# A Batch-Recursive Algorithm For Passive Ground Target Tracking

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**Abstract** – We propose a batch-recursive algorithm for tracking ground moving targets on constrained paths using bearings-only measurements in clutter collected by a ground moving observer. The proposed algorithm, in its batch stage, uses a fast and low-complexity procedure based on the maximum-likelihood method to obtain a raw estimate of the line-of-sight direction. Once this is achieved, such an estimate is employed at the recursive stage to initialize a *regularized particle filter*, which provides track maintenance by recursively estimating a modified polar coordinate representation of the state vector. The scenario represents a target moving along a realistic road network with junctions, roads branching or crossing, where the probability of having measured the line-of-sight bearing, among the multiple observed ones, is less than the unity. Realistic simulations are presented to support our findings.

## 1 Introduction

The problem of bearings-only tracking (BOT) is to track the kinematical parameters (i.e. position, velocity, etc.) of a transmitting target from passive measurements of the line-of-sight (LOS) bearings. The use of a single observer requires an observer maneuver in order to estimate the target range (see [1] for further reference). In practical situations, BOT is further complicated by the presence of spurious measurements due to clutter. Moreover, obstacles between target and observer may produce a temporally disappearance of the LOS bearing. When tracking ground moving targets, complex target motion models are usually used in order to handle its high capability to perform sudden maneuvers [2].

Due to the increasing development of digital maps containing terrain information, such as roads, open fields, hills, tunnels, etc., recent contributions to the target tracking problem consider this valuable source of information to improve accuracy (see [2] as example). In this paper we incorporate such information in a batch-recursive algorithm for tracking a ground moving target using bearings-only measurements in clutter. The scenario represents a target and an observer moving along a realistic road network with junctions, roads branching and crossing, where the probability of having measured the LOS bearing, among the multiple observed ones, is less than the unity. We use the maximum-likelihood (ML) method at the algorithm's batch stage to obtain an initial estimate of the LOS direction. At the recursive stage, such an estimate is used to propose initial particles in the *regularized particle filter* (RPF) which tracks the target. Our approach differs from [3], because i) we use the modified polar (MP) coordinates system to represent the state vector, ii) the batch procedure is not intended to obtain an initial estimate of the full state vector, but an estimate of the LOS direction only, which results in a faster and low-complexity procedure to initialize the recursive stage, iii) we incorporate road network information to improve accuracy and observability in the estimation of the state vector, which also permits the use of a simpler target motion model, and iv) target tracking is performed using the *regularized particle filter*.

The organization of the paper is as follows. In section 2, we present the state and measurement equations to road constrained motion. In section 3 we describe the RPF implementation. Section 4 presents the batch procedure to provide track initialization. In section 5 simulation results are given. Finally, in section 6 we give the conclusions.

## 2 Target State Equations

Commonly, road network information is modeled as a large collection of roads, each of which consists of a number of interconnected segments. Each segment is assumed to be a straight line between two georeferenced nodes. Our approach incorporates such information at three different stages: i) constraining the direction of the velocity components of the target state vector [4], ii) using the concept of *directional process noise* [2], that assumes for on-road targets more uncertainty along a road segment than orthogonal to it, and iii) considering the road network information as a pseudo-measurement [5].

### 2.1 Constrained Dynamic Model

Knowing the event that the target is evolving on a specific road segment $s$ and considering that its relative velocity vector is parallel to the direction of $s$ [4], it can be proved, by combining the noiseless approximate dynamic equations [6] and the cartesian to MP coordinates (and vice versa) transformation equations [1], that the constrained relative dynamics of a constant velocity target w.r.t. a maneuvering observer can be expressed as

$$\boldsymbol{\chi}_{k+1} = \boldsymbol{f}_s(\boldsymbol{\chi}_k) - \boldsymbol{\varrho}_k + \boldsymbol{v}_k \tag{1}$$

where

• $\boldsymbol{\chi}_k$ is the relative discrete target state vector at time $k$ in modified polar coordinates defined as

$$\boldsymbol{\chi}_k = \begin{bmatrix} \theta_k & \dot{\theta}_k & \xi_k & r_k \end{bmatrix}^T \tag{2}$$

where $\theta_k$ and $r_k$ stand, respectively, for the relative bearing angle and range, with first order derivatives $\dot{\theta}_k$ and $\dot{r}_k$, and where $\xi_k = \dot{r}_k / r_k$ is the normalized range rate,

