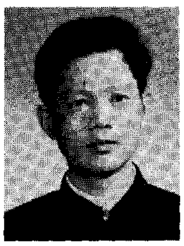
An algorithm for filtering multiplicative noise in wide range

Un algorithme pour le filtrage du bruit multiplicatif



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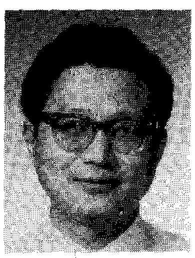

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Chi-Hau Chen received a BSEE degree from the National Taiwan University in 1959, an MSEE degree from the University of Tennessee in 1962 and a Ph. D. degree, also in Electrical Engineering, from Purdue University in 1965. From March 1965 to September 1968 he worked for ADCOM, Inc., Cambridge, MA, and AVCO Systems Division, Wilmington, MA, in the areas of communications sysrems and statistical data processing. Since September 1968 Dr. Chen has been teaching and doing research in pattern recognition, image and signal processing and communication theory at the Southeastern Massachusetts University, North Dartmouth, MA, where he is now Professor of Electrical and Computer Engineering and was also Department Chairman from 1978 to 1981. He is the author of the books, *Statistical Pattern Recognition* (Hayden Book Company, 1973), *Digital Waveform Processing and Recognition* (CRC Press, Inc., 1981) and *Nonlinear Maximum Entropy Spectral Analysis Methods for Signal Recognition* (John Wiley and Sons, 1982) and editor of the books *Pattern Recognition and Artificial Intelligence* (Academic Press, 1976), *Computer-Aided Seismic Analysis and Discrimination* (Elsevier Scientific, 1978), *Pattern Recognition and Signal Processing* (Sijthoff and Noordhoff, 1978), *Seismic Signal Analysis and Discrimination* (Elsevier Science, 1982), *Issues in Acoustic Signal-Image Processing and Recognition* (Springer-Verlag, 1983), and *Seismic Signal Analysis and Discrimination III* (Elsevier Science, 1984). Dr. Chen is a senior member of the Institute of Electrical and Electronic Engineers and a member of Eta Kappa Nu, the American Society for Engineering Education, the American Association for Artificial Intelligence, the Society for Exploration Geophysicists and the Pattern Recognition Society. He is also the founder and president of his own consulting firm, Information Research Laboratory, Inc., dealing with projects in pattern recognition, signal processing, image processing, and artificial intelligence.

SUMMARY

A new algorithm for filtering of multiplicative noise in image processing is considered. A triangular function is used to approximate the local probability density function of the image. Under this assumption the computation of necessary statistics is greatly simplified. An effective filtering algorithm for processing multiplicative noise is then developed. Experimental results show that the algorithm is effective in processing both small and large multiplicative noises. Also the computation and storage requirements are reasonable to implement in a minicomputer.

KEY WORDS

Algorithm, filtering. image processing, multiplicative noise.

RÉSUMÉ

Un nouvel algorithme de filtrage du bruit multiplicatif en traitement d'images est présenté. Une fonction triangulaire est utilisée pour approcher la densité de probabilité locale de l'image. Dans cet article le calcul de la statistique nécessaire est considérablement simplifiée. Un algorithme efficace de filtrage du bruit multiplicatif est alors développé. Les résultats expérimentaux montrent que l'algorithme est efficace aussi bien dans le cas de bruits faibles que dans celui de bruits importants. De plus, le volume des calculs et de stockage sont raisonnables et permettent l'implantation de l'algorithme sur des minicalculateurs.

MOTS CLÉS

Algorithme, filtrage, traitement d'images, bruit multiplicatif.

CONTENTS

1. Introduction

Much work has been done in filtering of additive noise when the noise is assumed to be independent of signal [1]. However many noise processes are inherently signal dependent. For an important class of such noise processes, the data is represented by an independent noise multiplied by a signal. For example, the effect of film-grain noise under the signal dependent assumption is to generate a multiplicative noise [2]. Frost, *et al.* [3] proposed an adaptive filter algorithm for removing multiplicative noise in radar imagery. Their algorithm can also be used to process optical images. Froehlich *et al.* [4] examined a more complicated model including multiplicative noise. Maximum *a posteriori* probability estimation was used to remove the signal dependent noise. The equation they derived was too complex to be solved hence no practical processing algorithm can be given. Lee [5, 6] suggested the method of using local statistics for enhancement and filtering of images with both additive and multiplicative noises. His algorithm for processing multiplicative noise was based on linear approximation of the nonlinear relation between signal and noise. The result was feasible only for small noises. More recently he introduced a computationally more efficient sigma filter [7] for smoothing multiplicative noise in the form of speckles. Again the filter is effective only for small noises.

