Signal Quantization using a Leaky Integrate-and-Fire neuron

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Résumé – Dans cet article, nous présentons un quantificateur basé sur le modèle neuronal Leaky-Integrate and Fire (LIF). Le LIF est un modèle simplifié du fonctionnement des cellules ganglionnaires. Les cellules ganglionnaires sont placées dans la couche de la rétine responsable du codage de l'information visuelle, avant qu'elle ne soit transmise au cerveau à travers le nerf optique. En général, le LIF quantifie les valeurs d'intensité selon une valeur seuil, un temps d'observation donné, la présence ou non d'une période réfractaire dans le neurone et les paramètres R et C caractérisant la résistance et la capacité du modèle neuronal. En variant la valeur du seuil et du temps d'observation, nous avons testé expérimentalement le comportement du quantificateur LIF à un signal d'entrée donné et nous présentons les résultats et la comparaison avec le quantificateur scalaire uniforme et le quantificateur Lloyd.

Abstract – In this paper we present a quantizer based on the Leaky-Integrate and Fire (LIF) neural model. The LIF is the model according to which function the ganglion cells. Ganglion cells are placed in the layer of the retina responsible for the encoding of visual information, before it is transmitted to the brain through the optic nerve. In general, the LIF quantizes intensity values according to a threshold value, a given observation time, the presence or not of a refractory period in the neuron and the parameters R and C characterizing the resistance and capacity of the neural model. Varying the value of the threshold and observation time we tested experimentally the behaviour of the LIF quantizer to a given input signal and through this work we present the results and the comparison to the uniform scalar quantizer and the Lloyd quantizer.

1 Introduction

As technology improves, the need for finding new ways for the efficient transmission and storage of information augments dramatically. It is generally believed, that nature provides the ideal means and methods for information processing through the use of chemical and biological reactions in the body. The system responsible for the information processing in the body is the nervous system which consists of neurons. Neurons, are cable-like cells which receive electrical impulses, and if the stimulus is important enough according to some threshold, they encode the stimulus into action potentials (spikes) that travel along the neuron axon to be transmitted to the next neuron.

As analytically described in [3], a great example of the neural coding activity, is the one performed by the ganglion cells in the ganglionic layer of the mammalian retina for the encoding of the visual information. Inspired by this particular encoding procedure, we carried an extensive study on the Leaky Integrate-and-Fire (LIF) neural model which characterizes the function of the ganglion cells. More specifically, we implemented a LIF quantizer for the quantization of input intensity values according to the number of spikes produced by the LIF neural model. We theoretically and experimentally studied the behavior of the quantizer for various parameter values and computed the accuracy and the efficiency of our quantizer by computing the rate-distortion curves and comparing them to existing quantizers.

With this paper, we aim to introduce the LIF quantizer and present the analytical theoretical study of the model and its experimental testing according to the variation of the values of the model's parameters. It is highly important to mention that unlike other existing quantizers, this particular model encodes input data in a dynamic way and exhibits an interestingly different behavior under the selection of the parameter values of the neural refractory period and the observation time. In section 2, we provide the necessary theoretical background describing the LIF neuron model and provide all the equations for the analytical computation of the model's characteristics such as the inter spike delay in the spike train and the model's firing rate. Furthermore, the characteristic functions and firing rate of the model are being presented. In section 3 we describe our experiments and discuss about finding the optimal parameters for the better behavior of the quantizer. We also make a comparison of the LIF quantizer to the scalar Uniform Quantizer and the non-uniform Lloyd Quantizer. Finally, in section 4 we provide conclusions according to our experiments.

2 LIF model

2.1 Theoretical background

As described in [1], the Leaky Integrate-and-fire neuron is a neural model which can be described by the circuit shown in figure 1.

The input current $I(t)$ is being divided in the current I_R, which passes through the resistor and the current I_C which

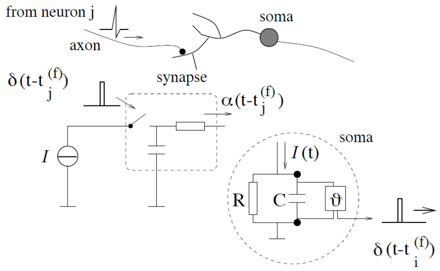

FIGURE 1: The LIF neuron circuit which consists of a resistance R in parallel with a capacitor C. (figure from the book of Gerstner-Kistler "Spiking Neurons" given in [1])

charges the capacitor. Given the Ohm's law for I_R and the definition of capacity as $C = q/u$ (where q is the charge and u the voltage) the total current can be written as

$$I(t) = I_R + I_C = \frac{u(t)}{R} + C\frac{du}{dt} \quad (1)$$

By multiplying (1) by R and by introducing a time constant $\tau_m = RC$ the equation becomes

