) can be estimated at time t with the introduction of an instantaneous frequency ω :

$$h(\omega) = \int H(\omega, t)\, dt \tag{2}$$

in some sense this is comparable to the power spectrum in Fourier analysis [1]. In fact, here the definition of instantaneous frequency is different with the one in Fourier frame and the interpretation and detailed physical meaning of $h(\omega)$ is still to be fully characterized.

3 Results

The original velocity time series is divided into 73 segments (without overlapping) of 2^{14} points each. After decomposition, the original velocity series is decomposed into several IMFs, each one having a different mean frequency, which is estimated by considering the (energy weighted) mean frequency in the Fourier power spectrum of each mode. The relation between mode number k and mean frequency [1] is displayed in Fig. 1. The straight line in log-linear plot which is obtained suggests the following relation :

$$\overline{f}(k) = f_0 \rho^{-k} \tag{3}$$

where $\overline{f}$ is the mean frequency, f_0 is a constant and ρ is very close to 2. This indicates that EMD acts as a dyadic filter bank in the frequency domain also for homogeneous turbulence, as was shown previously using stochastic simulations of Gaussian noise or fractional Gaussian noise [3,8].

When compared with the original Fourier spectrum of the turbulent time series (see Fig.2 and 3), these modes can be denoted as follows :

- the first mode, which has smallest time scale, corresponds to the measurement noise ;
- modes 2 and 3 are associated to the dissipation range of turbulence, for which a departure from the $-5/3$ power-law spectrum is visible [4] ;
- mode 4 corresponds to the smallest scale in the inertial range (the $-5/3$ scaling range) which is called the Kolmogorov scale ;

– modes 5 to 11 all belong to the inertial range ;
– larger modes belong to the large turbulent forcing scales.

This approach is thus very interesting to characterize the turbulent cascade of energy flux from large to smaller scales, which is a characteristic feature of fully developed turbulence.

Fig. 2 and 3 represent the Fourier power spectra of each mode and of the sum of the modes, respectively. They show (i) that each mode in the inertial range is narrow-banded ; (ii) that adding more and more modes corresponds to going farther and farther towards large scales in the inertial range, reconstituting the $-5/3$ Kolmogorov spectrum. This property can be very interesting to decompose a turbulent signal into a mean and small-scale fluctuations, as is often done for turbulence modelling purposes.

The Hilbert marginal spectrum $h(\omega)$ [1] of the velocity is displayed in Fig. 4 together with the Fourier spectrum. It is clear that the following relation

$$h(\omega) \sim \omega^{-\beta_H(\omega)} \tag{4}$$

holds in some range, with an exponent $\beta_H \approx 2$ different from the $-5/3$ Fourier exponent. We recall here that the frequency ω defined in EMD is different from the Fourier frequency. Here instantaneous frequency is used to represent the relation between time, frequency and energy. The precise physical meaning of Hilbert marginal spectrum is still to be explored, and the value of β_H to be interpreted, e.g. through dimensional analysis arguments as can be done for the value $-5/3$ in Fourier space. In the equation above, it was written as $\beta_H(\omega)$ to express the fact that the local slope seems to slightly depend on the frequency.

Finally, we know that the intermittency property of turbulent fluctuations, which is often characterized in the inertial range in a scale-invariant framework involving multifractal models and scaling moment functions, is one most important property of turbulence. Turbulent fluctuations are classically studied using structure functions, corresponding to study the statistical moments of local increments $|U(t+\tau) - U(t)|$, where U is the streamwise component of the velocity time series. Here we propose to consider the intermittency properties of each mode : this corresponds to consider the intermittency properties of the mode decomposition, instead of considering it using structure function decomposition. moments to characterize the intermittency of those modes. The results are shown in Fig. 5, indicating that the following behaviour seems to hold :

$$\langle |C_K(t)|^q \rangle = \psi(q)k + b(q) \tag{5}$$

where $\psi(q)$ is the slope of those modes' statistical moments in the inertial subrange. $\psi(q)$ and $b(q)$ will be precisely estimated and interpreted in future studies.

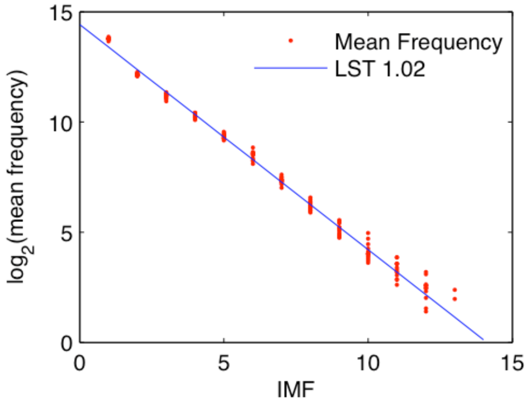

FIG. 1 – The relation between modes and mean frequency [1]. The slope is very close to 1, which indicates that EMD acts as a dyadic filter bank.

