Reflection and refraction of an acoustic beam from a water-sediment interface

Réflexion et réfraction d'un faisceau acoustique sur une interface eau-sédiment

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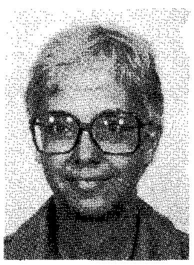
Mathématiques appliquées.
Acoustique linéaire.
Analyse numérique.



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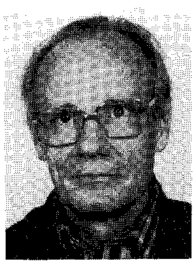

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SUMMARY

We study the reflection and the refraction of an acoustic beam at a water-sediment interface. Earlier works are extended to cover the case of a real (directional) source located in one of the two media. Both linear and parametric sources are considered. New results on the beam displacement are presented.

KEY WORDS

Bottom-interaction, parametric array, underwater acoustic.

RÉSUMÉ

On étudie la réflexion et la réfraction d'un faisceau acoustique sur une interface eau-sédiment. Les travaux antérieurs sont étendus de façon à couvrir le cas d'une source réelle (directive) placée dans l'un des milieux. Des sources linéaires et paramétriques sont considérées. Des résultats nouveaux sur le déplacement sont présentés.

MOTS CLÉS

Interaction avec le fond, antenne paramétrique, acoustique sous-marine.

CONTENTS

1. Previous work

Assuming that the water-sediment interface can be treated as a two-fluid interface, the starting equations are:

$$(1) \qquad (\nabla^2+\chi'^2)\,p_r=0, \qquad (\nabla^2+\chi^2)\,p_t=0.$$

$$(2) \qquad p_r+p_i=p_t \qquad \text{and} \qquad \frac{\partial p_r}{\partial z}+\frac{\partial p_i}{\partial z}=\frac{\rho'}{\rho}\frac{\partial p_t}{\partial z},$$

when $z=0$.

The notation is the same as in Reference [1, 2]: p_i, p_r, p_t pressure of the incident, reflected and transmitted sound field, respectively, $\chi'=k'+i\alpha'$, ρ', c' and $\chi=k+i\alpha$, ρ, c complex wave number at a given frequency, density and sound speed in water and sediment, respectively. The interface is located in the plane $z=0$, $z>0$ in the sediment. The horizontal distance in the plane of incidence is x, the point $x=z=0$ being on the acoustic axis of the incident beam. The incident angle is θ_i and $\theta_i^*=\sin^{-1}(k/k')$ is the critical angle. For a given source in water, p_i is known and $p_i(x, 0)=\exp(ik'\sin\theta_i x)\,f(x)$ can be considered as given, $f(x)$ being the profile of the incident beam at the interface. Due to diffraction effects, the phase fronts may not be plane and $f(x)$ is generally a complex function. Variations of p_i at the interface as a function of the variable y, i. e., transverse to the plane of incidence, are neglected, as they can be shown to play only little role in the vicinity of the interface when the incident angle is not too small [1].

Under the above assumptions, the boundary condition Equation (2) can be written:

$$(3) \quad \hat{p}_r(l, 0)=\mathrm{R}(l)\,\hat{p}_i(l, 0), \qquad \hat{p}_t(l, 0)=\mathrm{T}(l)\,\hat{p}_i(l, 0),$$

where $\mathrm{R}(l)$ and $\mathrm{T}(l)=1+\mathrm{R}(l)$ are the complex reflection and transmission coefficients, respectively, with:

$$(4) \qquad \mathrm{R}(l)=\frac{\sqrt{\chi'^2-l^2}-\rho'/\rho\sqrt{\chi^2-l^2}}{\sqrt{\chi'^2-l^2}+\rho'/\rho\sqrt{\chi^2-l^2}}.$$

