A Two-step Compressed Sensing Based Channel Estimation Solution for Millimeter Wave MIMO Systems

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Résumé – Les communications à ondes millimétriques (mmWave) s'appuient sur de grands réseaux d'antennes pour compenser l'affaiblissement de propagation élevé dans les liaisons d'accès cellulaire. L'estimation du canal est difficile pour les systèmes mmWave en raison du grand nombre d'antennes. La technique d'acquisition comprimée (CS) a été envisagée pour réduire le nombre requis de symboles pilotes. La complexité des algorithmes CS existants augmente linéairement avec le produit du nombre d'antennes à l'émetteur et au récepteur. Avec plusieurs dizaines à plusieurs centaines d'antennes de chaque côté, la complexité devient très élevée. Dans cet article, nous proposons un algrithme basé sur la technique CS qui estime les angles de départ pour des trajets constituant le canal dans une première étape, et ensuite estime les angles d'arrivée et les gains de ces trajets dans une deuxième étape. L'avantage principal de cette solution est une réduction de la complexité, qui devient une fonction affine du nombre d'antennes à l'émetteur et au récepteur. Les simulations montrent que les précisions de l'estimation du canal pour les solutions classiques et la solution proposée sont proches, tandis que la solution proposée a une complexité beaucoup plus faible.

Abstract – Millimeter wave (mmWave) wireless communications rely on large antenna arrays for compensating high path loss in cellular access links. Channel estimation is challenging for mmWave systems because of the large number of antennas. Compressed sensing (CS) has been considered for reducing the needed number of training pilots. The computational complexity of existing CS-based solutions is linearly increasing with the product of the number of antennas at transmitter and receiver. With several tens to several hundred antennas at both communication sides, the complexity becomes very high. In this paper, we propose a CS-based solution that estimates the angles-of-departure (AODs) for paths constituting the channel in a first step, and the angles of arrival (AOAs) and path gains for these paths in a second step. The main advantage of this solution is a reduction of complexity which becomes an affine function of the number of antennas at transmitter and receiver. Simulations show that channel estimation accuracy for the classical and proposed solutions are close, while the latter has a much lower complexity.

1 Introduction

Millimeter wave (mmWave) wireless communication is expected to be a key feature of future 5G cellular systems and complement the sub-6 GHz bands [1, 2]. The major benefit is the availability of much greater spectrum for higher data rates. However, due to the increased path loss compared to sub-6 GHz frequencies, highly directional beamforming achieved by means of large antenna arrays at both transmitter and receiver is inevitable. The small antenna size at mmWave makes it possible to construct large arrays with compact form factors [2]. In addition to the directional gain, the use of large arrays, a.k.a. massive MIMO, brings the benefit of increasing data rate by spatial multiplexing. These benefits could be leveraged by a careful design of precoders and combiners which relies on channel state information (CSI) obtained by channel estimation.

The common approach for channel estimation is through the use of training pilots. With a very large number of transmit antennas, training overhead could become prohibitive as it would require a large percentage of available resources. In mmWave propagation, the scattering is expected to be limited [3]. Therefore, the number of paths with significant gain is expected to be small compared to the large size of the channel matrix which, in turn, is expected to be sparse when expressed in the angular domain. Compressed sensing (CS) techniques allow sparse signal recovery from far fewer measurements than signal length [4]. By exploiting sparsity, channel estimation can be formulated as a CS problem and solved using CS tools with less pilots.

Several works have proposed CS-based algorithms for mmWave channel estimation [5, 6]. However, the computational complexity of these algorithms is proportional to the product of the number of antennas at transmitter and receiver, i.e., the size of the channel matrix. With several tens to several hundred antennas at both sides, the complexity becomes very high for practical implementation. In [7], a model for channel temporal correlation was assumed and exploited for reducing this complexity. In this work, we propose a two-step algorithm for CS-based channel estimation, with the objective of reducing the computational complexity. In the first step, the algorithm recovers angles-of-departure (AODs) of propagation paths, and in the second step it recovers the angles-of-arrival (AOAs) and path gains. This separation into two steps makes the complexity

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an affine function of the number of antennas at transmitter and receiver, but not a function of their product.

We use the following notation : $\mathbf{A}$ is a matrix, $\mathbf{a}$ is a vector, a is a scalar. $\mathbf{A}^T$,$\mathbf{A}^*$,$\overline{\mathbf{A}}$ are transpose, Hermitian and conjugate of $\mathbf{A}$, respectively. $\otimes$ denotes Kronecker product. $\|\mathbf{a}\|$ is the Euclidean l_2 norm, $\|\mathbf{a}\|_0$ is the l_0 norm, $|a|$ is the absolute value, and $\|\mathbf{A}\|_F$ is the Frobenius norm. $E\{\}$ denotes the expectation.

