Detection and Estimation of High Energy Physical Particles Using Monte Carlo Methods

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Résumé – Cet article est consacré à la détection de particules de très haute énergie, dites "muons", étudiées dans le cadre de l'observatoire international Pierre Auger. On propose une modélisation statistique originale des données qui permet de traiter efficacement la trace laissée par les muons sur les détecteurs en ignorant le signal de nuisance généré par d'autres particules plus nombreuses mais de moindre intérêt. Mme si elle requiert l'utilisation de méthodes de Monte Carlo par chaîne de Markov, l'approche proposée reste de complexité raisonnable, compte tenu des contraintes de l'application, et améliore significativement l'état de l'art.

Abstract – This paper addresses the problem of detection and estimation of physical particles called muons encountered in the Auger project. One of the main challenges in this work is to separate the signals generated by muons from the ones generated by electro-magnetic (EM) particles. In this paper, we provide a simple *approximate* model to capture the contributions of the EM particles which enables us to separate the two signals efficiently. Next, we design an Reversible Jump MCMC (RJ-MCMC) sampler to count muons and estimate their parameters. The results show the capability of the proposed model and sampler in detecting and estimating muons in mixed signals.

1 Introduction

The Auger project [1] is aimed at studying ultra-high energy cosmic rays, with energies in order of 10^{19}eV, the most energetic particles found so far in the universe. The long-term objective of this project is to study the nature of those ultra-high energy particles and determine their origin in the universe. Nevertheless, they are *not* observed directly. In fact, when they collide the earth's atmosphere, a host of secondary particles are generated, some of which, mostly "muons", finally reach the ground. To detect them, the Pierre Auger Observatory was built which consists of two independent detectors; an array of Surface Detectors (SD) and a number of Fluorescence Detectors (FD).

The number of muons and their arrival times can be used as indications of both the chemical composition and the origin of the primary particles (see [1] for more information). Here, we concentrate on the signal decomposition problem, where the goal is to count the number of muons and estimate their individual parameters from the signals observed by the SD detectors. To do so, one of the main challenges is to deal with the "background signal" generated by Electro-Magnetic (EM) particles making the problem of muon counting rather difficult.

Here, we follow a Bayesian paradigm and use the hierarchical model developed in [2, 4] for modeling the muonic signal. Since, the posterior distribution of the unknown parameters is only available up to a normalizing constant, we opt for approximating it using Monte Carlo simulation methods [6]. More precisely, we use the Reversible Jump MCMC (RJ-MCMC) sampler proposed by [3], which enables us to simultaneously count the muons and estimates their individual parameters.

In this work, we concentrate on the crucial issue of modeling the signal generated by the EM particles. For this purpose, one should note that modeling them "exactly" and counting them using the RJ-MCMC sampler is not an efficient solution, as they are numerous (their number can reach even a few hundreds). Moreover, they are less informative compared with muons when making inference about the parameters of the original particle. Hence, here, we propose an *approximate* model to capture their contribution in a less computationally expensive approach. In fact, in the Auger project, each primary particle generates traces in a few SD tanks (typically a few tens), and, thus, the processing of each single trace should remain manageable in terms of computation time.

This paper is organized as follows. Section 2 describes briefly the hierarchical model developed in [2, 4] for modeling the muonic signal and the proposed approximate model for describing EM contributions. Next, in section 3 we follow a Bayesian framework to jointly detect the muons and estimate their parameters using an RJ-MCMC sampler that we designed for this problem. Section 4 discusses the results showing the performance of the proposed algorithm. Finally, Section 5 concludes the paper and gives possible future directions.

2 Hierarchical Model

This section describes the hierarchical model we developed for modeling both the muonic signal [2, 4] and the contributions of the EM particles.

When a muon (or an EM particle) crosses a SD tank, it generates photoelectrons (PE's) along its track that are, then, captured by detectors and create a discrete observed signal. We denote the vector of length N of the observed signal[1], generated by both muons and the EM particles, by $\boldsymbol{n} = (n_1, \ldots, n_N) \in \mathbb{N}^N$, where the element n_i indicates the number of PE's deposited in the time interval

$$[t_{i-1}, t_i) \triangleq [t_0 + (i-1)t_\Delta, t_0 + i\, t_\Delta),$$

where t_0 is the absolute starting time of the signal and $t_\Delta =$ 25 ns is the signal resolution (length of one bin).