• $\boldsymbol{f}_s(\boldsymbol{\chi}_k)$ is a vector function describing the noiseless relative dynamics of a target, w.r.t a non-maneuvering observer, whose velocity vector is parallel to the road segment $s$, and which is given by

$$\boldsymbol{f}_s(\boldsymbol{\chi}_k) = \begin{bmatrix} \arctan(A_k/B_k) \\ E_k(m_s B_k - A_k) \\ E_k(B_k + m_s A_k) \\ r_k C_k \end{bmatrix} \tag{3}$$

with

$$\begin{aligned} A_k &= \sin\theta_k + m_s T D_k \\ B_k &= \cos\theta_k + T D_k \\ C_k^2 &= A_k^2 + B_k^2 \\ D_k &= \xi_k \cos\theta_k - \dot{\theta}_k \sin\theta_k \\ E_k &= D_k / C_k^2 \end{aligned}$$

where $m_s$ stands for the slope of the road segment $s$ and $T$ represents the sampling time,

• $\boldsymbol{\varrho}_k$ is a vector function accounting for a constant observer acceleration and given by

$$\boldsymbol{\varrho}_k = \frac{T}{r_k C_k^2} \begin{bmatrix} 0 \\ \gamma_y B_k - \gamma_x A_k \\ \gamma_y A_k + \gamma_x B_k \\ 0 \end{bmatrix} \tag{4}$$

where $(\gamma_x, \gamma_y)$ stand, respectively, for the known observer acceleration components in $x$ and $y$ directions, and

• $\boldsymbol{\upsilon}_k$ is the process noise used to model unpredictable target accelerations, assumed to be zero-mean, white and Gaussian whose covariance matrix is given by

$$\mathbf{Q}_{k,s} = \mathbf{J}_k \begin{bmatrix} \mathbf{Q}_{k,s}^p & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_{k,s}^v \end{bmatrix} \mathbf{J}_k^T \tag{5}$$

where $\mathbf{J}_k$ stands for the Jacobian matrix containing the partial derivatives of $\boldsymbol{\chi}_k$ w.r.t. to the position and velocity components, $\mathbf{Q}_{s,k}^p$ and $\mathbf{Q}_{s,k}^v$ stand, respectively, for the noise covariance matrices for the position and velocity components built using the *directional process noise* [2].

## 2.2 Measurement Equations

In the following, we present the measurement model corresponding to the bearings collected by the ground moving sensor and we introduce the concept of pseudo-measurement to constraint the position of the target to the road network.

### 2.2.1 Bearing Measurements

The $M_k$ bearing measurements available at time $k$, are disposed in vector $\mathbf{y}_k$, whose elements are given by

$$y_{j,k} = \begin{cases} \theta_k + w_{1,k} & \text{if } \psi_k = j, \\ u_k & \text{if } \psi_k \neq j \text{ or } \psi_k = 0 \end{cases} \tag{6}$$

where $j \in \{1, \ldots, M_k\}$ denotes the index element in vector $\mathbf{y}_k$, $w_{1,k}$ is a zero-mean independent Gaussian noise with variance $\sigma_\theta^2$, $u_k$ is a random variable accounting for the bearings due to clutter, assumed to be uniformly distributed in the interval $\mathcal{I} = [0, 2\pi]$ [7], and $\psi_k$ is a $\{0, 1, \ldots, M_k\}$-valued random variable with probability $p(\psi_k = i) = 1 - P_D$ if $i = 0$ and $p(\psi_k = i) = P_D / M_k$ if $i \neq 0$, where $P_D$ is the prior probability of target detection. It should be noticed that for $\psi_k \neq 0$, $\psi_k$ denotes the index of the LOS bearing in vector $\mathbf{y}_k$, and for $\psi_k = 0$, it represents the absence of LOS bearing in $\mathbf{y}_k$.