The reflection coefficient of a plane wave with incident angle θ_i is $\mathrm{R}_0=\mathrm{R}(k'\sin\theta_i)=|\mathrm{R}_0|\exp(i\,\varphi_{\mathrm{R}_0})$, $\varphi(l)$ being the phase of $\mathrm{R}(l)$. Here $\hat{p}(l)$ is the Fourier transform of $p(x)$, l being the Fourier variable. When a is the 3dB-width of the incident beam in the region of impact, $a_s=a/\cos\theta_i$ is a characteristic length for the spot insonified on the interface. Assuming ka_s large, we have among others derived analytical solutions which are valid at various ranges from the spot, including the nearfield and the very (extreme) farfield. (A good knowledge of the transmitted beam at depths ranging from zero to a few wavelengths is particularly important for experimental and practical purposes.) The main conclusion in Reference [1] was that transmission at incidence around or above critical is possible provided the incident beam is highly directional and that its ka_s is not too high, i. e., not higher than about 20. (These requirements are contradictory

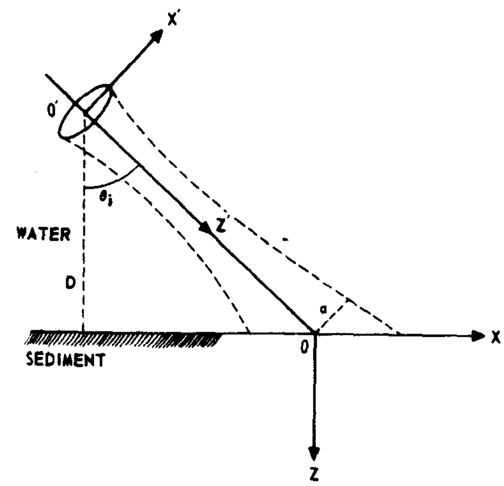

Fig. 1

for a linear source. It is well known, however, that they can be satisfied by using a parametric acoustic source.) This result has recently been theoretically confirmed by other authors using similar approaches to obtain very farfield asymptotic solutions [3, 4], or using fast Fourier transforms in order to compute the nearfield [4]. In the experiment of Reference [5], no transmission is observed at or above critical incidence. The reason for this is probably a too high value of ka_s. The work by Muir *et al.* [6] seems to give the only reported experimental evidence of transmission above critical incidence. They observed penetration of a parametrically generated beam, ka being about 9.

2. Beam displacement

The results obtained in our previous work [1, 2] show that there is a displacement of the acoustic axis, *see*, for example, Figure 2 which corresponds to a gaussian profile for $f(x)$ with $ka_s \simeq 16$ and $\theta_i=50°$ or $60.7°$ $(=\theta_i^*)$ (The acoustical axis is indicated by the double arrow. On this figure, $\xi=xk'\sin\theta_i/2\pi$, $\zeta=zk'\sin\theta_i/2\pi$). Numerical results using various parameters seem to indicate that this displacement increases when θ_i increases and when ka decreases. Another way to define a displacement is to measure the distance between the $x=0$ axis and the point of maximum sound level at a given depth z. With this definition however, there is little or no displacement in Figure 2.

The results in Reference 1 and 2 also show that the reflected beam closely follows Snell's law, and that there is no displacement of its acoustic axis. This result may seem to contradict the result obtained by