Each particle has two component-specific parameters, namely, the arrival time $t_\star$ and the signal amplitude $a_\star$; in what follows, replace the subscript $\star$ by μ for muons and by EM for the EM particles. The absorption process of the PE's deposited by either muons or EM's is modeled by a non-homogeneous Poisson point process with the expected number of PE's in the bin i given as [2, Section 2.2]

$$\bar{n}_{\star,i}(a_\star, t_\star) = a_\star \int_{t_{i-1}}^{t_i} p_{\tau,t_d}(t - t_\star)\mathrm{d}t, \qquad (1)$$

where $p_{\tau,t_d}(t)$ is the time response distribution, t_d is the rise-time and τ is the exponential decay (both measured in ns). Note that the total expected number of PE's is $\bar{n}_i = \bar{n}_{\mu,i} + \bar{n}_{\mathrm{EM},i}$, for $i = 1, \ldots, N$.

2.1 Model for the muonic signal

Assuming independence of muons, the expected number of PE's in the i^{th} bin, i.e., $\bar{n}_{\mu,i}$, given N_μ, $\boldsymbol{t}_\mu$, and $\boldsymbol{a}_\mu$ becomes

$$\bar{n}_{\mu,i} = \sum_{j=1}^{N_\mu} \bar{n}_{\mu,i}(a_{\mu,j}, t_{\mu,j}), \qquad (2)$$

where $\bar{n}_{\mu,i}(a_{\mu,j}, t_{\mu,j})$ is defined in (1).

To complete the model, following [2, 4], we assume the muons' arrival times and the signal's amplitudes are independent *a priori*. The muonic signal amplitude a_μ depends on muon's energy and its track-length inside the water tank. The muon's track-length distribution has a geometrical expression based on the zenith angle θ and water tanks geometry which is derived in [5].

The prior distribution of muon's arrival time, i.e., $p(t_\mu)$, is assumed to be a log-Normal distribution $\mathcal{LN}(a, b)$ shifted by t_0 ns to the right. Finally, a truncated Poisson with mean $\bar{N}_\mu$ is assigned on the number N_μ of muons.

[1]Note that here we use the number of observed PE's as the observed signal for simplicity.

2.2 Approximate model for the EM signal

Due to the fact that EM particles are often numerous compared to the muons (in the order of a few hundreds in some cases), handling them using the RJ-MCMC sampler is not realistic. In addition, counting them accurately is of less concern, as they are barely informative in estimating the characteristics of the original particle. Therefore, in this paper, we propose to approximate their contribution, which is a marked Poisson point process, by a simpler process which is controlled by the hyperparameters of the prior distributions assigned over EM parameters.

More precisely, we approximate the contribution of EM's, i.e., $\bar{\boldsymbol{n}}_{\mathrm{EM}}$, and we assume that it is drawn from a distribution $\mathcal{D}(\boldsymbol{m}_{\mathrm{EM}}, \boldsymbol{\Sigma}_{\mathrm{EM}})$—a distribution with the mean vector $\boldsymbol{m}_{\mathrm{EM}}$ and covariance matrix $\boldsymbol{\Sigma}_{\mathrm{EM}}$.

For the sake of efficiency of the sampler and at the price of losing correlation between adjacent bins, we assume that the contribution of the EM particles are *independent* between the bins. Therefore, we write the expected number of PE's in the bin i, $i = 1, \ldots, N$, as $\bar{n}_i = \bar{n}_{\mu,i} + \bar{n}_{\mathrm{EM},i}$, where $\bar{n}_{\mathrm{EM},i} \sim \mathcal{D}\left(m_{\mathrm{EM},i}, \sigma^2_{\mathrm{EM},i}\right)$.

The means $m_{\mathrm{EM},i}$ and variances $\sigma^2_{\mathrm{EM},i}$ are derived by calculating the first and second moments of the marked Poisson point process according to which $\bar{n}_{\mathrm{EM},i}$ is distributed; see (1). This process is controlled by the hyperparameters of EM's. To proceed, we assign log-normal $\mathcal{LN}(a, b)$ prior distributions over both their arrival times t_{EM} and their signal's amplitude a_{EM}. We denote by $\boldsymbol{\psi}_{\mathrm{EM}} = (\bar{N}_{\mathrm{EM}}, a_{a_{\mathrm{EM}}}, b_{a_{\mathrm{EM}}}, a_{t_{\mathrm{EM}}}, b_{t_{\mathrm{EM}}})$ the vector of hyperparameters of the EM particles, which control the distribution $\mathcal{D}\left(m_{\mathrm{EM},i}, \sigma^2_{\mathrm{EM},i}\right)$.