Another commonly used technique in filtering of multiplicative noise is homomorphic filtering [8]. This method which is usually implemented in frequency domain requires extensive storage and computation in spite of using FFT algorithm. Intuitively, homomorphic filter can also be implemented in the spatial domain by using local statistics. Because of the nonlinear characteristics of the logarithmic operations, the transformed logarithmic image cannot be assumed as Gaussian as is done typically and the computation of statistical parameters can be difficult. Other approaches reported to be effective for multiplicative noise reduction include the edge-preserving nonlinear restoration [9] and the adaptive noise smoothing filter [10] based on a nonstationary image model suitable for various types of signal-dependent noises.

In this paper, the proposed filter is based on a combination of the homomorphic filtering and the use of local statistics as discussed by Lee. The resulting filter is adaptive to the local signal and noise properties. The algorithm allows evaluating the necessary statistics from the original image instead of the logarithmic image. The main advantage of the algorithm is that it is applicable to a wide range of multiplicative noise as the linear approximation is not used.

2. Principle of the algorithm

Let X_{ij} be the original gray level and U_{ij} be an independent noise. The gray level of a degraded pixel can be represented as

(1) $$Z_{ij}=X_{ij}U_{ij}$$

which after taking logarithm becomes

$$\log z_{ij}=\log X_{ij}+\log U_{ij}$$

The linear minimum mean square error (mmse) estimate of $\log X_{ij}$ can be shown as

(2) $$\widehat{\log X_{ij}}=E(\log X)+K(\log Z_{ij}-E(\log Z))$$

$$\widehat{X_{ij}}=\exp(\widehat{\log X_{ij}})$$

where

(3) $$K=(Q_z-Q_u)/Q_z$$

Q_z is the local variance of degraded image after taking logarithm, and Q_u is the variance of the logarithm of the noise.

Also $E(\log Z) = E(\log X) + E(\log U)$

$$Q_z = Q_x + Q_u$$

All parameters are estimated locally from a given window size. In order to evaluate Q_z and $E(\log X)$ the main problem is to determine the probability density function of the original image, $p(x)$. Under the assumption that the distribution of the original image is Gaussian, the first and second moments are enough to determine $p(x)$. It can be easily shown that

$$\bar{X} = \frac{\bar{Z}}{U}$$

$$\bar{X}^2 = \frac{\sigma_z^2 - \sigma_u^2 \bar{X}^2}{\sigma_u^2 + \bar{u}^2}$$

where $\bar{x}$, $\bar{z}$, $\bar{u}$ are the local means of the original image, degraded image and noise respectively, and σ_x^2, σ_z^2, σ_u^2 are corresponding variances.

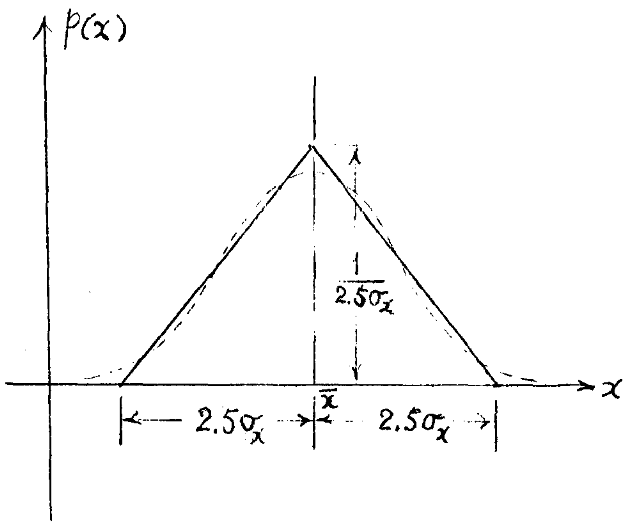


Fig. 1. — Approximation of Gaussian density by a triangular function.