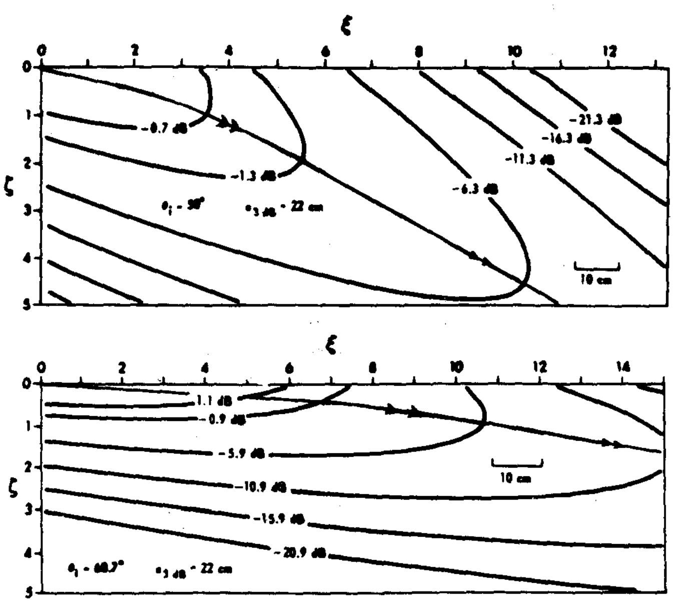

Fig. 2

Schoch [7] and Brekhovskikh [8]. They found that the reflected beam is displaced (translated) by:

(5) $$\Delta_R \simeq -\partial\varphi_R/\partial l \ (l=k' \sin\theta_i).$$

This result is obtained when $ka_s \gg 1$ so that $|R(l)| \simeq |R_0| \simeq 1$ when $l \lesssim 0(a_s^{-1})$. Further, it presupposes that θ_i is not close to θ_i^* or $\pi/2$, as Δ_R diverges at these angles. Following this theory the transmitted beam should be displaced by $\Delta_T \simeq \Delta_R/2$, *see* the ------ curves on Figure 3.

In our previous theory, Δ_R given by Equation (5) is zero, because $R(l)$ and $T(l)$ are approximated by R_0 and T_0, respectively, as a consequence of the $ka_s \gg 1$ assumption. This approximation, on the other hand, although consistent, may be less good numerically at moderate values of ka_s (say 10-20), especially in the phase of $R(l)$ and $T(l)$. Figure 4 shows results obtained on the interface for the reflected and transmitted sound field, either using the exact Equation (3) (curves ——) or using Equation (3) with $\varphi_R(l)$, $\varphi_T(l)$ replaced by φ_{R_0}, φ_{T_0} (curves ------). Here $\theta_i = 76°$, $a_s = 22$ cm, $k = 70$ m^{-1}, $\theta_i^* = 60.7°$, and the incident profile is gaussian. The reflected beam is displaced by about 25 cm, which is also the value predicted by Equation (5). There is, however, practi-

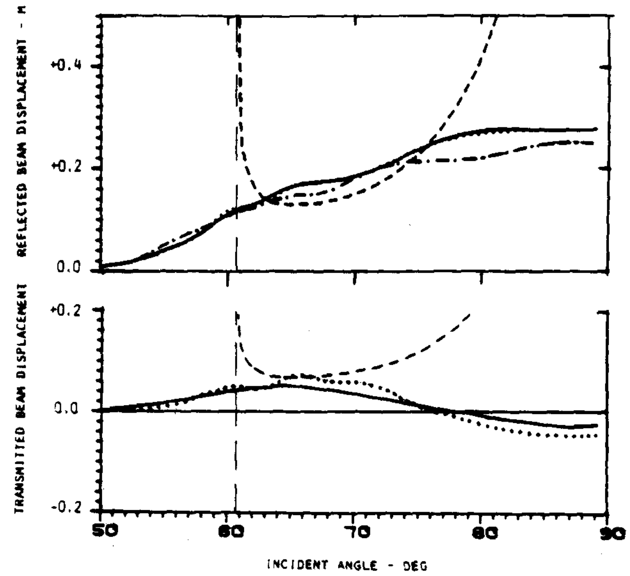

Fig. 3

cally no displacement of the transmitted beam. Figure 3 shows the displacement computed using the exact Equation (3) for a gaussian profile, (——), a profile in $[1+(x/B)^2]^{-1}$, B = constant, (. . . . . .), and a step function (-·-·-·). [There is a numerital incertainty in the result for a step function at high θ_i.] In all cases $a_s \simeq 22$ cm, $k = 70$ m^{-1}, $\theta_i^* = 60.7°$. It is seen that the beam displacement depends little on the shape of the incident beam profile. The numerical results for $\theta_i = \theta_i^*$ and $\theta_i \to \pi/2$ are in good agreement with asymptotic expressions which we have found in these domains.