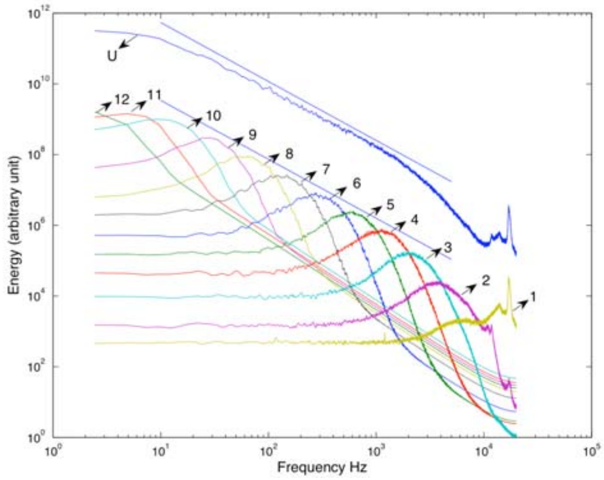

FIG. 2 – Fourier spectrum of each mode showing that they are narrow-banded (from 1 to 12). The slope of the reference line is $-5/3$.

4 Conclusion

In present paper, we apply Empirical Mode Decomposition to analyze a high Reynolds number, $Re_\lambda = 720$, turbulence experiment time series. After decomposition, the original velocity time series is separated into several intrinsic modes. It is found that this method acts as a dyadic filter bank in frequency domain (in Fourier frame). Comparing the Fourier spectrum of each mode, we can draw that the first mode contains the smallest scale and the most noise of the measurement, and several modes are associated to the inertial subrange. Finally, when the Fourier spectrum of each mode is compared with the original ones, these modes can be divided into three terms : the smallest scales corresponding to the dissipation range, the moderate scales corresponding to the inertial subrange and the large scales corresponding to the coherent structures (energy-contain structures). When all these modes are added step by step, it illustrates a clearly asympto-

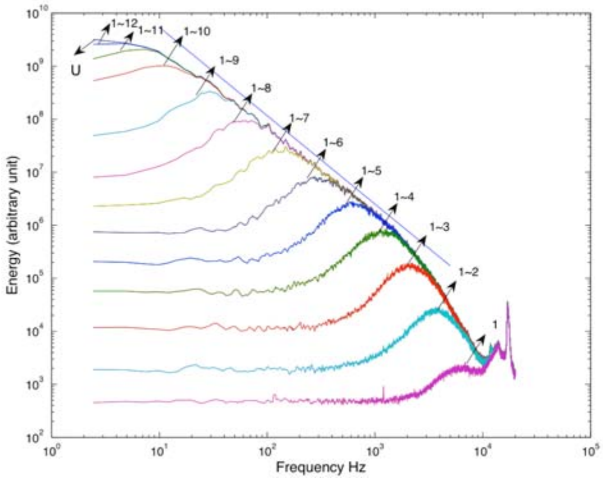

FIG. 3 – Fourier spectrum of the sum of modes, 1 to p, with $p = 2, 3...12$. It shows a clear asymptotic behavior. The slope of reference line is $-5/3$.

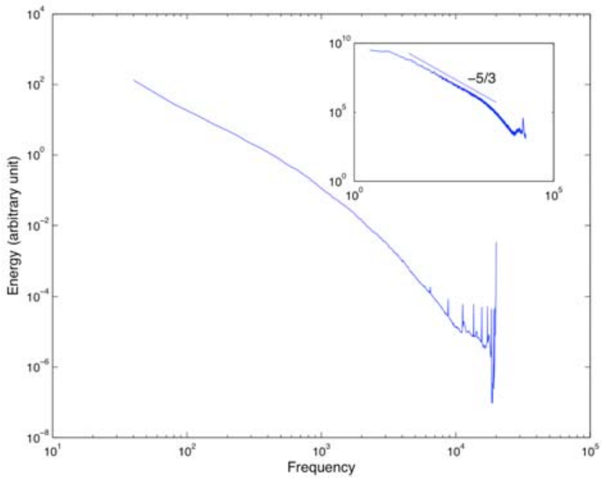

FIG. 4 – Hilbert Marginal Spectrum of Velocity U. For comparison Fourier spectrum is displayed in the up-right pannel. It may suggest an approximate (or generalized) power-law.

tic approximation behavior. This can be very useful for turbulence modeling : some model parameters may be adjusted based on these results. This provides also a possible way to establish a low dimensional dynamical system modelling [9].

The Hilbert marginal spectrum displayed an approximate power-law, which is different with the Fourier spectrum $-5/3$. We also considered high order statistical moments to measure the intermittency for each mode ; this could provide a new way to characterize the scale-dependence of turbulent intermittency.

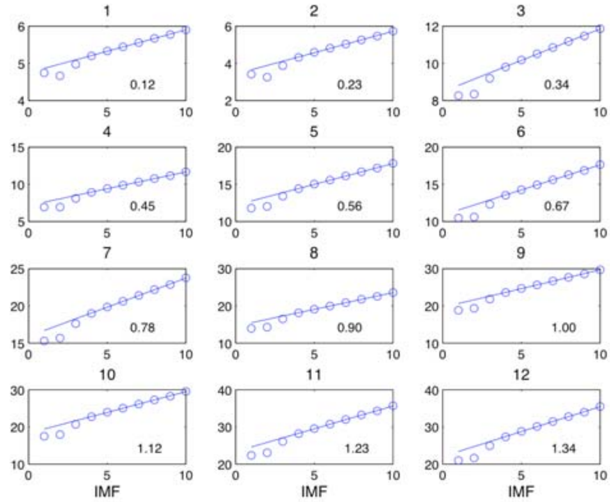

FIG. 5 – The statistical moments of order q of each IMF, $\langle |C_k|^q \rangle$.

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