### 2.2.2 Pseudo-Measurement

An alternative approach to incorporate road network information is through the use of pseudo-measurements [5]. In this paper we propose to use them for three purposes; i) to define the road segment in which the target is evolving, ii) to handle target transitions between road segments (e.i. when the target approaches a node) and iii) to penalize the target evolution far away from the road network. Thus, let define

$$d_k = h(p_{\mathbf{x}_k}, s) + w_{2,k} \tag{7}$$

as an independent pseudo-measurement of the minimum Euclidian distance between the target's position $p_{\mathbf{x}_k}$ at time $k$ and the nearest road segment $s$ [5], where $h(\cdot)$ denotes the non linear function providing the distance, and $\omega_{2,k}$ is the measurement noise, assumed to be zero-mean white Gaussian process with variance $\sigma_d^2$. We consider that the dynamics of the target state is constrained to the road segment $s$, if $s$ is the nearest road segment to the current position of the target.

# 3 Regularized Particle Filter

Consider the system described by equations (1), (6) and (7) and let denote the set of available observations at time $k$ by $\mathbf{Z}_k = \{\mathbf{z}_0, \ldots, \mathbf{z}_k\}$, with $\mathbf{z}_k = [\ \mathbf{y}_k^T \quad d_k\ ]^T$. From a Bayesian perspective, the tracking problem is to recursively calculate some degree of belief in the state $\boldsymbol{\chi}_k$ at time $k$, taking different values, given the data $\mathbf{Z}_k$ up to time $k$. Hence, it is required to construct the posterior density $p(\boldsymbol{\chi}_k | \mathbf{Z}_k)$. In this procedure, it is assumed that the initial density $p(\boldsymbol{\chi}_0) = p(\boldsymbol{\chi}_0 | \mathbf{Z}_0)$ is available.

One simple method to approximate the posterior density is by means of particle filters [8]. Thus, starting with a weighted set of samples (particles) $\{\boldsymbol{\chi}_{k-1}^i, \omega_{k-1}^i\}_{i=1}^N$ approximately distributed according to $p(\boldsymbol{\chi}_{k-1} | \mathbf{Z}_{k-1})$, new samples are drawn from a suitable proposal distribution, which may depend on the previous state and the new measurements, however, for simplicity it is often chosen to be the prior, i.e. $\boldsymbol{\chi}_k^i \sim p(\boldsymbol{\chi}_k | \boldsymbol{\chi}_{k-1})$. In order to maintain a consistent sample, the new importance weights are set to $\omega_k^i \propto \omega_{k-1}^i p(\mathbf{Z}_k | \boldsymbol{\chi}_k^i)$ where $\sum_{i=1}^N \omega_k^i = 1$. Thus, the new particle set $\{\boldsymbol{\chi}_k^i, \omega_k^i\}_{i=1}^N$ is then approximately distributed according to $p(\boldsymbol{\chi}_k | \mathbf{Z}_k)$ and, therefore, an estimate of the state can be obtained using, for instance, the minimum mean square (MMS) [5]. It should be noticed that in order to consider multiple bearing measurements and pseudo-measurements, the likelihood of the observations $p(\mathbf{Z}_k|\boldsymbol{\chi}_k^i)$ may be written as

$$p(\mathbf{Z}_k|\boldsymbol{\chi}_k^i) \quad = p(d_k^i|\boldsymbol{\chi}_k^i) \sum_{\psi_k=0}^{M_k} p(\mathbf{y}_k|\boldsymbol{\chi}_k^i, \psi_k) p(\psi_k) \qquad (8)$$

where $d_k^i$ is the minimum Euclidian distance from particle $i$ to the nearest road segment, eq. (7).

To avoid *degeneracy* and *sample impoverishment* [8] we draw new samples if $1/\sum_i(\omega_k^i)^2 < N_{\text{thr}}$, from a continuous approximation of the posterior density $p(\boldsymbol{\chi}_k|\mathbf{Z}_k) \approx \sum_{i=1}^N \omega_k^i K_h(\boldsymbol{\chi}_k - \boldsymbol{\chi}_k^i)$ where $K_h(\boldsymbol{\chi}) = \frac{1}{h^n} K(\frac{\boldsymbol{\chi}}{h})$ is the rescaled Epanechnikov kernel $K(\cdot)$, $h = ((8(n+4)(2\sqrt{\pi})^n)/c_n)^{\frac{1}{n+4}} N^{-\frac{1}{n+4}}$ is the Kernel bandwidth and $n$ is the dimension of the state vector $\boldsymbol{\chi}$, with $c_n$ denoting the volume of the unit sphere in $\Re^n$[8].