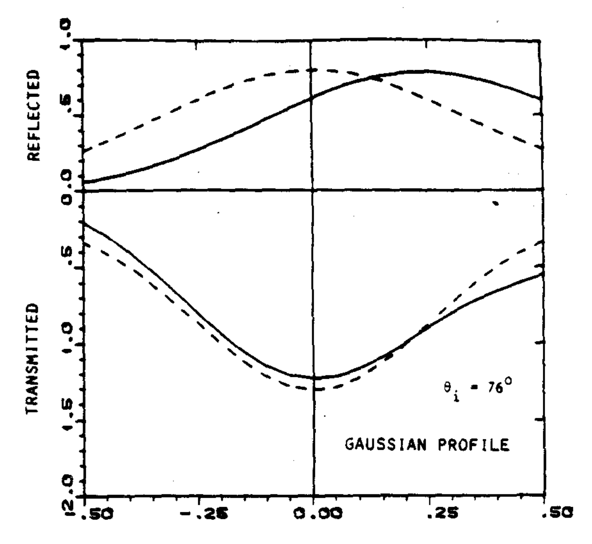

Fig. 4

3. Real sources

In the above computations, the incident beam has a plane wave front, i. e., $f(x)$ is real. A real source, however, generates an incident beam that has curved wave fronts. As an example, Figure 5 shows the amplitude and phase profile of a parametrically generated beam ($f_- = 20$ kHz) computed at 15 m from the transducer, using quasilinear theory [9]. The phase front is nearly spherical, as for a linearly generated beam. At other ranges (nearfield), however, the phase front may have a different shape [9]. The effect of having a real source (parametric or linear) can be studied by replacing $f(x)$ by its computed value, using, for example, the formulas and programs of References [9, 10]. This may affect the displacement and even change its sign, see also Reference [4]. More numerical work is needed here.

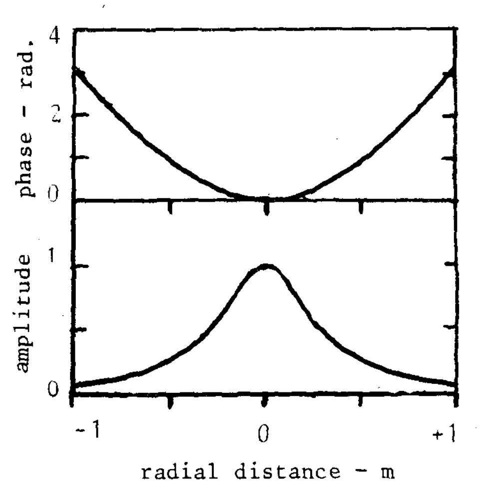

Fig. 5

Similar considerations might help to interprete the reflection and transmission of sound beams at fluid-solid interfaces, *see* for example, References [11 and 12]. For other works treating especially the penetration of a water-sediment interface by a parametric beam, *see* Reference [13].

A proper evaluation of the beam displacement is important in order to improve ray calculation in shallow water [14].

4. Conclusion

The variation in phase of the reflection coefficient R (l) causes a displacement of the reflected beam. The displacement is approximately predicted by Equation (5) only when θ_i differs appreciably from θ_i^* and $\pi/2$. This effect is small for the transmitted beam, and therefore it is a good approximation to replace T (l) by a constant. Highly directional (parametrically generated) beams with a moderate ka are transmitted at incidences around or above critical. The acoustic axis of the transmitted beam (defined from sound level curves) is displaced. More experimental work is needed to verify these results.

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