2 Channel model

We adopt the double-directional impulse response channel model [3]. Consider a MIMO system where transmitter and receiver are equipped with N_t and N_r antennas, respectively. We restrict ourselves to 2D uniform linear arrays (ULA). The extension to other array geometries is straightforward. Let $\mathbf{a}_t(\phi) \in \mathbb{C}^{N_t \times 1}$ and $\mathbf{a}_r(\phi) \in \mathbb{C}^{N_r \times 1}$ denote the steering vectors associated to transmitter and receiver arrays, respectively. For a ULA of size N antennas, the steering vector can be written as

$$\mathbf{a}(\phi) = \frac{1}{\sqrt{N}}[1, e^{-j2\pi\nu \sin(\phi)}, \cdots, e^{-j2\pi\nu(N-1)\sin(\phi)}]^T, \quad (1)$$

where $\nu = d/\lambda$, λ is the wavelength at operating frequency, d is the antenna separation, and ϕ is the azimuth angle.

The narrowband channel matrix can be expressed as

$$\mathbf{H} = \sum_{p=1}^{P} \sqrt{N_t N_r} a_p \mathbf{a}_r(\phi_{r,p}) \mathbf{a}_t^*(\phi_{t,p}) \quad (2)$$

where P denotes the number of paths, a_p is a complex amplitude, and $\phi_{r,p}$ and $\phi_{t,p}$ are azimuth angles at receiver and transmitter, respectively.

2.1 Angular domain representation and channel sparsity

Due to high directivity of large arrays, a natural choice is to represent the channel in the angular domain [8] with overcomplete dictionaries of steering directions

$$\mathbf{H} = \mathbf{D}_r \mathbf{H}_a \mathbf{D}_t^*, \quad (3)$$

where $\mathbf{D}_r \in \mathbb{C}^{N_r \times G_r}$ and $\mathbf{D}_t \in \mathbb{C}^{N_t \times G_t}$ are two dictionaries of size $G_r \geq N_r$ and $G_t \geq N_t$, respectively. It is an approximation of the channel matrix by replacing the AOD and AOA of a path with their nearest dictionary columns. The larger the dictionary size, the more accurate is this approximation. An example of a dictionary $\mathbf{D} \in \mathbb{C}^{N \times 2N}$ of size $G = 2N$ for a ULA of N antennas and $d = \lambda/2$ is given by $\mathbf{D}(k,l) = 1/\sqrt{N} e^{-j2\pi kl/2N}$, $l = 0, \ldots, 2N-1$. This dictionary corresponds to an overlap of two unitary basis, and can be generated by taking the top N rows of a DFT matrix of size $2N$.

As scattering is limited at mmWave frequencies [3], it can be expected that the number of paths significantly contributing to the channel in (2) is relatively small. Furthermore, one can discard paths having small amplitudes without much accuracy loss. Thus, matrix $\mathbf{H}_a$ is expected to be sparse. Let $\mathbf{h} = \text{vec}(\mathbf{H}) \in \mathbb{C}^{N_t N_r \times 1}$ be the vectorization of the channel matrix. Equation (3) can be recast as

$$\begin{aligned} \mathbf{h} &= (\overline{\mathbf{D}}_t \otimes \mathbf{D}_r)\text{vec}(\mathbf{H}_a) \\ &= \mathbf{\Psi}\mathbf{h}_a, \end{aligned} \quad (4)$$

where $\mathbf{h}_a = \text{vec}(\mathbf{H}_a) \in \mathbb{C}^{G_t G_r \times 1}$. Matrix $\mathbf{\Psi} \in \mathbb{C}^{N_t N_r \times G_t G_r}$ is a dictionary for the channel expressed in vector form. $\mathbf{h}_a$ is said to be K-sparse if it has at most K non-zero entries.

3 Channel estimation

Measurements for channel estimation are obtained by transmitting pilot signals over time-frequency resources, and processing the received signals with combiners. For one pilot $\mathbf{p}_j \in \mathbb{C}^{N_t \times 1}$ and one combiner $\mathbf{q}_i \in \mathbb{C}^{N_r \times 1}$, the corresponding scalar measurement is given by

$$y_{i,j} = \mathbf{q}_i^* \mathbf{H} \mathbf{p}_j + \mathbf{q}_i^* \mathbf{n}_j, \quad (5)$$

where $\mathbf{n}_j \in \mathbb{C}^{N_r \times 1}$ is the noise vector over time-frequency resource of $\mathbf{p}_j$. In the absence of prior information on channel statistics, least-squares (LS) is the classical solution for linear estimation. The number of measurements needed for applying LS is $N_t N_r$. Considering the sparse nature of mmWave channels, CS techniques can be used for drastically reducing the needed number of measurements.