# 4 Track Initialization

Whereas spreading particles over the entire road network may be the simplest way to initialize the RPF it may present some serious drawbacks: i) the number of particles required to cover road networks with a large collection of roads may be quiet hight, and ii) for low target detection probabilities, it may occur that the first available observations do not include the LOS bearings, producing particles to concentrate on roads that may be far away from the one on which the target is actually evolving. In such a situation, it may happen that those particles be resampled, and then become very unlikely and soon die out as the new observations are available. Therefore, a procedure to spread particles only in the direction of the LOS bearing becomes necessary to properly track the target. Thus, we propose to estimate the LOS direction by means of a batch procedure. In order to deduce a tractable expression to compute such a direction we assume that the relative position of the target with respect to the observer does not change significatively over a short period of time, implying that the pdf of the measurements vector $\mathbf{y}_k$ at time $k$ with elements given by (6) may be written as

$$p(\mathbf{y}_k) = \prod_{j=1}^{M_k} \left( \frac{1-P_T}{2\pi} + P_T \phi(y_{j,k}; \theta_{LOS}, \sigma^2) \right)^1 \qquad (9)$$

where $P_T = P_D/M_k$, and where $\phi(z; m, \sigma^2)$ stands for the pdf of a Gaussian random variable $z$ with mean $m$ and variance $\sigma^2$. Therefore, collecting $N_m$ measurements within such a short period of time, and using the maximum-likelihood method it can be proved that the estimated direction of the LOS bearing and its estimated variance are given by

$$(\hat{\theta}_{LOS}, \hat{\sigma}^2) \approx \underset{\theta_{LOS}, \sigma^2}{\operatorname{argmax}} \left( \sum_{k=1}^{N_m} \log p(\mathbf{y}_k) \right) \qquad (10)$$

[1] It should be noticed that, in order to make (9) exact the LOS bearing $\theta_{LOS}$ should be a function of $k$. However, since we are interested in spreading particles only in the direction of the LOS bearing, approximation (10) not only results precisely enough to do so, but also computationally inexpensive compared to the approaches where at the batch stage obtain an estimate of the full target state vector.

Thus, initial particles may be uniformly spread within a circular sector such that $\theta^i \sim \mathcal{U}[\hat{\theta}_{LOS} - \hat{\sigma}, \hat{\theta}_{LOS} + \hat{\sigma}]$ and $r^i \sim \mathcal{U}[0, R]$, setting $\dot{\theta}_k^i = 0$, and $\xi_k^i = 0$, for $i = 1:N$, where $R$ stands for the maximum range which guaranties the coverage of the surveillance region and $\mathcal{U}[a,b]$ stands for the uniform distribution in the interval $[a,b]$.

# 5 Simulation Results

The scenario used to demonstrate the performance of the proposed algorithm is depicted in figure 1. The target initially situated at node 16 maintains a constant velocity course, $v_T = 15$ m/s, changing its direction at nodes $\{15, 2, 3, 4, 18\}$ and describing the solid line trajectory at the bottom of figure 1. The observer departs from node 12 with a constant velocity of $v_o = 17$ m/s and undergoes a constant acceleration of $a_o = 0.3$ m/s$^2$ in the interval 0.5-1.0 minutes, afterwards it maintains the initial constant velocity changing its course at nodes $\{13, 9, 10, 6, 7\}$ and describing the doted line trajectory at the top of figure 1. Three bearing measurements are received at each sensor scan $T = 0.5$ $s$, for an approximated observation period of 5.70 minutes. When present, the LOS bearings are measured with an accuracy of $\sigma_\theta = 0.5$ deg.

The batch stage used $N_m = 50$ measurements to estimate the LOS direction, and the following nominal filter parameters were used at the recursive stage: the *directional process noise* standard deviations (STDs) for the position and velocity components orthogonal to the road were respectively set to $\sigma_{o,p} = 5.5$ m and $\sigma_{o,v} = 3$ m/s. The corresponding STDs for the components along the direction of the road were $\sigma_{a,p} = \sqrt{10}\sigma_{o,p}$ and $\sigma_{a,v} = \sqrt{2}\sigma_{o,v}$. The pseudo-measurement STD was $\sigma_d = 5.5$ m and the regularized particle filter used $N = 1000$ particles, carrying out resampling for $N_{\text{thr}} = 2/3N$.