If we assume $p(x)$ is Gaussian it is still difficult to calculate $E(\log X)$ and $E(\log^2 X)$ for the complexity of integration. For simplicity we consider a triangular distribution as shown in Figure 1 as an approximation to the Gaussian distribution. With this assumption, the following integrals can be evaluated [11], and the derivation is provided in Appendix.

$$\int \log x \, dx = x \log x - x$$

$$\int \log^2 x \, dx = x \log^2 x - 2x \log x + 2x,$$

$$\int x \log x \, dx = x^2 (1/2 \log x - 1/4) + c,$$

$$\int x \log^2 x \, dx = \frac{x^2}{2} \log x - x^2 (1/2 \log x - 1/4)$$

where

(4 *a*) $$c = 1 - V/(T - S^2 + V)$$

(4 *b*) $$S = E(\log x) = \frac{F}{2} - H - 1.5$$

(4 *c*) $$T = E(\log^2 x) = \frac{G}{2} - \frac{3F}{2} + H(3 - \log \bar{x}) + 3.5$$

(4 *d*) $$H = \left(\frac{\bar{x}}{2.5\sigma_x}\right)^2 \log \bar{x};$$

where se set $\sqrt{2\pi} = 2.5$

(4 *e*) $$V = Q_u = E(\log^2 u) - (E \log u)^2$$

(4 *f*) $$F = \left(\frac{D}{2.5\sigma_x}\right)^2 \log D + \left(\frac{A}{2.5\sigma_x}\right)^2 \log A$$

(4 *g*) $$G = \left(\frac{D \log D}{2.5\sigma_x}\right)^2 + \left(\frac{A \log A}{2.5\sigma_x}\right)^2$$

with

$D = \bar{x} - 2.5\sigma_x$

$$A = \bar{x} + 2.5\sigma_x$$

The algorithm thus involves the computation of the minimum mean square error estimate of X_{ij} given by

$$\widehat{X_{ij}} = \exp(\widehat{\log X_{ij}})$$

with

$$\widehat{\log X_{ij}} = c \log Z_{ij} + k$$

where C is calculated from equation (4) and

$$k = S - C(S + E(\log u))$$

It is noted that the calculations in equation (4) are made possible with the triangular density assumption. Although equation (4) can be computed numerically with the Gaussian probability density for $p(x)$, the processed images obtained in experiments have shown that the results are much worse than the use of triangular density assumption.

3. Experimental results

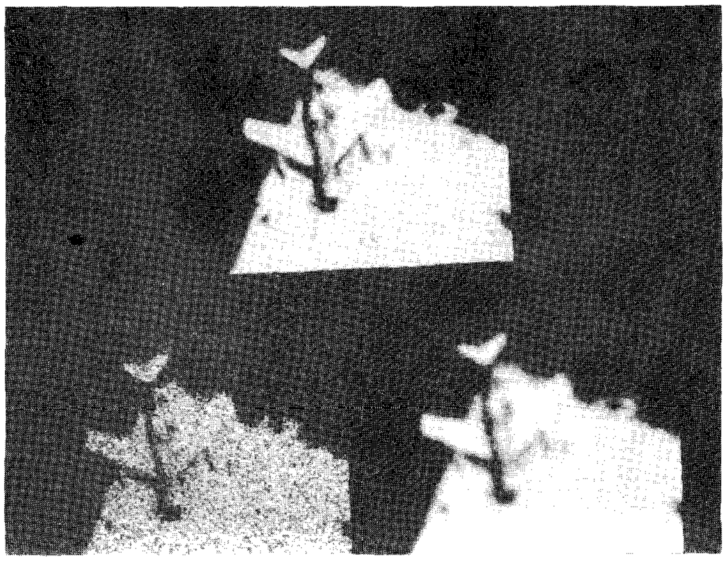
A wide range of multiplicative noise is tested for different images. Also different window sizes (3×3, 5×5, 7×7 and 9×9) are tested and analyzed. In Figures 2, 3, 4 the multiplicative noise added is (0.2-1.0) uniform distribution, with original image in the top, degraded image in lower left and filtered image in lower right, with corresponding histograms also shown in the figures. Pictures for Figures 2 and 4 are from the USC image data base. Figure 3 is from our reconnaissance image data base. Figure 5 is a Seasat SAR image which contains certain amount of multiplicative noise. The histogram of the processed image clearly shows that the noise has been eliminated. The processed image is obviously better than the median filtering result (3×3 applied once) shown in Figure 6 *a* and is comparable to the image processed by using the automatic spatial clustering algorithm [12] as shown in Figure 6 *b*. The edge pre-