$$\tau_m \frac{du}{dt} = -u(t) + RI(t) \quad (2)$$

In the integrate-and-fire model, the form of an action potential is not described explicitly. Spikes are generated at a firing time $t^{(f)}$. This firing time is defined by the following threshold criterion

$$t^{(f)} : u(t) = \theta.$$

Immediately after $t^{(f)}$ the potential is set to a new value $u_r < \theta$,

$$\lim_{t \to t^{(f)}; t > t^{(f)}} u(t) = u_r$$

While $t < t^{(f)}$ the dynamics is given by equation (2) until the next threshold crossing occurs. The leaky integrate-and-fire neuron may also incorporate an absolute refractory period. In this case, if u reaches the threshold at time $t = t^{(f)}$, the dynamics is interrupted during an absolute refractory time Δ^{abs} and the integration restarts at time $t + \Delta^{abs}$ with a new initial condition.

Let's consider the simple case of a constant input current stimulus $I(t) = I_0$. For the sake of simplicity we will assume a reset potential $u_r = 0$. Assuming a spike has occurred at time $t = t^k$ the trajectory of the membrane potential is given by integrating (2) with the initial condition $u(t) = u_r = 0$. The solution is given by the relation

$$u(t) = RI_0 \left[1 - exp\left(-\frac{t - t^k}{\tau_m}\right)\right] \quad (3)$$

After each spike, the potential is reset to the value $u_r = 0$ and the integration process starts again. The condition $u(t) = \theta$ is satisfied for $t = t^{k+1}$, where t^{k+1} denotes the time in which the next spike occurs. Then, equation (3) can be written as

$$u(t^{k+1}) = \theta = RI_0 \left[1 - exp\left(-\frac{t^{k+1} - t^k}{\tau_m}\right)\right] \quad (4)$$

We assume $d(u) = t^{k+1} - t^k$, the inter-spike delay of an integrate-and-fire neuron with no refractory period, which is the time between two occurring spikes.

Consequently, solving (4) for the delay $d(u)$ yields

$$d(u) = \begin{cases} \infty, & u < \theta \\ h(u;\theta) = \tau_m \ln \frac{u}{u-\theta}, & u \geq \theta \end{cases} \quad (5)$$

At this point it is important to denote that for the case of a neuron with an absolute refractory period, the occurrence of the next spike will be delayed by the duration of the refractory period Δ^{abs}. So, in this case, the inter-spike delay $d'(u)$ is given by $d'(u) = d(u) + \Delta^{abs}$.

2.2 The LIF Quantizer

2.2.1 The encoder

As described in the previous section, the LIF neuron produces a number of spikes given an input intensity value. This means that the LIF model is able of signal quantization, by assigning a particular spike number value to each signal sample. More specifically, as described in figure 2, for each signal sample, the LIF quantizer first computes the membrane potential u by multiplying the input current value I to the resistance R, according to Ohm's law. The integration delay $d(u)$ is calculated using equation (5), given the parameters of the threshold θ. In the case where there is a non zero refractory period it is added to the interspike delay $d'(u) = d(u) + \Delta^{abs}$. Then, using the relation $N_s = \lfloor \frac{t_{obs}}{d'(u)} \rfloor$, we get the number of spikes N_s produced by the LIF. As a result, the output of our system will be a sequence of numbers of spikes for each input sample, and this is the encoded data produced by the encoder.

In order to compute the amount of information provided by the encoded data, we will compute the entropy H of the encoded signal as

$$H = -\sum_{i=1}^{n} p(N_{s_i}) \log_2 p(N_{s_i}), \quad (6)$$

which is expressed in bits/sample and where $p(Ns_i)$ corresponds to the probability of occurrence of the different values of number of spikes.

2.2.2 The decoder

After we have produced the encoded sequence, the data is being transmitted to the decoder in order to reconstruct the initial signal. In [2] it has been proven that after the encoding of the data into a sequence of spikes, an estimation of the firing period $\tilde{d}$ can be found using $\tilde{d}(u) = \frac{t_{obs}}{N_s}$, where t_{obs} is the observation time of the firing process, and N_s the number of spikes produced by the LIF Quantizer. Finally, computing the inverse function of equation (5) we get

$$\tilde{u} = \begin{cases} 0, & \tilde{d}(u) = \infty \\ h^{-1}(\tilde{d}(u), \theta) = \frac{\theta}{1 - exp\left(-\frac{\tilde{d}}{\tau_m}\right)}, & \tilde{d}(u) < \infty \end{cases} \quad (7)$$