The estimation performance of the proposed algorithm is provided in terms of the root-mean-square (RMS) position errors using 100MC runs. Figure 2 shows the RMS error curves corresponding to four different target detection probabilities $P_D = \{1, 0.9, 0.8, 0.7\}$. As expected, low target detection probabilities leads to a degradation on the accuracy of the target tracking. However, in spite of multiple spurious bearing measurements at each sensor scan, a target detection probability of less than the unity and of the use of a low-complexity target motion model, the proposed algorithm exhibits a comparable performance with respect to the algorithms studied in [9] (RMS errors between 0.20-1.04kms) for a maneuvering target in a typical BOT scenario, i.e. without spurious measurements and a target detection probability equal to one. Figure 3 depicts the average over the trajectories followed by the target for 100MC runs. As we can see, even for a target detection probability of 0.7, in most of the cases the estimated trajectory is the right one.

# 6 Conclusions

We proposed a new batch-recursive bearings-only tracking algorithm for ground moving targets constrained to roads, able to handle multiple bearing measurements in clutter. Observer maneuver may not be a requirement because the information provided by the road network improves observability of the state vector. Simulation results showed an improvement on the accuracy of target tracking over those using complex target evolution models, even for a probability of target detection less than the unity.

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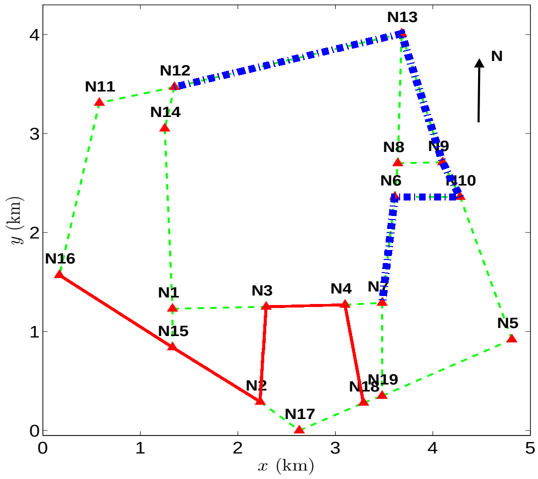


FIG. 1: Simulation scenario: dashed lines and solid triangles represent respectively the road segments and nodes of the road network. Bottom solid trajectory represents the target path. Top doted trajectory is the observer's path.

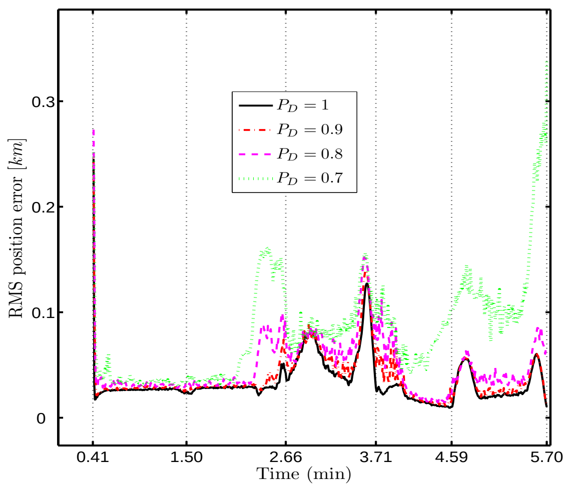


FIG. 2: RMS position error versus time. Vertical dashed lines represent transition instants between road segments in the target trajectory.

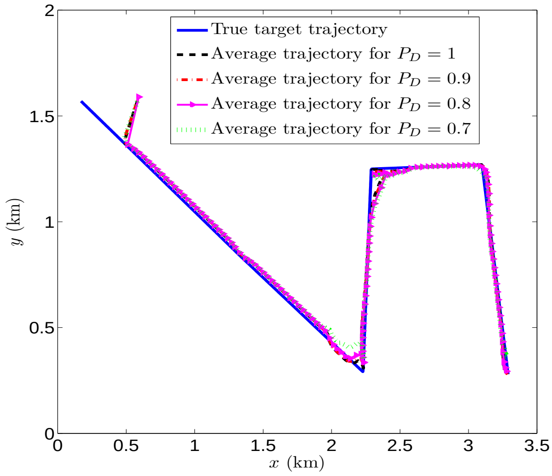


FIG. 3: Actual and average trajectories obtained from 100MC runs.

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