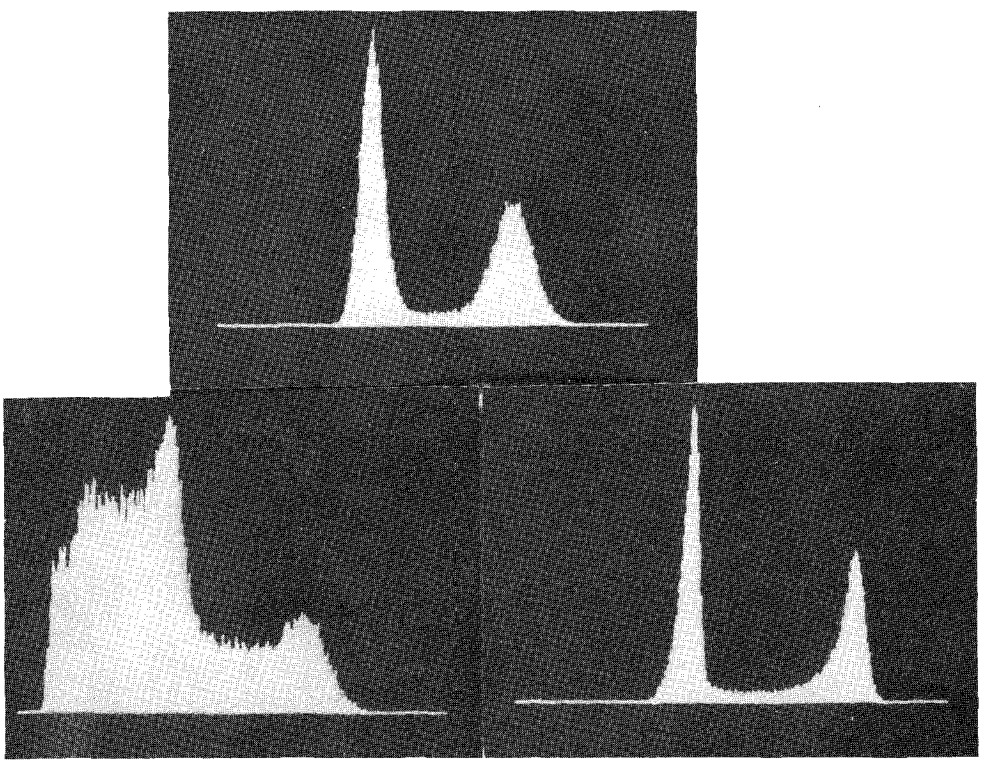
(a)

(b)

Fig. 2. — (*a*) A 256 × 256 picture of a girl's face and its processing results. (*b*) Corresponding histograms.

(a)

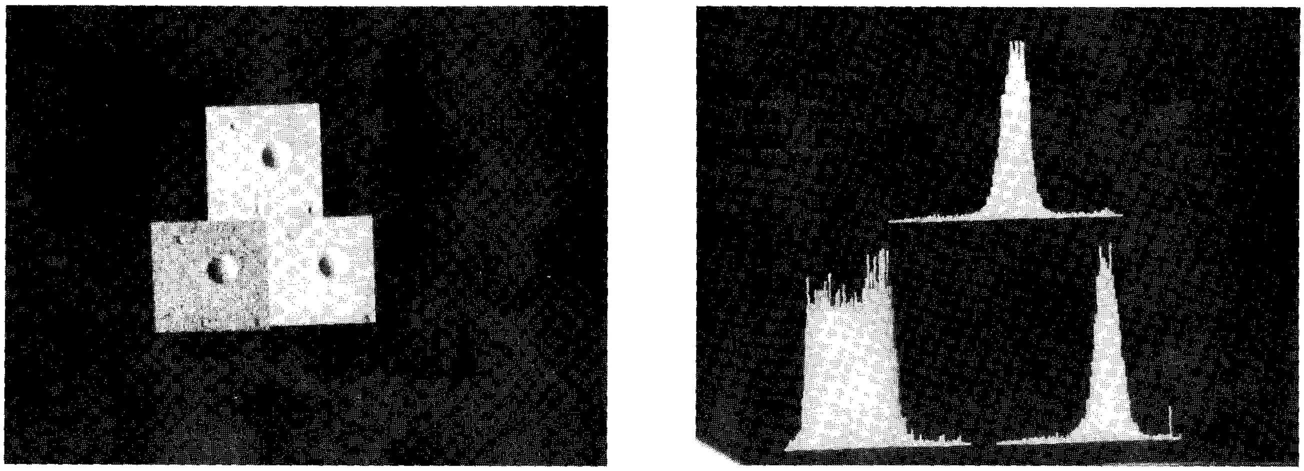




(b)

Fig. 3. — (*a*) A 256 × 256 airfield scene and its processing results.
(*b*) Corresponding histograms.