3.1 CS-based channel estimation

Let $\mathbf{P} = [\mathbf{p}_1, \cdots, \mathbf{p}_{N_p}]$ be a matrix of N_p pilots and $\mathbf{Q} = [\mathbf{q}_1, \cdots, \mathbf{q}_{N_c}]$ be a matrix of N_c combiners. By applying the N_c combiners of $\mathbf{Q}$ to every pilot, it is possible to write the measurements in matrix form as follows

$$\begin{aligned} \mathbf{Y} &= \mathbf{Q}^* \mathbf{H} \mathbf{P} + \mathbf{Q}^* \mathbf{N} \\ &= \mathbf{Q}^* \mathbf{D}_r \mathbf{H}_a \mathbf{D}_t^* \mathbf{P} + \mathbf{Q}^* \mathbf{N} \end{aligned} \quad (6)$$

where $\mathbf{N} = [\mathbf{n}_1, \cdots, \mathbf{n}_{N_p}]$. Vectorizing this equation gives

$$\begin{aligned} \mathbf{y} &= (\mathbf{P}^T \bar{\mathbf{D}}_t \otimes \mathbf{Q}^* \mathbf{D}_r)\mathbf{h}_a + \mathbf{n} \\ &= \tilde{\mathbf{A}} \mathbf{h}_a + \mathbf{n}, \end{aligned} \quad (7)$$

where size of $\tilde{\mathbf{A}}$ is $m \times G_t G_r$, and $m = N_p N_c$ being the number of measurements.

Assuming $\mathbf{h}_a$ is K-sparse, the channel estimation can be formulated as a CS optimization problem as follows [4]

$$\hat{\mathbf{h}}_a^{CS} = \arg\min_{\mathbf{h}_a} \|\mathbf{y} - \tilde{\mathbf{A}}\mathbf{h}_a\|, \text{ s.t. } \|\mathbf{h}_a\|_0 \leq K. \quad (8)$$

When sparsity level K is small, only a few measurements ($m \ll N_t N_r$) are needed for channel estimation. A simple and fast algorithm for sparse signal recovery is orthogonal matching pursuit (OMP) [4]. OMP is an iterative algorithm where a new non-zero entry of $\mathbf{h}_a$ is identified at each iteration. The complexity order of one OMP iteration is upper bounded by $\mathcal{O}((N_p + N_c)G_t G_r)$. For K iterations, the complexity order is $\mathcal{O}(K(N_p + N_c)G_t G_r)$. This complexity is very high for large antenna arrays. For example, taking $N_p = N_c = 32$, $G_t = 256$ and $G_r = 128$, the application of OMP requires about $2^{20} \approx 10^6$ operations per iteration.

3.2 Two-step CS-based channel estimation

The transmitter and one receiver combiner can be seen as a multiple-input-single-output (MISO) system. The estimation of the MISO equivalent channel vector has lower computational complexity than the estimation of the MIMO channel matrix. The MISO channel is expected to be sparse in the angular domain, where non-zero entries correspond to AODs. Since MIMO and MISO channels share the same AODs, the recovery of a MISO channel could be exploited for recovering the MIMO channel. The two-step CS based channel estimation solution relies on this issue and starts by recovering AODs at the first step. We now describe how this solution is processed.

3.2.1 The first step estimation

The Hermitian of $\mathbf{Y}$ in (6) can be written as

$$\mathbf{Y}^* = \mathbf{P}^*\mathbf{D}_t\mathbf{H}_a^*\mathbf{D}_r^*\mathbf{Q} + \mathbf{N}^*\mathbf{Q} \tag{9}$$

The rows of $\mathbf{H}_a^*$ correspond to AOD directions. By assuming sparsity, most of these rows are equal to zero.

For a combiner vector $\mathbf{q}_i$, let

$$\mathbf{w}_i = \mathbf{H}_a^*\mathbf{D}_r^*\mathbf{q}_i. \tag{10}$$

The vector $\mathbf{w}_i \in \mathbb{C}^{G_t\times 1}$ corresponds to the MISO channel vector expressed in angular domain. The non-zero entries of $\mathbf{w}_i$ correspond to non-zero rows of $\mathbf{H}_a^*$. Thus, $\mathbf{w}_i$ is sparse.