To choose $\mathcal{D}\left(m_{\mathrm{EM},i}, \sigma^2_{\mathrm{EM},i}\right)$ from the parametric family of distributions, one should note that it should live on $\mathbb{R}_+$, as $\bar{n}_i$ cannot take negative values. Moreover, we want a distribution that alleviates the sampling steps. Therefore, we opt for using a Gamma distribution $\mathcal{G}(\alpha_{\mathrm{EM},i}, \beta_{\mathrm{EM},i})$, with the shape $\alpha_{\mathrm{EM},i}$ and the scale $\beta_{EM,i}$, for $\bar{n}_{\mathrm{EM},i}$; since Gamma is a conjugate distribution for the mean parameter of a Poisson distribution (note that $p(n_i \mid \bar{n}_i)$ is Poisson). By a simple reparametrization, we have $\alpha_{\mathrm{EM},i} = m^2_{\mathrm{EM},i}/\sigma^2_{\mathrm{EM},i}$ and $\beta_{\mathrm{EM},i} = \sigma^2_{\mathrm{EM},i}/m_{\mathrm{EM},i}$.

Therefore, we approximate distribution of $\bar{\boldsymbol{n}}$ by

$$p(\bar{\boldsymbol{n}} \mid \boldsymbol{m}_{\mathrm{EM}}, \boldsymbol{\sigma}^2_{\mathrm{EM}}, \boldsymbol{a}_\mu, \boldsymbol{t}_\mu, N_\mu) = \prod_{i=1}^{N} p(\bar{n}_i | m_{\mathrm{EM},i}, \sigma^2_{\mathrm{EM},i}, \boldsymbol{a}_\mu, \boldsymbol{t}_\mu, N_\mu) = \prod_{i=1}^{N} \mathcal{G}(\alpha'_i, \beta'_i),$$

where

$$\alpha'_i = \frac{(m_{\mathrm{EM},i} + n_{\mu,i})^2}{\sigma^2_{\mathrm{EM},i}} \quad \text{and} \quad \beta'_i = \frac{\sigma^2_{\mathrm{EM},i}}{m_{\mathrm{EM},i} + n_{\mu,i}}.$$

Using the conjugacy of the Gamma prior distribution over the mean of the Poisson distribution, we can integrate $\bar{n}_i$ out and derive the likelihood for bin i as

$$p(n_i \mid m_{\mathrm{EM},i}, \sigma^2_{\mathrm{EM},i}, \boldsymbol{a}_\mu, \boldsymbol{t}_\mu, N_\mu) = \frac{1}{n_i!} \cdot \frac{(\beta'_i/(\beta'_i+1))^{(n_i+\alpha'_i)}}{(\beta'_i)^{\alpha'_i}} \cdot \frac{\Gamma(n_i+\alpha'_i)}{\Gamma(\alpha'_i)}. \tag{3}$$

This makes the sampling faster, as we do not need to generate $\bar{n}_i$, for $i = 1, \ldots, N$.

Finally, the joint posterior distribution of all the unknown parameters is

$$p(N_\mu, \boldsymbol{a}_\mu, \boldsymbol{t}_\mu, \boldsymbol{\psi}_{\mathrm{EM}} \mid \boldsymbol{n}) \propto p(\boldsymbol{n} \mid \boldsymbol{m}_{\mathrm{EM}}, \boldsymbol{\sigma}^2_{\mathrm{EM}}, \boldsymbol{a}_\mu, \boldsymbol{t}_\mu, N_\mu) \; p(\boldsymbol{t}_\mu, \boldsymbol{a}_\mu \mid N_\mu)\, p(N_\mu) f(\boldsymbol{m}_{\mathrm{EM}}, \boldsymbol{\sigma}^2_{\mathrm{EM}} \mid \boldsymbol{\psi}_{\mathrm{EM}})\, p(\boldsymbol{\psi}_{\mathrm{EM}}), \tag{4}$$

where $f(\boldsymbol{m}_{\mathrm{EM}}, \boldsymbol{\sigma}^2_{\mathrm{EM}} \mid \boldsymbol{\psi}_{\mathrm{EM}})$ denotes the relation between the moments and hyperparameters of the EM particles. The posterior (4) is considered as the target distribution for the RJ-MCMC sampler described in the next section.

3 Estimating the parameters

In this section, we describe the RJ-MCMC sampler developed for simulating from the posterior distribution (4). Figure 1 describes one iteration of such a sampler.