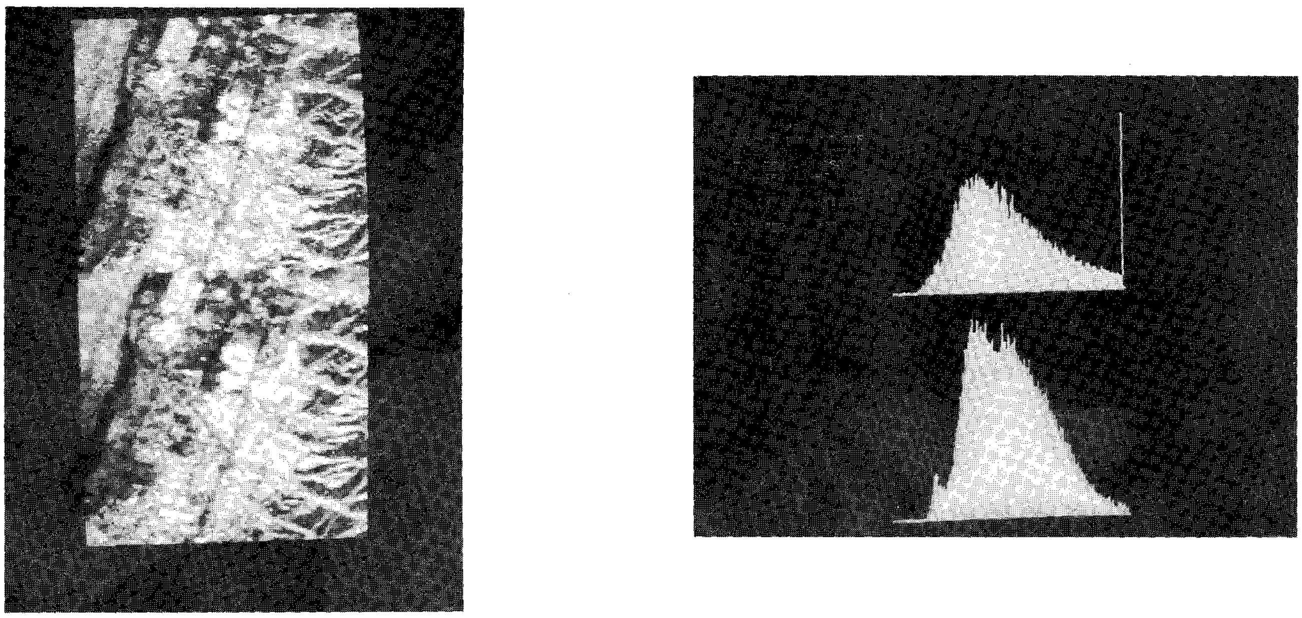
(a)



(b)

Fig. 4. — (*a*) A 100 × 100 scene of moon surface and its processing results. (*b*) Corresponding histograms.