The i^{th} column of $\mathbf{Y}^*$, denoted by $\tilde{\mathbf{y}}_i$, can be written as

$$\tilde{\mathbf{y}}_i = \mathbf{P}^*\mathbf{D}_t\mathbf{w}_i + \mathbf{N}^*\mathbf{q}_i. \tag{11}$$

It is possible to apply OMP on $\tilde{\mathbf{y}}_i$ to estimate $\mathbf{w}_i$ and recover the AODs associated to channel paths. Due to high path loss and since $\mathbf{q}_i$ is randomly selected and not aligned with AOAs, the signal to noise ratio (SNR) is expected to be low, resulting in a poor recovery performance. However, notice that we have N_c vectors $\mathbf{w}_i$, $i = 1, \cdots, N_c$, and all these vectors share the same support of non-zero entries. By jointly recovering the support for all these vectors, one expects to improve the recovery accuracy. An algorithm for joint recovery that is suitable for our problem is simultaneous OMP (SOMP) [9]. SOMP executes the same operations of OMP with only difference in the identification of a new entry at each iteration. Let $\mathbf{r}_i^{k-1}$ be the residual vector remaining from $\tilde{\mathbf{y}}_i$ at iteration $k-1$. Also let $\mathbf{R}^{k-1} = [\mathbf{r}_1^{k-1}, \cdots, \mathbf{r}_{N_c}^{k-1}]$, and $\tilde{\mathbf{p}}_j$ denote the j^{th} column of $\mathbf{P}^*\mathbf{D}_t$. At iteration k, SOMP selects a new entry according to

$$t_k = \underset{j\in\{1,\cdots,G_t\}}{\arg\max} \|\tilde{\mathbf{p}}_j^*\mathbf{R}^{k-1}\|. \tag{12}$$

The complexity of this operation is $\mathcal{O}(N_pN_cG_t)$. For K iterations, the complexity of step 1 is $\mathcal{O}(KN_pN_cG_t)$.

The output of step 1 is K-sparse estimate of $\mathbf{w}_i$, denoted as $\hat{\mathbf{w}}_i$, $i = 1, \cdots, N_c$.

3.2.2 The second step estimation

Using (10), the l^{th} non-zero entry of $\hat{\mathbf{w}}_i$, out of K, can be written as

$$\hat{\mathbf{w}}_{i,l}^* = \mathbf{q}_i^*\mathbf{D}_r\mathbf{H}_{a,l} + e_{i,l}, \tag{13}$$

where $\mathbf{H}_{a,l}$ is the l^{th} column of $\mathbf{H}_a$ identified as not equal to zero, and $e_{i,l}$ is the estimation noise. Define the column vector $\mathbf{v}_l$

$$\mathbf{v}_l = [\hat{\mathbf{w}}_{1,l}, \cdots, \hat{\mathbf{w}}_{N_c,l}]^*.$$

It can be expressed as

$$\mathbf{v}_l = \mathbf{Q}^*\mathbf{D}_r\mathbf{H}_{a,l} + \mathbf{e}_l, \tag{14}$$

where $\mathbf{e}_l = [e_{1,l}, \cdots, e_{N_c,l}]$. Vector $\mathbf{v}_l$ can be seen as a measurement vector, and OMP can be applied to recover the sparse vector $\mathbf{H}_{a,l}$. Repeating this operation for $l = 1, \cdots, K$, we end up with an estimation of $\mathbf{H}_a$. As a remark, the K OMP runs are parallelizable. The complexity of one OMP run for K' iterations at this step is $\mathcal{O}(K'N_cG_r)$. Since $\mathbf{H}_{a,l}$ corresponds to one AOD, it is expected to be sparser than $\mathbf{w}_i$.

The overall complexity of the two-step solution is equal to the summation of complexities in each step. Assuming the number of OMP iterations at the second step is fixed to K', this complexity is given by

$$\mathcal{O}(KN_pN_cG_t) + \mathcal{O}(KK'N_cG_r). \tag{15}$$

When $G_t > G_r$ the complexity of the first step is dominant. Compared to the classical solution, the complexity is reduced by a factor of $G_r/\max(N_p, N_c)$. It is possible to reverse the order of recovery and start with AOAs at the first step. In this case, the complexity becomes $\mathcal{O}(KN_pN_cG_r)+\mathcal{O}(KK'N_pG_t)$. Reversing the order could improve or deteriorate the recovery accuracy, depending on the values of N_p, N_c, N_t and N_r. In fact, it might be better to start with AODs when the number of pilots is sufficiently large, and do the reverse when the number of pilots is small and the number of combiners is large.