This sampler is divided into two main blocks; first block is a conventional RJ-MCMC sampler which aims at sampling from the joint posterior distribution of the number N_μ of muons and their parameters assuming hyperparameters of the EM particles $\boldsymbol{\psi}_{\mathrm{EM}}$ are fixed, i.e., targeting $p(N_\mu, \boldsymbol{a}_\mu, \boldsymbol{t}_\mu \mid \boldsymbol{\psi}_{\mathrm{EM}}, \boldsymbol{n})$. It consists of two move types; a within-model move which updates the muon parameters, i.e., $\boldsymbol{t}_\mu$ and $\boldsymbol{a}_\mu$, assuming the number N_μ of muons is known and a between models move that proposes a jump in the model space by proposing birth or death of a component [3, 4].

The second block simulates the hyperparameters of the EM particles conditioning on the parameters of muon[2]. Since these hyperparameters are in the highest level of hierarchy in the model, they get very little information from the observed data. Hence, an accurate estimate of them is not the purpose of this step. The main goal here is to separate the background signal from the muonic signal, to have a better inference on the number N_μ of muons and their arrival times.

To proceed, first, we need to assign prior distributions over the EM's hyperparameters. It is known from physics [4] that the distribution of arrival times of the EM particles has a heavier tail than the one of arrival time of muons. Moreover, the distribution of their amplitudes is concentrated on small values compared to the ones of muonic signal amplitudes. Constrained uniform distributions were assigned as prior distributions over $\boldsymbol{\psi}_{\mathrm{EM}}$. These constraints prevents some undesired behavior of the sampler, such as, the background signal taking parts of muonic signal. Finally, to update $\boldsymbol{\psi}_{\mathrm{EM}}$, a mixture of two normal random walk kernels are used.

[2]Other option would be to maximize the likelihood $p(\boldsymbol{n} \mid \boldsymbol{\psi}_{\mathrm{EM}})$ in the spirit of empirical Bayes framework using a Monte Carlo Expectation Maximization algorithm. However, we found that sampling $\boldsymbol{\psi}_{\mathrm{EM}}$ is a much simpler approach.

At the $(m+1)^{\mathrm{th}}$ iteration of the sampler

Update Muon (EM Fixed): Update muon parameters assuming EM parameters are fixed;

- generate $(N_\mu^{(m+1)}, \boldsymbol{a}_\mu^{(m+1)}, \boldsymbol{t}_\mu^{(m+1)})$ from the target posterior distribution $p(N_\mu, \boldsymbol{a}_\mu, \boldsymbol{t}_\mu \mid \boldsymbol{\psi}_{\mathrm{EM}}^{(m)}, \boldsymbol{n})$ using the RJ-MCMC sampler.

Update EM (Muon Fixed): Update EM parameters assuming muon parameters are fixed;

- generate $\boldsymbol{\psi}_{\mathrm{EM}}^{(m+1)}$ from the target posterior distribution $p(\boldsymbol{\psi}_{\mathrm{EM}} \mid N_\mu^{(m+1)}, \boldsymbol{a}_\mu^{(m+1)}, \boldsymbol{t}_\mu^{(m+1)}, \boldsymbol{n})$ using Metropolis-Hastings (MH).

FIG. 1: One iteration of the proposed sampler designed for generating samples from the target distribution (4).

4 Results and discussion

In this section, we show the capability of the proposed model and the designed sampler in detecting muons and estimating their parameters. The data we used consist of simulated showers with Proton and Iron as their chemical composition of the original particle. For Proton and Iron, we have access to 3382 signals from 89 and 1221 signals from 30 showers, respectively. The energy of showers are close to 10^{19} eV and their zenith angle $\theta \in [45°, 60°]$.

In order to illustrate the pattern of the observed signals we are dealing with, as well as the obtained results using the proposed Bayesian method, an example with eleven muons is presented in Figure 2; (a) top panel shows the observed signal, while bottom panel shows the muonic signal in red and the EM signal in blue. It can be seen that certain peaks in the EM signal (in blue) are very similar to the ones generated by muons (in red), such that the total signal (on top) is difficult to analyze (recall that the objective is to separate the two signals and count the muons). Figure 2 (b) shows the marginal posterior distributions of the number N_μ of muons (left) and sorted arrival times given N_μ (right), obtained using 40 000 samples generated by the designed RJ-MCMC sampler. Each line corresponds to a value of N_μ, for $8 \leq N_\mu \leq 11$. The