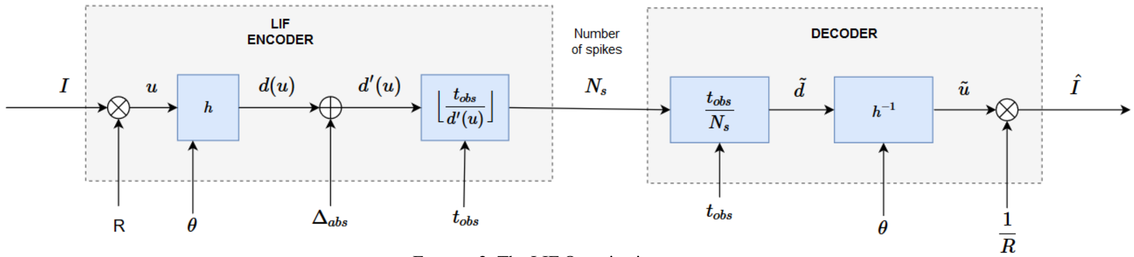

FIGURE 2: The LIF Quantization process.

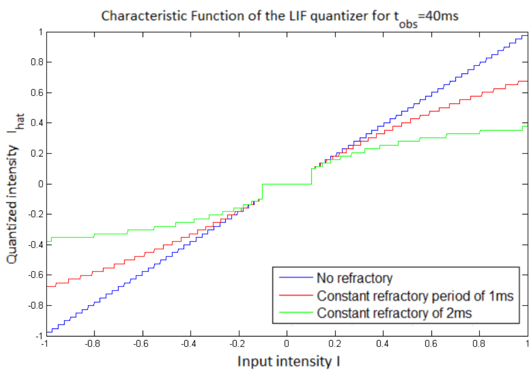

FIGURE 3: Characteristic functions of the LIF quantizer for $\theta = 0.1$ for different values of refractory period for $t_{obs} = 40ms$.

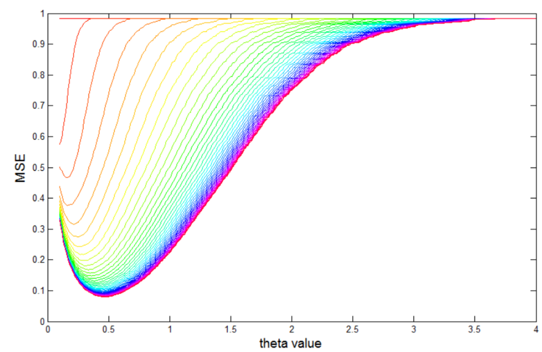

FIGURE 4: MSE of the LIF quantizer in function of the threshold θ, for t_obs varying from 1ms to 40ms for a model with a refractory period $\Delta^{abs} = 1ms$

which computes an approximation of the input value of the membrane potential u. Dividing the action potential by the value of the resistance R we obtain an approximation $\hat{I}$ of the input current value I. The system of encoding using the LIF quantizer is described in figure 2.

2.3 Characteristic functions

We tested experimentally the LIF quantizer using a constant stimulus for each firing process. For the experiments we used a linear sample of input intensities varying from -1 to 1 with a sampling step of 0.001 and constructed the corresponding characteristic functions for the LIF Quantizer with no refractory period and with a constant refractory period of 1ms and 2 ms. The results are presented in figure 3. We can easily observe in figure 4, that in the absence of a refractory period, the LIF quantizer behaves similarly to the scalar uniform quantizer, while in the presence of a constant refractory period, the LIF quantizer is non uniform with an augmenting quantization step. We can also see that our quantizer has a deadzone of a size of 2θ in which no spikes are being produced, since the input intensities are lower than the threshold. Consequently, all the intensities lower than this threshold are being quantized to zero.

3 Experiments

We experimentally tested our quantizer using an input signal of 10000 samples following a random Gaussian distribution of a standard deviation $\sigma = 1$. Our experiments were carried out for values of observation time t_{obs} between 1ms and 40ms, varying the threshold value θ between 0 and 4 with a step of 0.01. We repeated the same experiments for different values of refractory period. First we selected the quantizer to have no refractory period and then we selected both a low refractory value equal to 0.01ms and a high value of 0.2ms. Using this parameter variation we used the LIF quantizer to encode the random signal and then we compute the entropy of the encoded data using the equation (6). For the evaluation of our results we compute the Mean Squared Error(MSE) given by the sum of squared absolute difference between the input value and the quantized value estimated by the LIF quantizer. Figure 4, shows the curve of the MSE in function of the threshold θ, for a LIF quantizer, in the presence of a constant refractory period. The different colors denote the different values of observation time t_{obs}.

From the curve we observe that in the presence of a refractory period the quantizer exhibits overload noise, which is not the case when there is no refractory period. This means that for each observation time, there is a threshold value which minimizes the MSE, which is varying according to the observation time.