4 Simulation results

We use the channel model proposed in [3]. The operating frequency is 28 GHz and the environment is NLOS outdoor. Hybrid analog/digital architecture based on phase shifters [1] is considered at transmitter and receiver with $N_t = 128$ and $N_r = 64$ antennas, and $L_t = 16$ and $L_r = 8$ RF chains. The antenna separation in ULA is $\lambda/2$. Pilots and combiners phases are randomly selected from the set $\{1, -1, j, -j\}$. The power of a transmitted pilot $\mathbf{p}_i$ is given by $\|\mathbf{p}_i\|^2 = \rho$, and the noise variance is equal to one. After a channel is generated, path coefficients are normalized to make the omni-directional power equal to one, i.e., $\sum_{p=1}^{P} |a_p|^2 = 1$, resulting in an SNR equal to ρ. The sizes of dictionaries used for angular domain representation are $G_t = 2N_t$ and $G_r = 2N_r$.

A performance metric for channel estimation is the normalized mean squared error (NMSE) given by

$$\text{NMSE} = E\{\|\mathbf{H} - \hat{\mathbf{H}}\|_F/\|\mathbf{H}\|_F\}. \tag{16}$$

Figure 1 shows the variation of the NMSE with the number of measurements where SNR is 0 dB. The total number of measurements that can be obtained during one symbol transmission period is $L_t L_r = 128$. More measurements can be obtained by sending different pilots over multiple symbol periods. We only recover four paths. It reveals that classical CS outperforms the two-step solution. One reason of this performance difference is the propagation of recovery error from the first to the second step in two-step CS. NMSE is decreasing with the number of measurements. A NMSE saturation can be observed.This is essentially due to the limited number of recovered paths.

Figure 2 shows the variation of the complexity order with the number of measurements for the two solutions.The two-step solution has a clear complexity improvement over classical CS. The ratio of these complexities is approximately 10.

Figure 3 shows the variation of spectral efficiency with SNR for the transmission of one beamformed data stream. The number of measurements is $2L_t L_r = 256$. The precoder and combiner are computed based on singular vectors of the estimated channel. The gap between the two solutions is insignificant. The reason is that the strongest channel path contributing to mode 1 is well recovered by both solutions.

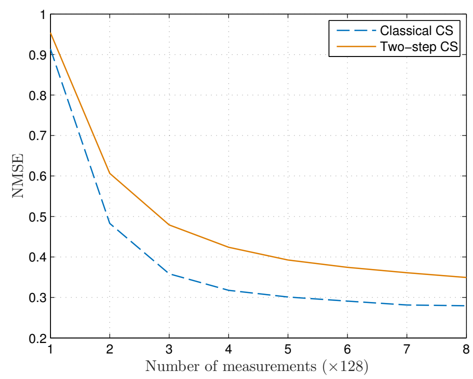

FIGURE 1 – NMSE vs. number of measurements. SNR=$0\ dB$. Recovery 4 paths.

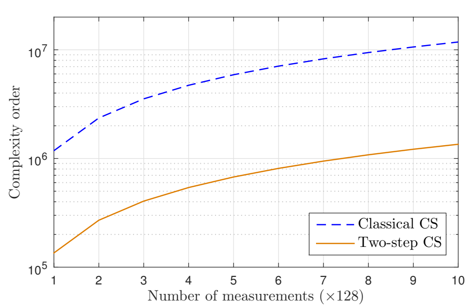

FIGURE 2 – Complexity order vs. number of measurements.

5 Conclusion

In this paper we proposed a new solution for mmWave massive MIMO channel estimation based on CS. Compared to classical CS solutions, the proposed solution significantly reduces the computational complexity and has a small degradation in

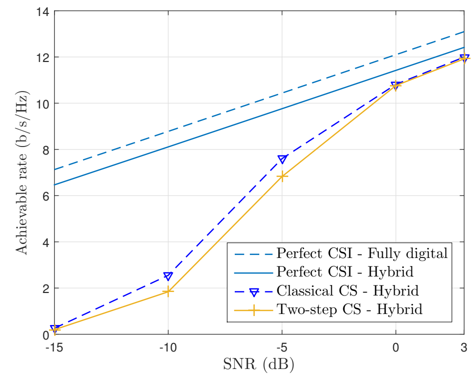

FIGURE 3 – Achievable rate vs. SNR. Transmission of one beamformed data stream.

estimation accuracy and achievable data rate.

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