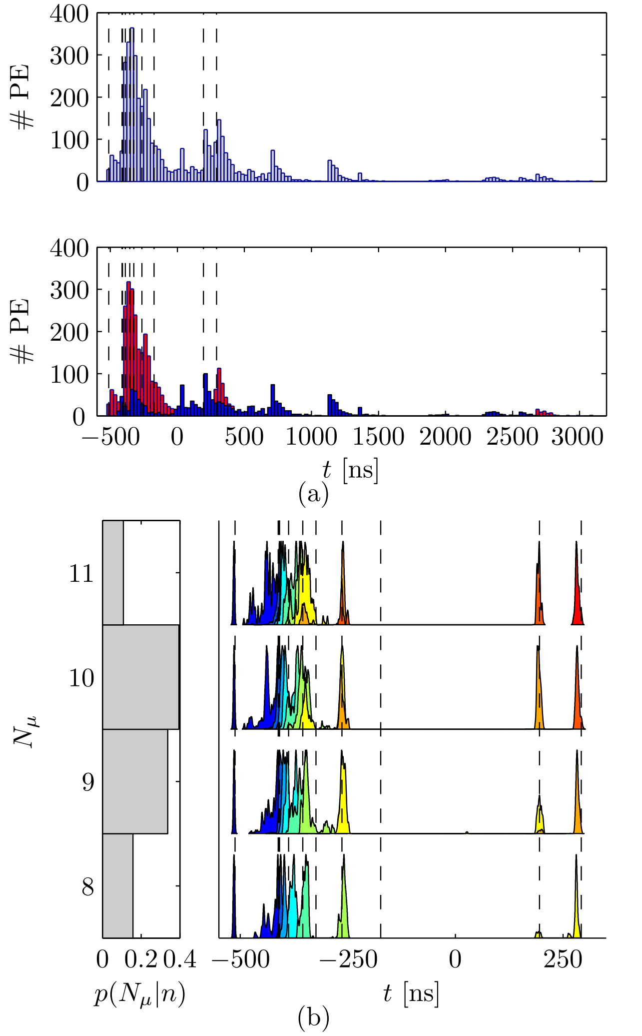

FIG. 2: An example with eleven muons; (a) top panel shows the observed signal, while bottom panel shows the muonic signal in red and EM signal in blue. (b) marginal posterior distributions of the number N_μ of muons (left) and sorted arrival times given N_μ (right).

true arrival times are indicated by vertical dashed lines.

From figure 2 (b) it can be observed that the RJ-MCMC sampler has explored the models $\mathcal{M}_8$ to $\mathcal{M}_{11}$, $\mathcal{M}_{10}$ has the greatest posterior probability, $p(N_\mu = 10 \,|\, \boldsymbol{n}) \simeq 0,40$. More precisely, all the eleven muons have been detected except the one located at $t = -174$ ns.

Next, we show the average performance of the sampler on all available signals in a systematic way. To obtain these results, 40 000 iterations of the sampler described in the previous section were run and the first half of the chains was discarded as burn-in period. Figure 3 presents normalized muon detection error defined as $(\widehat{N_\mu} - N_\mu)/N_\mu$, where $\widehat{N_\mu}$ corresponds to the model with the highest posterior probability. The mean±standard deviation of the normalized errors is 0.12±0.21 and 0.1±0.15, for Proton and Iron, respectively.

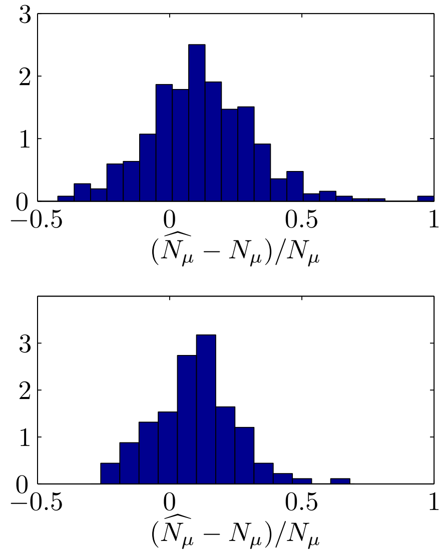

FIG. 3: Distribution of normalized muon detection error; top panel Proton (3382 signals from 89 showers) and bottom panel Iron (1221 signals from 30 showers).

5 Conclusion

In this paper, we proposed a hierarchical model and an RJ-MCMC sampler for the problem of joint detection and estimation of muons. Presented results showed the efficiency of the proposed Bayesian method.

Recall that so far we used the observed number of PE's as signal for simplification. Hence as a future work, we need to generalize the designed sampler to be able to process the "actual" Auger signal, which are also affected by measurement noise.

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