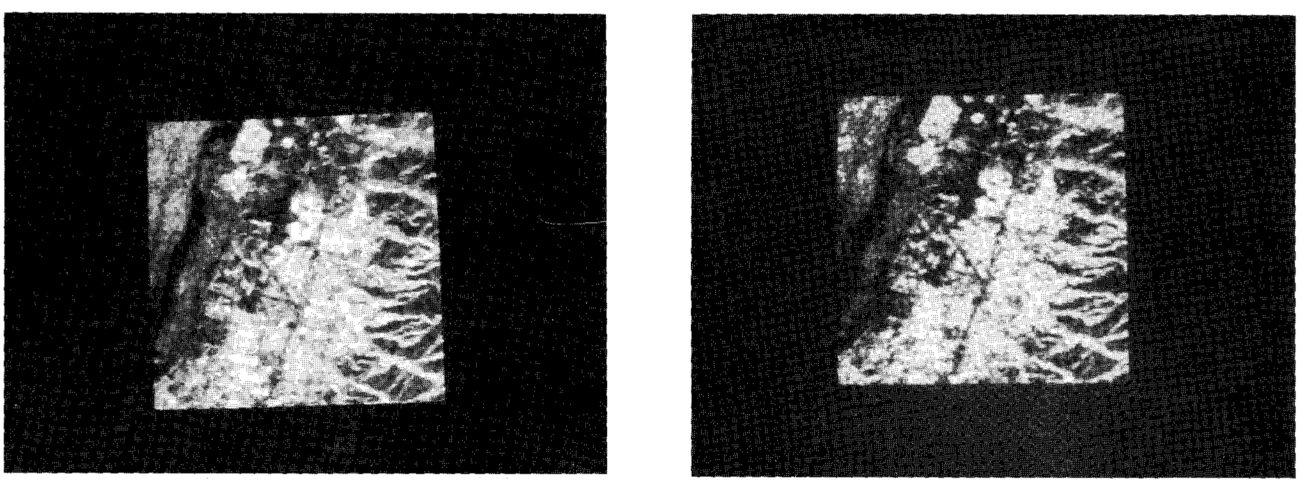
(a)



(b)

Fig. 5. — (*a*) Seasat SAR Image (*upper*) and its processing result (*lower*). (*b*) Corresponding histograms before (*upper*) and after (*lower*) processing.

(a)



(b)

Fig. 6. — (*a*) Median filtering of the Seasat Image. (*b*) Automatic spatial clustering of the image.

serving three sigma filter idea suggested by Lee can easily be incorporated in our proposed algorithm to offer possibly futher improvement.

4. Discussion

The proposed algorithm is effective for filtering of multiplicative noise. In comparison with other method, it can be used in a wide range of multiplicative noises. This advantage can be attributed to the homomorphic filtering. However the algorithm is implemented entirely in the spatial domain, so that computing time and storage are saved in comparison with the conventional homomorphic filtering employing two-dimensional Fourier transforms.

Different window sizes will greatly effect the quality of processed images. If the window is too small, the filtering effect is lowered. If the window is too large, subtle details of the image will be blurred. For small noise or images with plenty of details smaller window is suitable. For large noise larger window is suggested. A good choice of window size, from our experience, is 7×7 that provides sufficient statistical information and filtering effect.

5. Acknowledgment

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Appendix: derivation of equation (4)

From Figure 1, the density function $p(x)$ is

$$p(x)=\begin{cases} \dfrac{x-(\bar{x}-2.5\,\sigma_x)}{(2.5\,\sigma_x)^2}, & x\leqq\bar{x} \\[2ex] \dfrac{\bar{x}+2.50\,\sigma_x-x}{(2.5\,\sigma_x)^2}, & x\geqq\bar{x} \end{cases}$$

Let $D=\bar{x}-2.5\,\sigma_x$; $A=\bar{x}+2.5\,\sigma_x$

$$S=E[\log x]$$

$$=\int_D^{\bar{x}}\frac{X-D}{(2.5\,\sigma_x)^2}\log x\,dx+\int_{\bar{x}}^{A}\frac{A-x}{(2.5\,\sigma_x)^2}\log x\,dx$$

$$=\frac{1}{(2.5\,\sigma_x)^2}\left[\int_D^{\bar{x}}x\log x\,dx-D\int_D^{\bar{x}}\log x\,dx+A\int_{\bar{x}}^{A}\log x\,dx-\int_{\bar{x}}^{A}x\log x\,dx\right]$$

After some manipulation we have

$$S=\frac{1}{(2.5\,\sigma_x)^2}\left[\frac{D^2\log D}{2}+\frac{A^2\log A}{2}-\bar{x}^2\log\bar{x}-1.5(2.5\,\sigma_x)^2\right]=\frac{F}{2}-H-1.5$$

where F and H are defined by equations (4 *f*) and (4 *d*) respectively. Similarly,

$$T=E[\log^2 x]=\frac{1}{(2.5\,\sigma_x)^2}\left[\int_D^{\bar{x}}x\log^2 x\,dx-D\int_D^{\bar{x}}\log^2 x\,dx+A\int_{\bar{x}}^{A}\log^2 x\,dx-\int_{\bar{x}}^{A}x\log^2 x\,dx\right]$$

After some manipulation we have

$$T=\frac{G}{2}-\frac{3F}{2}+H(3-\log\bar{x})+3.5$$

where G is given by equation (4 *g*).