3.1 LIF with no refractory period

As described in section 2.3, in the absence of a refractory period, the LIF quantizer behaves similarly to the uniform scalar quantizer with a deadzone λ. In this section, we compare those two quantizers, using the same random input signal and varying the value of θ for the LIF quantizer and the step size q for the Uniform quantizer. For lower observation times, the dead zone of the LIF tends to become equal to the quantization step size. In our experiments, we compared the LIF to the

Uniform Quantizer without a deadzone, varying the stepsize similarly with the variation of θ for the LIF. For the comparison we plot the curve of the MSE in function of the Entropy shown in figure 5. We observe that for lower observation times t_{obs} the LIF quantizer exhibits a better performance than the Uniform, while for higher observation times the results for the LIF quantizer are slightly inferior but still comparable to the ones of the Uniform.

3.2 LIF with a constant refractory period

In this section we introduce a constant refractory period in our LIF quantizer and we compare the results to the Lloyd Quantizer, which is the most widely used non-uniform quantizer. As clearly observed in section 2.3, in the presence of a refractory period, the quantizer exhibits overload noise. Figure 4, reveals that there is an optimal theta which minimizes the MSE for each value of t_{obs}. Since the Lloyd quantizer uses an optimization algorithm to find the best partition which minimises the MSE, in order to be fair to the comparison we will also chose the optimal theta value which minimizes the MSE for our LIF quantizer. In the experiments, we quantize the input signal with the LIF, for different observation times. Then, for each observation time we chose the threshold value θ which minimizes the MSE, and for this value, we compute the entropy and the rate. Similarly, for the Lloyd quantizer, we first use a Gaussian training set, different from the signal that will be quantized, for the learning procedure of the Lloyd algorithm, in order to obtain the optimal partition into levels and the codebook containing the quantization values. Afterwards, we use the same input signal used for the LIF quantization, to be quantized by the Lloyd quantizer and compute the MSE and the entropy. The MSE-Entropy curve is presented in figure 6.

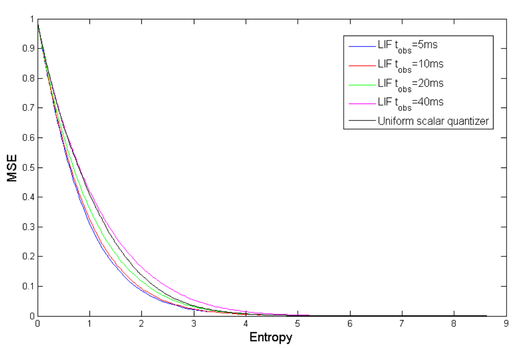

FIGURE 5: Comparison of the LIF and the Uniform scalar quantizer in the absence of a refractory period

It is very important to mention, that apart from the advantage of using dynamic quantization, the LIF only needs to minimize the value of θ, which is computationally cheaper compared to the Lloyd's clustering procedure in the learning algorithm. Finally, for the quantization using the LIF, there is no need of a codebook transmission to the decoder which reduces the size of the bandwidth needed for the transmission of the data.

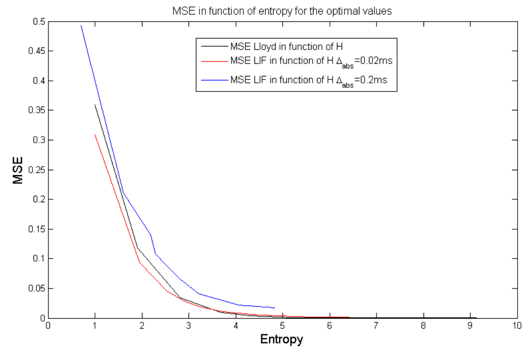

FIGURE 6: Comparison of the LIF and the Lloyd quantizer - The red curve describes the LIF quantizer for a low refractory period of 0.02 ms, the blue one describes the LIF for a higher refractory period of 0.2 ms and the black curve denotes the Lloyd quantizer

3.3 Conclusions and future work

In this paper, we presented a dynamical way of quantization using the LIF neural model. After an extended theoretical and experimental study of the model, we observed that according to the absence or not of a refractory period, the model switches from a uniform to a non-uniform quantization procedure. Comparing our model to existing quantizers, we observed that when there is no refractory period, the LIF is comparable to the uniform scalar quantizer with a dead-zone equal to two times the threshold value of the LIF. For lower values of observation time though, the LIF can even outperform the uniform scalar quantizer. In the presence of a refractory period, the LIF behaves similarly to the Lloyd quantizer, with the Lloyd outperforming the LIF for higher values of refractory period. However, for lower values of refractory and higher values of observation time the LIF quantizer outperforms the Lloyd algorithm, Furthermore, some of the LIF characteristics, such as the dynamical quantization and the fact that there is no need for a learning procedure or a codebook transmission, make the LIF quantizer a very promising quantization procedure to be further studied and analysed.

Motivated by [2], we aim to study the application of this quantization process to image and video coding taking a step further in the 2D and 3D quantization process.

Références

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